Discussion on Celestial Amplitudes @ Corfu

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Thanks to all the speakers for great talks
and suggesting many *open-ended* topics for discussion!

(no references, sorry)

Change of basis from definite-momentum to definite-boost wavefunctions

$$\mathrm{Poinc}_4 = T^4 \rtimes \mathrm{SL}(2,\mathbb{C})$$

$$|\omega, z\rangle_J = \int \mathrm{d}\omega\, \mathrm{d}^2 z'\, K_{\Delta,J}(\omega, z, z')\, |z'\rangle_{\Delta,J}, \qquad K_{\Delta,J}(\omega, z, z') = \begin{cases} \omega^{\Delta-1}\delta^2(z-z') \\ \dfrac{\omega^{\Delta-1}}{|z-z'|^{2(2-\Delta)}}\left(\dfrac{z-z'}{\bar{z}-\bar{z}'}\right)^J \end{cases}$$

Identify z's with the coordinate on a celestial sphere integrated over $\mathcal{I}^{\pm}$

(Repeat this for $\mathrm{SU}(2) \subset \mathrm{SL}(2,\mathbb{C})$ induced representations?)

Of course, that's not yet any more holographic
than the traditional formulation of the S matrix

The real claim to fame of celestial holography
is the existence of a well-defined *theory* at null infinity

What not just formulate a theory on $\mathcal{I}^{\pm}$?

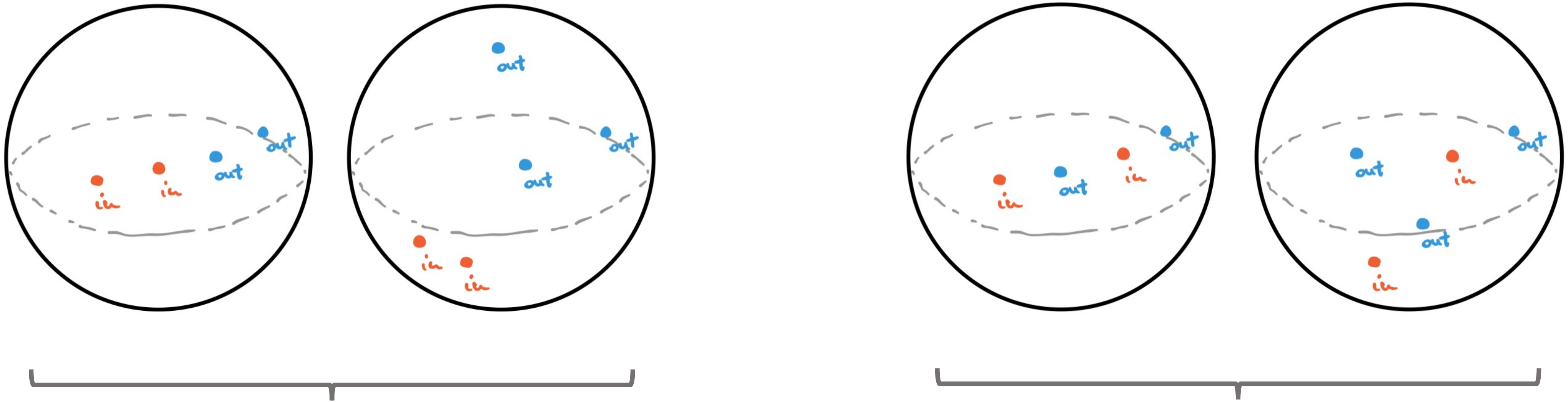
It’s common to say the theory will have to be “exotic”

(from CFT point of view, $\Delta \in 1 + i\mathbb{R}$)

But the real exoticness of the theory comes from somewhere else…

Real scattering angles

$\Rightarrow$ correlators supported only on patches of $\mathbb{CP}^1$



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(Glauberg gluon effect: should OPE between in and out states even exist?)

By now a classic example at tree level:

$$\delta^4(\textstyle\sum_i p_i)\left(\frac{1}{s-m^2}+\text{perm}\right) \quad \rightsquigarrow \quad \frac{\delta(i(z-\bar{z}))}{\sin(\pi\Delta/2)}\underline{\theta(z-1)}z^2 m^{\Delta-6}\left[e^{\pi i\Delta/2}+z^{\Delta/2-2}+\left(\frac{z}{z-1}\right)^{\Delta/2-2}\right]$$

Four ways out:

- Formulate a CFT with such selection rules?
- Analytic continuation to (covering space of) $\mathbb{CP}^1_z \times \mathbb{CP}^1_{\bar{z}}$?
- Casali-Puhm representation?
- Shadow correlators?

What we do with the shadows

$$\langle\tilde{\phi}\phi\phi\phi\rangle \sim \sum \cdots {}_2F_1(\cdots;z)\overline{{}_2F_1(\cdots;z)}$$

- What happens in string theory?
- Need for higher-derivative corrections, loop-level, general-multiplicity statements

How viable is non-perturbative celestial bootstrap?

We need:

- Translation of bulk constraints: causality, locality, unitarity, high-energy growth, …
- Boundary constraints: spectrum, OPE's, crossing, …

How redundant should we expect them to be?

If there was a bulk-boundary duality,
should we expect to be weak-weak or strong-week?

Lesson from the S-matrix theory:

- It's useful to compute in (2,2) signature, as long as it is continuously related to (1,3)

- Knowledge of analyticity properties didn't come out of vacuum: LSZ-based approaches, low-loop examples, …

Catch-22:

- On the one hand, we need examples to understand what constraints we should be imposing

- On the one hand, only UV soft theories are thought to have well-defined celestial amplitudes
(and there's only one example)

The role of (ambi)twistor string

- Worldsheet $\mathbb{CP}^1_\sigma$ branched cover of $\mathbb{CP}^1_z$ pinned at

$$\sigma_i = \{z_i, \bar{z}_i, \ldots\}$$

$$\underbrace{\sigma_i = \{z_i,}_{n=4}$$

$$\underbrace{\sigma_i = \{z_i, \bar{z}_i,}_{n=5}$$

$$\underbrace{\sigma_i = \{z_i, \bar{z}_i, \ldots\}}_{n \geqslant 6}$$

- Continue discussion on celestial OPE's and $w_{1+\infty}$ symmetries

Where to next?

Using worldsheet theory, can we make predictions that can't simply be deduced from symmetries or by Mellin transforming momentum-space results?

Low-energy EFT/swampland constraints:

Can't ignore graviton loops in a theory of quantum gravity!

- Already a problem in the plane-wave basis
- Solution so far: hard IR cut-off
- Avoid IR issues in D > 4?
- Maximal SUGRA (no R^2 and R^3 operators)?

Constraints at fixed-angle scattering have been notoriously difficult
to implement in plane-wave basis,
but in celestial amplitudes they are built-in

Possibly a real place to shine!

Thank you!