SOFT PIONS, CURRENT ALGEBRAS, AND SUM RULES

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- Preliminaries
- Soft Pions
- Current Algebras
- Sum Rules
- Further Soft Pion Applications
- Deep Inelastic Scattering Sum Rules
- Anomalies and PCAC
- Historical Significance
- Updates and Revivals

**Preliminaries**

**Noether's Theorem**

\[ L(x) = L \left[ \Phi_j, \lambda, \Phi_\lambda \right] \]

\[ \Phi_{j,\lambda} \rightarrow \Phi_j(x) + \lambda(x) \, G_j [\Phi_\lambda] \]

\[ S_L = \frac{\int_\Lambda L \, d\lambda}{S_\lambda} + \frac{\int_\Lambda \lambda \, d\lambda}{S_\lambda} \]

**Current**

\[ j^\alpha = -\frac{\delta L}{\delta (\partial_\alpha \lambda)} = -\frac{\delta L}{\delta (\partial_\alpha \Phi_\lambda)} \, G_j \]

**Obeys**

\[ \partial_\lambda j^\alpha = -\frac{\delta L}{\delta \lambda} \]

*If \( L \) is invariant for constant \( \lambda \), then \( \delta x / \delta \lambda = 0 \) and the current is conserved.*

*If \( \delta x / \delta \lambda \) is small, the current is partially conserved.*
APPLICATION TO THE DIRAC LAGRANGIAN

\[ \mathcal{L} = \overline{\psi} \gamma^\mu \partial_\mu \psi + i m \overline{\psi} \gamma^0 \psi \]

\[ \overline{\psi} = \psi^\dagger \gamma^0 \]

1. \[ \psi \rightarrow e^{i \theta \gamma^5} \psi \]

\[ \psi \text{ is invariant } \Rightarrow \text{ conserved vector current } \quad v^\mu = \overline{\psi} \gamma^\mu \psi \]

\[ \partial_\mu v^\mu = 0 \]

2. \[ \psi \rightarrow e^{i \theta \gamma_5} \psi \]

\[ [\gamma_5, \gamma_\mu \gamma^\mu] = 0, \quad [\gamma_5, \gamma^\nu] \neq 0 \]

\[ \psi \text{ is invariant only when } m = 0 \]

\[ \Rightarrow \text{ partially conserved axial-vector current } \quad a^\mu = \overline{\psi} \gamma^\mu \gamma_5 \psi \]

\[ \partial_\mu a^\mu = i m \overline{\psi} \gamma_5 \psi \]
Decay of the Pi Meson

M. L. Goldberger and S. R. Treiman
Princeton Physical Laboratory, Princeton University, Princeton, New Jersey
(Received February 10, 1958)

**Goldberger-Treiman Relation**

\[ f_\pi = \text{charged pion decay constant} \]
\[ g_A = \text{nucleon axial-vector coupling constant} \]
\[ g_\pi = \text{pion-nucleon coupling constant} \]
\[ M_N = \text{nucleon mass} \]

\[ f_\pi = \sqrt{2} \frac{M_N g_A}{g_\pi} \quad \text{(This is PDG} \ f_\pi = \text{Adler-Bashen} \ f_\pi/M_\pi^2) \]

Amazing - Good to around 6% (now 2.3%)
REINTERPRETATION AS (PARTIALLY CONSERVED) AXIAL-VECTOR CURRENT

\[ \phi_{\pi^+} = C \phi_{\pi^-} = C \phi_0 \]

\[ M_{\pi^0} \approx 2 M_{\pi^0} = C \frac{g_n}{g_\pi} \]

( NAMBU, BEANSTEIN, FUBINI, CELL-MANN + THIRRING; CELL-MANN + LÉVY; BERNSTEIN, CELL-MANN + MICHEL; CHOU; ALL 1960)

EARLY APPLICATIONS

\[ \chi = \text{NUCLEON + PION} \]

AXIAL CHARGE

\[ = -i \int J_\mu \, d^3x \]

\[ \langle \alpha^{(m)} | \chi | \alpha^{(m)} \rangle = \langle \alpha^{(m)} | \chi | \alpha^{(m)} \rangle \]

PHYSICAL REVIEW

VOLUME 133, NUMBER 4
FEBRUARY 15, 1962

Chirality Conservation and Soft Pion Production*

Y. NAMBU and D. LURIÉ
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, University of Chicago, Chicago, Illinois
(Received September 25, 1961)

PHYSICAL REVIEW

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24 AUGUST 1964

Tests of the Conserved Vector Current and Partially Conserved Axial-Vector Current Hypotheses in High-Energy Neutrino Reactions*

STEPHEN L. ADLER
Princeton University, Princeton, New Jersey
(Received 8 April 1964)
SOFT PION THEOREMS AS A PRECISION TOOL

Consistency Conditions on the Strong Interactions Implied by a Partially Conserved Axial-Vector Current

Stephen L. Adler
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey and Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 13 September 1964)

$$A^{\text{TNN}} = \text{SYMMETRIC ISOSPIN PION-NUCLEON SCATTERING AMPLITUDE}$$

$$\Lambda^{\text{TNN}}(V = V_A = 0) \approx \frac{g^2}{M_N} \quad \text{GOOD TO } \sim 10\%$$

$$A^{\text{TNN}}(S = t = u = -M_N^2, \not{p} = 0) = 0 \quad \text{"ADLER ZERO"}$$

3 PIONS IN-SHELL 4TH PION OFF-SHELL

Consistency Conditions on the Strong Interactions Implied by a Partially Conserved Axial-Vector Current. II

Stephen L. Adler
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 26 March 1965)

GENERAL EXTERNAL LINE INSERTION RULES

NUCLEON LINE: NONZERO INSERTION

PION LINE: ZERO INSERTION
THE SYMMETRY GROUP OF VECTOR AND AXIAL VECTOR CURRENTS*

MURRAY GELL-MANN
California Institute of Technology, Pasadena, California

(Received 25 May 1964)

\[ F_i(t) = -i \int \mathcal{F}_{14} \, d^3x, \]

\[ F_i^s(t) = -i \int \mathcal{F}_{14}^s \, d^3x, \]

\[ [F_i(t), F_j(t)] = i \epsilon_{ijk} F_k(t), \]

\[ [F_i^s(t), F_j^s(t)] = i \epsilon_{ijk} F_k^s(t). \]

(c) The commutation rules of the operators \( F_i^s(t) \) close the algebraic system by giving

\[ [F_i^s(t), F_j^s(t)] = i \epsilon_{ijk} F_k^s(t). \]

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RENORMALIZATION EFFECTS FOR PARTIALLY CONSERVED CURRENTS*

S. PUBINI
Istituto de Fisica dell'Università-Torino and CERN, Geneva

G. PURLAN**
CERN, Geneva

(Received 23 December 1964)
SUM RULES

CALCULATION OF THE AXIAL-VECTOR COUPLING CONSTANT RENORMALIZATION IN β DECAY

Stephen L. Adler

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 17 May 1965)

RENORMALIZATION OF THE WEAK AXIAL-VECTOR COUPLING CONSTANT

William I. Weisberger

Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 26 May 1965)

\[ g_A^2 = 1 + \frac{f^2}{\pi} \frac{2}{\pi} \int_{M^2_{\pi N} + M^2_{\pi \pi}}^{\infty} \frac{WdW}{W^2 - M^2_{\pi \pi}} \left[ \sigma_{T^+}^+(W) - \sigma_{T^+}^-(W) \right] \]

\text{given } g_A = 1.24 \quad \text{exp. } g_A = 1.272

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SUM RULES for the Axial-Vector Coupling-Constant Renormalization in γ Decay

Stephen L. Adler

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 7 June 1965)

- **πππ Sum Rule**
  \[ l = \frac{f^2}{\pi} \frac{1}{\pi} \int_{2M_{\pi \pi}}^{\infty} \frac{WdW}{W^2 - M^2_{\pi \pi}} \left[ \sigma_{T^+}^+(W) - \sigma_{T^+}^-(W) \right] \]

- **Sum Rule for forward lepton \( k^2 \approx 0 \) inclusive \( \pi N \) scattering**

Need large \( I=0 \)

5-wave \( \pi \pi \) cross section at low energy to saturate sum rule
Further soft pion applications (many!)

- Callan & Treiman \( K \to \pi \ell^+ \nu \) related to \( K \to \pi \nu \)
- Weinberg \( K \to 2\pi \ell^+ \nu \) importance of \( \pi \) pole terms
- Weinberg \( \pi \pi^* \) scattering length, multiple \( \pi \) production

→ These are reprinted in Adler & Dashen, "Current Algebras and Applications to Particle Physics," W. A. Benjamin (1969)

→ See also Coleman, "Aspects of Symmetry," Cambridge (1985), Chapter 2 on "Soft Pions"

- Chiral Lagrangians as generating functions for soft pion theorems were systematized by Weinberg; and by Callan, Coleman, Wess & Zumino

- Chiral \( SU(2) \times SU(3) \) as a strong interaction symmetry was developed by Dashen; Goldstone bosons
DEEP INELASTIC SCATTERING SUM RULES

Sum Rules Giving Tests of Local Current Commutation Relations in High-Energy Neutrino Reactions

Stephen L. Adler
CERN, Geneva, Switzerland and Lyman Laboratory, Harvard University, Cambridge, Massachusetts
(Received 6 October 1965)

\[ 2 = \int_0^\infty dv \left[ W_2^{\nu\nu}(q^2, \nu) - W_2^{\nu\nu}(q^2, \nu) \right] \]

\[ \frac{Q^2}{2} = k_v - k_\ell \]

\[ \nu = \nu_v - \nu_\ell \]

INEQUALITY FOR ELECTRON AND MUON SCATTERING FROM NUCLEONS

J. D. Bjorken
Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 7 February 1966)

\[ \lim_{E_\nu \rightarrow \infty} \frac{d (E_\nu + E_\nu)}{d \phi^2} \geq \frac{2\pi d^2}{q^2} \]

Asymptotic Sum Rules at Infinite Momentum

J. D. Bjorken
Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 30 September 1966)

\[ \text{Scaling: } u W_2(q^2, \nu) \Rightarrow F_2(x) \quad x = \frac{q^2}{2M_\nu \nu} \]
ANOMALIES AND PCAC

Axial-Vector Vertex in Spinor Electrodynamics

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(Received 24 September 1968)

IL NUOVO CIMENTO
Vol. LXI, N. 1
16 Marzo 1960

A PCAC Puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the $\sigma$-Model.

J. S. BELL
CERN - Geneva
R. JACKIW (*)
CERN - Geneva

Jefferson Laboratory of Physics, Harvard University - Cambridge, Mass.

(ricevuto 11 Settembre 1968)

$$\gamma^\mu F_{3\mu} = \frac{2\pi}{\sqrt{2}} \phi_{\pi^0} + \frac{\sqrt{5}}{\sqrt{10}} F_{\gamma\gamma} F_{\gamma\gamma} \varepsilon_{\gamma\gamma} \varepsilon_{\gamma\gamma} \quad S = \sum_q g_q Q_q^2$$

$\pi^0 \gamma\gamma$ matrix element for charge $2/3$, $-1/3$ quarks. Factor of 3 too small.

Absence of Higher-Order Corrections in the Anomalous Axial-Vector Divergence Equation

STEPHENV L. ADLER and WILLIAM A. BARDEEN
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 24 February 1969)

$S$ has no higher loop corrections - does count quarks.
HISTORICAL SIGNIFICANCE

- QUANTUM FIELD THEORY, NOT RECIPROCAL BOOTSTRAP!

- PCAL ➔ APPROXIMATE CHIRAL INVARIANCE OF STRONG INTERACTIONS

- CURRENT ALGEBRA ➔ NON-ABELIAN (YANG-MILLS) GAUGE STRUCTURE OF ELECTROWEAK SECTOR

- DEEP INELASTIC SUM RULES, SCALING ➔ REALITY OF QUARKS

- ANOMALY CALCULATION OF π^0 ➔ 2γ ➔ COLOR TRIPLET OF QUARKS

FOR AN EXCELLENT BOOK-LENGTH DISCUSSION, SEE:

TIAN YU CAO "FROM CURRENT ALGEBRA TO QUANTUM CHROMODYNAMICS: A CASE FOR STRUCTURAL REALISM"
Cambridge University Press (2010)
Evaluation of the axial-vector commutator sum rule for pion-pion scattering

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F.J. Ynduráin
Departamento de Física Teórica, C-XI, Universidad Autónoma de Madrid, Canto Blanco, E-28049, Madrid, Spain
(Received 10 April 2007; published 4 June 2007)

Updated evaluation of A-W and TH sum rules and GT relation satisfied to better than 6%.

Adler sum rule - Scholarpedia

Updated derivation using the full 3 family CKM matrix for weak mixing


The Axion - Very light pseudo-Nambu-Goldstone boson
Neutral pseudoscalar pion analog
Arises (Weinberg; Wilczek) from spontaneous breaking of Peccei-Quinn symmetry introduced to solve strong CP problem
Possible light dark matter candidate