IMPLICATIONS OF A FRAME DEPENDENT DARK ENERGY FOR THE SPACETIME METRIC AND "HUBBLE TENSION"

Stephen L. Adler
Institute for Advanced Study adler@ias.edu
arXiv: 1905.08228

OUTLINE

- STANDARD VS Weyl INVARIANT DARK ENERGY
- MOTIVATIONS
- NEW INSIGHTS ON OLD PROBLEMS
- IMPLICATIONS FOR COSMOLOGY - GENERAL
- NOTATIONS
- METRIC PERTURBATION ANALYSIS
- CONNECTION TO $H_0$, RESCALINGS
- RESULTS FOR LATE TIME COSMOLOGY
- SUGGESTIONS FOR FURTHER WORK
**STANDARD VERSUS WEYL SCALING INVARIANT DARK ENERGY ACTION**

**STANDARD COSMOLOGICAL ACTION**

\[
S_{\text{cosm}} = -\frac{\Lambda}{8\pi G} \int d^4x \ (\mathbf{g})^{1/2}
\]

\[
\Lambda = 3H_0^2
\]

- FOUR-SPACE GENERAL COORDINATE INVARIANT
- "VACUUM ENERGY" - ALL FIELD CONTRIBUTE

**ALTERNATIVE DARK ENERGY ACTION**

\[
S_{\text{eff}} = -\frac{\Lambda}{8\pi G} \int d^4x \ (\mathbf{g})^{1/2} \left( g_{00} \right)^{-2}
\]

- IN FRW WITH \( \ell_0 = 1 \), MINKSI
- \( S_{\text{cosm}} \)
- THREE-SPACE GENERAL COORDINATE INVARIANT
- WEYL SCALING INVARIANT
- \( g_{00} \rightarrow \lambda \ g_{00} \)
- FRAME DEPENDENT

FRAME DEPENDENT PHYSICS
IC ALLOWED - CMB DIPOL
PICKS A PREFERRED FRAME

\[ \rightarrow \]
PHENOMONOLOGY TO DISTINGUISH BETWEEN TNEM

\[ \mathcal{S}_f = \left( 1 - f \right) \mathcal{S}_{\text{com}} + f \mathcal{S}_{\text{eff}} = -\frac{\Lambda}{8\pi G} \int d^3x \left( \phi^2 \right)^{1/4} \left[ 1 - f + f \left( \phi_0 \right)^2 \right] \]

\[ f = 0 \quad \text{only} \quad \mathcal{S}_{\text{com}} \]

\[ f = 1 \quad \text{only} \quad \mathcal{S}_{\text{eff}} \]

For FRW, \( \phi_0 = 1 \) if drops out

\( f \) only enters at level of metric perturbations
MOTIVATIONS

- HISTORICAL -

Weyl proposed invariance \( \mathcal{J}_W(x) \rightarrow \lambda(x) \mathcal{J}_W(x) \)

\( \lambda(x) = \) conformal factor

Called this a "gauge transformation", scale
a local property of metric

- RECENT FOLLOW-UPS -

hep-th/0307199 Forger & Römer - detailed field theory analysis

1410.6675 't Hooft - gravitational essay advocating Weyl
invariance as "the missing symmetry component for space and time"
1306.0492  S.L.N. - "INTEGRATING GRAVITY INTO TRAC0 DYNAMICS: THE INDUCED GRAVITATIONAL ACTION"

**Basic Idea:** Quantum Field Theory as Thermodynamics

**Canonical Ensemble Average of Underlying Dynamics**

\[ \rho \propto e^{-\frac{\mathcal{H}}{T_0}} = e^{-\frac{1}{T_0} \int_0^\infty d\mathcal{L} \mathcal{L}(\mathcal{A})} \]

- Hamiltonian \( \mathcal{H} \) = Frame Dependence
- Three-Space General Coordinate Invariant
- For Massless Underlying Fields, \( T_0 \) Global Weyl Invariant
- Induced Gravitational Action Inherits These Properties to Zeroth Order in Derivatives

\[ S[g] \propto \int d^3 x (g_{ij})^{\frac{1}{2}} (\delta_{00})^2 \]

This is also local Weyl Invariant
NEW INSIGHTS ON OLD PROBLEMS

THE COSMOLOGICAL CONSTANT PROBLEM \( k = c = 1 \)

\( S_{\text{cosm}} \) INTERPRETS \( \Lambda \) AS VACUUM ENERGY DENSITY

\[
\rho_{\text{vac}} = \frac{\Lambda}{8\pi G} = (2.2 \times 10^{-3} \text{ eV})^4
\]

PLANCHE SCALE ZERO POINT ENERGIES CORRESPOND TO

\[
\rho_{\text{vac}} \sim m_{\text{pl}}^4 = (1.2 \times 10^{-20} \text{ eV})^4
\]

So \( \frac{\rho_{\Lambda}}{\rho_{\text{vac}}} \sim 1.1 \times 10^{-12.3} \) TREMENDOUS FINE TUNING FOR PARTICLE VACUUM ENERGIES TO CANCEL DOWN TO \( \rho_{\text{vac}} \)

SUPPOSE UNDERLYING PHYSICS REQUIRES \( S[9] \) TO BE WEYL SCALING INVARIANT \( S_{\text{cosm}} \propto \sqrt{d^4(x)^h} \) IS NOT AN EXACT SUM RULE REQUIRING VACUUM ENERGIES TO SUM TO 0

\[
\sqrt{d^4(x)^h} \propto g_s^{-2}
\]

IS ALLOWED, BUT IS NOT VACUUM ENERGY \( \Lambda \) STILL SMALL, BUT NO LARGE DENOMINATOR \( M_{\text{pl}} \) TO COMPARE IT TO
THE BLACK HOLE INFORMATION "PARADOX"

WHAT HAPPENS TO INFORMATION THAT FALLS THROUGH BLACK HOLE HORIZON WHEN BLACK HOLE EVAPORATES VIA HAWKING RADIATION?

\[ \text{S} + \text{E} \implies \text{SCHWARZSCHILD DE SITTER BLACK HOLE STILL NOT NEAR EVENT HORIZON} \]

WHAT ABOUT \( S + S \) ?

1308.1448 S. L. R. & FETTI RAMAZANÖGLU

ANALYTIC AND NUMERICAL STUDY OF SPHERICALLY SYMMETRICAL VACUUM SOLUTIONS: NO HORIZON \( S_{00} \rightarrow 0 \) FOR \( \nu > 0 \)

IN ISOTROPIC COORDINATES, SOLUTION IS SMOOTH TO \( \nu = 0 \) SINGULARITY OUTSIDE \( \sqrt{\nu \text{SCHWARZSCHILD} + 10^{-17} \left( \frac{M_{\text{HOLE}}}{M_{\odot}} \right)^2} \) UNI... UP TO COSMOLOGICAL DISTANCES, SOLUTION CLOSE TO SCHWARZSCHILD SOLUTION, SO EXACT ASTRONOMICAL UNALTERED. WHAT ABOUT INFORMATION PARADOX?
Homogeneous, isotropic, zero spatial curvature line element with physics invariant under three space general coordinate transformation

\[ ds^2 = \alpha^2 (t) dt^2 - \eta^2 (k) dx^2 = g_{00} \, dt^2 + g_{ij} \, dx^i \, dx^j \]
\[ g_{00} = \alpha^2 (t) \quad g_{ij} = - \frac{1}{\kappa} \eta^2 (k) \]

The case cannot be reduced to \( t = 0 \) by redefining the time variable

Define proper time \( \tau \) by \( d\tau = \alpha (t) \, dt \)

\[ d\tau^2 = dt^2 - \eta^2 (k) \, dx^2 \]

where \( \eta (x (\tau)) = \alpha (\tau) \eta (k) \)

For \( \alpha (\tau) , \eta (k) \) both positive, \( \left( \frac{\eta}{k} \right)^\gamma = \alpha (\tau) \eta^2 (k) \)

\[ S \alpha = - \frac{\Lambda}{8 \pi G} \int d^4 x \, \alpha (k) \eta^2 (k) \left[ 1 - \chi + \chi (h^2)^\kappa \right] = - \frac{\Lambda}{8 \pi G} \int d^4 x \, \eta^2 (k) \left[ 1 - \chi + \chi (h^2)^\kappa \right] \]
• Works out the \( f \) contribution to metric perturbations around uniform \( f \) background with \( f \neq 0 \) still no scalar propagating gravitational waves.

• Frame dependence of \( f \) term \& \( \frac{SSA}{SSg} \) gives \( T^i_j \)

To get full \( T^\mu_\nu \) impose covariant conservation

\[ D^\mu T^\mu_\nu = 0 \]

• Black hole with spherical symmetry: algebraic condition
• FW perturbations: ordinary differential equations
• General metric: partial differential equations
NOTATIONS

\(\theta(t), \bar{\theta}(t), \tilde{\theta}(t)\)

\(\alpha(t) = 1 + \bar{\theta}(t)\)

\(\Lambda(t) = \frac{\alpha(t)}{\theta(t)} / \theta(0)\)

\(\Theta(t) = 1 - \bar{\theta}(t)\)

\(\alpha_0 = \text{STANDARD FRW EXPANSION FACTOR}\)

FOR MATTER-DOMINATED ERA

\(\alpha(t) = \left(\frac{\rho_m}{\rho_m}\right)^{1/3} \left(\sinh(x)\right)^{2/3}\)

\(\Lambda = 1 - \Lambda\)

\(x = \frac{3}{2} \sqrt{\frac{\rho_m}{\Lambda}} \text{ H}_0 t \quad \text{DIMENSIONLESS TIME VARIABLE}\)

PRESENT ERA  \(\Lambda(t_0) = 1 \Rightarrow x_0 = \frac{\rho_0}{\Lambda} \text{ sinh} \left(\sqrt{\frac{\rho_m}{\Lambda}}\right) = 0.169\)

\(\Rightarrow \text{H}_0 x_0 = \frac{2}{3} x_0 \sqrt{\frac{\rho_m}{\Lambda}} = 0.946\)

\(\Rightarrow \Lambda = 0.679\)

\(\rho_m = 0.321\)

WILL SEE LATER THAT \(\text{H}_0 = \alpha(0) \text{H}_0\)
HUBBLE PARAMETER $H(a)$ DEFINED BY STANDARD EW FLOW IS

\[ H(a) = \frac{\dot{a}(a)}{a(a)} = \frac{\dot{\phi}(x)}{\phi(x)} \]

\[ = H_0 \sqrt{\Omega_{\Lambda}} \cos H^{-1} \phi = H_0 \left[ \Omega_m (1 + \tilde{z})^3 + \Omega_{\Lambda} \right]^{1/2} \quad 1 + \tilde{z} = \frac{1}{a(a)} \]

HUBBLE PARAMETER ARISING FROM $\phi$-DYNAMICS IS

\[ H_{\phi\phi}[\sqrt{x}] = \frac{d \phi[\sqrt{x}]}{d \sqrt{x}} = H_{\phi\phi}(x) = \frac{d \phi(x)}{\phi(x)} = \frac{d \psi(t)}{a(t) \psi(t)} \]
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MÉTRIC PERTURBATION ANALYSIS

- Matter dominated era: $\rho = c\rho = 0 \quad \dot{\rho} \neq 0$

Take linear combination of perturbation equations that eliminates $\dot{\sigma}$ to get

$$\ddot{\sigma} + 4 \frac{\dot{a}}{a} \dot{\sigma} + \left(2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) \sigma = 2A \gamma \ddot{\sigma} \quad (f=0 \text{ case in Mukhanov text})$$

- Steps to analyze this equation

  1. Change to dimensionless variable $x$ to get

$$\frac{d^2 \sigma}{dx^2} + \frac{2}{3} c^2 A(x) \frac{d \sigma}{dx} = \frac{4}{3} (2f-1) \sigma$$

  2. Large $x$ behavior: $c^2 A(x) \to 1$

Get equation with constant coefficients, solved by

$$\sigma(x) = C_1 e^{\mu_+ x} + C_2 e^{\mu_- x}$$

$$\mu_{\pm} = -\frac{2}{3} \left[2 \pm (6f+1)^{1/2}\right]$$
CROSSOVER IN BEHAVIOR AT $f = \frac{1}{2}$

$f < \frac{1}{2}$ BOTH EXPONENTS NEGATIVE

$f > \frac{1}{2}$ ONE EXPONENT POSITIVE, $\Phi \to \infty$ AS $x \to \infty$

$\Rightarrow$ FOR $f = 1$, THE CASE OF SCALE INVARIANT DARK ENERGY ACTION, METCAL PERTURBATION GROW WITH TIME

(3) SMALL $x$ BEHAVIOR

AS $x \to 0$, LEFT HAND SIDE DOMINATES, SO EQUATION IS APPROXIMATED BY

$$\frac{d^2 \Phi}{dx^2} + \frac{2}{3} \frac{1}{x} \frac{d \Phi}{dx} = 0$$

SOLVED BY

$$\Phi(x) = C_3 + C_4 x^{-5/3}$$

SO GET A UNIQUE SOLUTION BY REQUIRING REGULARITY AT $x = 0$, AND GIVING $\Phi(0)$, THE SOLE PARAMETER OF THE MODEL
NUMERICAL SOLUTION

INTEGRAL EQUATION

Define normalized perturbation \( \hat{\varphi}(x) = \frac{\varphi(x)}{\varphi(0)} \) which obeys

\[
\frac{\hat{\varphi}}{3} x = 1 + \frac{2}{3} \left( 2x - 1 \right) \int_0^x \frac{dw}{\sinh(w)} \left[ -\frac{1}{13} \int_0^w \frac{du}{\sinh(u)} \right] \hat{\varphi}(u)
\]

Iterate starting from initial assumption \( \hat{\varphi}(x) = 1 \).

FORWARD STEPPING

Equation is invariant under \( x \to -x \), so regular solution will have \( \left. \frac{\partial \varphi}{\partial x} \right|_{x=0} = 0 \) \( \Rightarrow \) starting from given \( \varphi(0) \) and \( \varphi'(0) = 0 \).

Write equation as

\[ \varphi'' = G(\varphi', \varphi) = -\frac{2}{3} \coth(x) \frac{\partial^2 \varphi}{\partial x^2} + \frac{4}{3} (2x - 1) \frac{\partial \varphi}{\partial x} \]

And stepwise iterate

\[
\varphi'(x + h) = \varphi'(x) + G(\varphi'(x), \varphi(x)) h
\]

\[
\varphi(x + h) = \varphi(x) + \varphi'(x) h
\]

Solution even in \( x \gg \) bubble deviations will be quadratic in \( x/x_0 \).
**Connection to \( N^\nu \), Rescalings**

*Numerical Solution* for \( \hat{\nu}(x) \) is well approximated by

\[
\hat{\nu}(x) = 1 + C \left( \frac{X}{X_0} \right)^2 \quad C = 0.244
\]

Since \( \frac{X_{\text{decoupling}}}{X_0} \approx 3 \times 10^{-4} \), at and before decoupling,

\[
\nu = \nu(0) \hat{\nu} = \nu(0) \quad \text{to high accuracy}
\]

\( \nu(0) \) is the one parameter of the model

*Rescalings to connect to Planck, WMAP fits*

- Recall \( ds^2 = \alpha^2(t) dt^2 - \nu^2(t) dx^2 \)
- At and before decoupling
  \[
  \alpha(t) = 1 + \nu(0) \approx \alpha(0) = 1 + \nu(0)
  \]
- Proper time \( \tau = \alpha(0) t \) just a constant rescaling of \( t \)
- \( \alpha(t) = \alpha(0) \) to first order \( \approx \theta(t) = 1 - \nu(t) \approx 1 - \nu(0) \approx \theta(0) \)
- \( \nu(t) = \alpha(t) \theta(t) \theta(0) \approx \alpha(t) = \alpha(t) \frac{\nu}{\alpha(0)} \)
- LINE ELEMENT FOR PLANCK ANALYSIS (ZEROTH ORDER) IS
  \[ ds^2 = d\tau^2 - \Psi[\chi] d\chi^2 = d\tau^2 - \chi^2 (\chi/\chi_0) d\chi^2 \]

- SO RELATION BETWEEN \( H_0^{PL} \) AND \( H_0 \) IS JUST A RESCALING BY \( \Psi(\chi) \)
  \[ H_0^{PL} = H_0 / \Psi(\chi) \quad \text{THAT IS} \quad H_0^{PL} \chi = H_0 \chi_0 \]

- SIMILARLY, \( \chi_0^{PL} \), AGE OF UNIVERSE MEASURED BY PLANCK
  IS RELATED TO \( \chi_0 \) BY \( \chi_0^{PL} = \kappa_0 \chi(\chi) \)
  \[ H_0^{PL} \chi_0 = H_0 \chi_0 \]

- EFFECT OF RESCALINGS IS TO MAKE PHYSICAL PROCESSES
  DEPEND ON \( \hat{\chi}(\tau) = \chi(\tau/\chi_0) \)
  \[ C \chi(\chi/\chi_0) \]
  SO IF \( C \) WERE ZERO THERE WOULD BE NO PHYSICAL EFFECTS

  THIS IS ESSENTIAL BECAUSE WITH \( C=0 \) THE RELATION BETWEEN
  COORDINATE TIME \( \tau \) AND PROPER TIME \( \chi \) IS A CONSTANT
  RESCALING, AND A CHANGE IN UNITS IN WHICH TIME IS MEASURED
  SHOULD HAVE NO EFFECT ON PHYSICAL CONSEQUENCES OF THE EQUATIONS.
RESULTS FOR LATE TIME COSMOLOGY

EFFECTIVE HUBBLE CONSTANT

\[ H_{\text{eff}}(z) = \frac{d \Lambda(z)/dt}{\Lambda(z) \Omega(t)} = \frac{d(\theta(t) \Lambda(z))}{dt} = \frac{l}{\Omega(t)} \left[ \frac{\Lambda(z)}{\Lambda(1)} + \frac{\theta(t)}{\theta(1)} \right] \]

\[ = \frac{1}{\Omega(t)} \Lambda(t) - \frac{\theta(t)}{\theta(1)} \quad \text{using} \quad \Lambda(t) \theta(1) = 1 \]

\[ \Omega(z) = \Omega(0) \left[ 1 + \frac{\Omega(z)}{\Omega(0)} \right] \Rightarrow \Omega(z) = \Omega(0) \left[ 1 + C \right] \]

\[ \frac{\Omega(z)}{\Omega(0)} = 2 \Omega(z) C (1 + C) \]

\[ H(z) = H_0 = \frac{\Lambda(0) N_{oL}}{\Omega(z)} \quad \frac{1}{\Omega(z)} = 1 - \Omega(z) = 1 - \Omega(0) \left( 1 + C \right) \]

\[ H_{\text{eff}}(z) = \left[ 1 - \Omega(0) - \Omega(z) C \right] N_{oL} - 2 \Omega(z) C (1 + C) \]

\[ = H_{oL} - \Omega(z) C \left[ H_0 + 2 / z_0 \right] \quad \text{using} \quad (1 - \Omega(z)) N_{oL} = H_{oL} \]

\[ H_{\text{eff}}(z_0) = 1 - \Omega(0) C \left[ 1 + 2 / z_0 \right] \approx 1 - \Omega(0) C \left[ 1 + 3 \sqrt{\Omega(z) / z_0} \right] \approx 1 - \Omega(0) 0.76 \]
RIESS: \[ \frac{H_{\text{local}}}{H_0} = 1.100 \pm 0.023 \] \text{Fit by } \Phi(0) = -0.133 \pm 0.031

- **Reduction in Age of Universe**

- **Age of Universe Fixed**, in our model, by \( \Phi(0)/\theta = 1 \)

  (Analog of FRW \( \theta(\theta_0) = 1 \))

  \[ \tau_{\theta} - \tau_{\theta,\text{pl}} = \frac{\Phi(0) \tau}{H_0^{\text{pl}}} \left( 1 + \frac{2 \tau_{\theta_0}}{9 H_0^{\text{pl}}} \right) \approx 1.315 \frac{\Phi(0) \tau}{H_0^{\text{pl}}} \]

  \[ \tau_{\theta} = 13.83 \text{ Gyr}, \quad \frac{1}{H_0^{\text{pl}}} = \frac{\tau_{\theta}}{9 H_0^{\text{pl}}} = 14.62 \text{ Gyr}, \quad \Phi(0) = -0.133 \pm 0.031 \]

  \( \Rightarrow \tau_{\theta} - \tau_{\theta,\text{pl}} = -0.62 \pm 0.14 \text{ Gyr} \)

- **Model is Larger Than BAO Hubble Parameter at } \geq 0.51 \text{ by } 1 \text{ to } 1.4 \text{ standard deviations}
Suggestions for Further Work

- $\frac{\Delta}{T}(x)$ drops from 1.244 at $z = 0$ to 1.122 at $z \geq 0.35$
  - Divide no measurements into:
    - Small $z$: $z < 0.75$
    - Large $z$: $z \geq 0.75$
  - Get mean and standard deviation for each group.
  - What is statistical significance of No small - No large?

- Existence proof of conserving completion of $T_{ij}$ for general metal?

- Black hole - now is "information paradox" modified?

- Repeat BAO, Cambridge Group (Lemos et al.) analysis with quadratic parameterization suggested by my model.
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- Long lived stars - How does "Methuselah star" analysis change?

- Lensing discrepancy in CMB - Does faster late time expansion have any effect?

- Other astrophysics that could be altered by accelerated late time expansion?