Impact of memory on human dynamics

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Abstract

Our experience of web access slowing down is a consequence of the aggregated web access pattern of web users. This is just one example among several human-oriented services which are strongly affected by human activity patterns. Recent empirical evidence is indicating that human activity patterns are characterized by power-law distributions of inter-event times, where large fluctuations rather than regularity is the common case. I show that this temporal heterogeneity can be explained by two mechanisms: (i) humans have some perception of their past activity rate and (ii) based on that they react by accelerating or reducing their activity rate. Using these two mechanisms I explain the inter-event time statistics of Darwin’s and Einstein’s correspondence and the email activity within an university environment. Moreover, they are typical examples of the accelerating and reducing class, respectively. These results are relevant to the system design of human-oriented services.

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Human activity patterns are inherently stochastic at the single individual level. Understanding this dynamics is crucial to design efficient systems dealing with the aggregated activity of several humans. A typical example is a call center design, where we save resources by taking into account that all workers will no call or receive calls at the same time \cite{1,2}. There are several other examples including the design of communication networks in general, web servers, road systems and strategies to halt epidemic outbreaks \cite{3,4}.

The stochasticity present in the human dynamics has been generally modeled by a Poisson process characterized by a constant rate of activity execution \cite{1–3}. Generalizations to non-stationary Poisson processes have also been considered taking into account the effects of seasonality \cite{5}. Yet, these approaches fail when confronted with recent empirical data for the inter-event time statistics of different human activities \cite{6–12}. I show that the missing mechanism is a key human attribute, memory.

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1. The model

Consider an individual and a specific activity in which he/she is frequently involved, such as sending emails. The chance that the individual execute that activity (event) at a given time depends on the previous activity history. More precisely, (i) humans have a perception of their past activity rate and (ii) based on that they react by accelerating or reducing their activity rate. Although it is obvious that we remember what we have done it is more difficult to quantify this perception. In a first approximation I assume that the perception of our past activity is given by the mean activity rate. I also assume that based on this perception we then decide to accelerate or reduce our activity rate. In mathematical terms this means that if \( \dot{\lambda}(t) \, dt \) is the probability that the individual performs the activity between time \( t \) and \( t + dt \) then

\[
\dot{\lambda}(t) = a \frac{1}{t} \int_0^t \, d\tau \dot{\lambda}(\tau),
\]

(1)

where the parameter \( a > 0 \) controls the degree and type of reaction to the past perception. When \( a = 1 \) we obtain \( \dot{\lambda}(t) = \dot{\lambda}(0) \) and the process is stationary. On the other hand, when \( a \neq 1 \) the process is non-stationary with acceleration (\( a > 1 \)) or reduction (\( a < 1 \)).

Implicitly in (1) is the assumption of an starting time (\( t = 0 \)). For the case of daily activities this can be taken as the time we wake up or we arrive to work. More generally, it is a reflection of our bounded memory, meaning that we do not remember or do not consider relevant what took place before that time. For instance, we usually check for new emails every day after arriving at work no matter what we did the day before.

Eq. (1) can be solved for any \( a \) resulting in

\[
\dot{\lambda}(t) = \lambda_0 a \left( \frac{t}{T} \right)^{a-1},
\]

(2)

where \( \lambda_0 \) is the mean number of events in the time period under consideration \( T \). Due to the stochastic nature of this process the inter-event time \( X \) between the two consecutive task executions is a random variable. We denote by \( F(\tau) = \text{Prob}(X < \tau) \) and \( f(\tau) = \dot{F}(\tau) \) the inter-event distribution and probability density function, respectively. Within short time intervals \( \lambda(t) \) is approximately constant and the dynamics follows a Poisson process characterized by an exponential distribution \( F(\tau, \lambda(t)) = 1 - e^{-\lambda(t)\tau} \). Furthermore, the mean fraction of events taking place within this short time interval is \( \dot{\lambda}(t) \, dt / \lambda_0 T \). Integrating over the whole time period we finally obtain

\[
F(\tau) = \int_0^T \frac{\dot{\lambda}(t)}{\lambda_0 \tau} \, dt \left( 1 - e^{-\lambda(t)\tau} \right).
\]

(3)

For the stationary process (\( a = 1 \)), we recover the exponential distribution \( F(\tau) = 1 - e^{-\lambda_0 \tau} \) characteristic of a Poisson process. More generally, substituting (2) into (3) we obtain

\[
F(\tau) = \begin{cases} 
1 - \exp \left( -\frac{\tau}{\tau_0} \right) + \left( \frac{\tau}{\tau_0} \right)^{a/(1-a)} \Gamma \left( \frac{1-2a}{1-a}, \frac{\tau}{\tau_0} \right), & a < 1, \\
1 - \exp \left( -\frac{\tau}{\tau_0} \right), & a = 1, \\
1 - \exp \left( -\frac{\tau}{\tau_0} \right) + \left( \frac{\tau}{\tau_0} \right)^{-a/(1-a)} \left[ \Gamma \left( \frac{2a-1}{a-1}, \frac{\tau}{\tau_0} \right) - \Gamma \left( \frac{2a-1}{a-1}, \frac{\tau_0}{\tau_0} \right) \right], & a > 1,
\end{cases}
\]

(4)

where \( 0 \leq \tau \leq T, \Gamma(\beta, y) = \int_y^\infty dx \, x^{\beta-1} e^{-x} \) is the incomplete gamma function and

\[
\tau_0 = \frac{1}{a \lambda_0}.
\]

(5)

\( a > 1 \): In the acceleration regime the probability density function exhibits the power law

\[
f(\tau) = \frac{1}{\tau_0} \frac{a}{a-1} \Gamma \left( \frac{2a-1}{a-1}, \frac{\tau}{\tau_0} \right)^{-2},
\]

(6)
for $\tau_0 \ll \tau < T$, where

$$\alpha = 2 + \frac{1}{a - 1}. \quad (7)$$

This approximation is valid provided that $\tau_0 \ll T$, i.e., when a large number of events is registered in the period $T$.

$\frac{1}{2} < a < 1$: In this case $f(\tau)$ does not exhibit any power-law behavior.

$0 < a < \frac{1}{2}$: In the reduction regime the probability density function exhibits a power law as well

$$f(\tau) = \frac{1}{\tau_0} \frac{a}{1 - a} \left( \frac{1 - 2a}{1 - a} \right) \left( \frac{\tau}{\tau_0} \right)^{-\alpha}, \quad (8)$$

but in the range $\tau \ll \tau_0$ and with exponent

$$\alpha = 1 - \frac{a}{1 - a}. \quad (9)$$

This approximation is particularly good for $\tau_0 \gg T$, i.e., when a small number of events is registered in the period $T$.

2. Comparison with empirical data

To check the validity of our predictions we analyze the regular mail correspondence of Darwin and Einstein [7] and an email data set containing the email exchange among 3188 users in an university environment for a period of three months [13].

**Regular mail:** In Fig. 1a we plot the cumulative number letters sent by Darwin and Einstein as a function of time, measured from the moment the first letter was recorded. In both cases we observe a growth tendency faster than linear, which is well approximated by the power-law growth $N(t) \sim t^{\alpha}$. Since $N(t) = \int_0^t dt' \lambda(t')$ this observation corresponds with a letter sending rate (2) with $a = 3.7$. Furthermore, both Darwin and Einstein sent more than 6000 letters during the time period considered by this data set. In this case $(a > 1, \tau_0 \ll T)$, we predict that the inter-event time distribution follows the power-law behavior (6) with $\alpha \approx 2.4 \pm 0.1$ (7). This prediction is confronted in Fig. 1b with the empirical data resulting in a very good agreement.

**Email:** Determining the time dependency of $\lambda(t)$ is more challenging for the email data. If we restrict our analysis to single users there are only 21 users that sent more than 500 emails. Among them a few sent more than 1000 emails but it is questionable how well they represent the average email user. Therefore, for about 99% of the users we do not count with sufficient data to make conclusions about their individual behavior, being forced to analyze their aggregated data. Furthermore, email activity patterns are strongly affected by the circadian rhythm ($T = 1$ day) and therefore, we can also aggregate data obtained for different days. In Fig. 2a we plot the email sending rate averaged over different days and over all users in the data set as a function of time. The characteristic features of this plot are: an abrupt increase following the start of the working hours, two maximums corresponding with the morning and afternoon activity peaks and a final decay associated with the end of the working hours.

It is important to note that large inter-event times are associated with low values of $\lambda$. Therefore, the decrease in the email sending rate after the working hours determines the tail of the inter-event time distribution. Based on this we predict that the email activity belongs to the rate reduction class ($a < 1$). Furthermore, in average each user sends an email every 2 days. In this case $(a < 1, \tau_0 < T)$, we predict that the inter-event time distribution should exhibit a power-law behavior (8) with $0 < \alpha < 1$ (9). This prediction is confirmed by the empirical data for the inter-event time distribution resulting in $\alpha = 0.9 \pm 0.1$ (see Fig. 2b and Refs. [6,9]).

3. Discussion and conclusions

The inter-event times statistics should not be confused with that of response or waiting times studied in Refs. [6,7]. For instance, in the context of email activity the response time is the time interval between the
arrival of an email to our Inbox and the time we answer that particular email. On the other hand, the inter-event time is the time interval between consecutive emails independent of the recipient. For practical applications such as the design of call centers, web servers, road systems and strategies to halt epidemic outbreaks the relevant magnitude is the inter-event time.

I have shown that acceleration/reduction tendencies together with some perception of our past activity rate (1) are sufficient elements to explain the power-law inter-event time distributions observed in two empirical data sets. Regarding the regular mail correspondence of Darwin and Einstein the acceleration is probably due to the increase of their popularity over time. In the case of the email data the rate reduction could have different origins. We could stop checking emails because we should do something else or because after we check for new emails the likelihood that we do it again decreases. The second alternative has a psychological origin, associated with our expectation that new emails will not arrive shortly. In practice, the reduction rate of sending emails may be a combination of these two factors.

In a more general perspective, this work indicates that a minimal model to characterize human activity patterns is given by two factors: (i) humans have a perception of their past activity rate and (ii) based on that

Fig. 1. Regular mail activity: Statistical properties of the Darwin’s and Einstein’s correspondence. (a) Cumulative number of letters sent by Darwin (open circles) and Einstein (solid squares). The solid line corresponds with a power-law growth \( N(t) \sim t^a \) with \( a = 2.7 \). (b) The inter-event time distribution associated with the data sets shown in (a). The solid line represents the power-law decay \( f(\tau) \sim \tau^{-\alpha} \), where the exponent \( \alpha \) was obtained using (7) and the value of \( a \) obtained from (a).
they react by accelerating or reducing their activity rate. From the mathematical point of view memory implies that the progression of the activity rate is described by integral equations. This is the key element leading to the power-law behavior. These results are relevant to other human activities where power-law inter-event time distributions have been observed [8,10–12]. Before making any general statement, further research is required to test the validity of the model assumptions case by case.

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