Statistical Mechanics (graduates) **Name:** ______________________

Fall 2017

Midterm

18 Oct 2017

Time Limit: 2 hours Teaching Assistant: Dr. Sandipan Dutta

This exam contains 1 page(s) and 4 questions. Total of points is 100.

1. (25 points) Study the statistical mechanics in the microcanonical ensemble of an extreme relativistic gas characterized by the single-particle energy states

$$\epsilon(n_x, n_y, n_z) = \frac{hc}{2L}\left(n_x^2 + n_y^2 + n_z^2\right)^{1/2}$$

Show that the ratio C_P/C_V in this case is 4/3 instead of 5/3 in the non-relativistic limit. (1.7)

2. (25 points) Show that the partition function $Q_N(V,T)$ of an extreme relativistic gas consisting of N monatomic molecules with energy–momentum relationship $\epsilon = pc$, c being the speed of light, is given by

$$Q_N(V,T) = \frac{1}{N!}\left[8\pi V\left(\frac{T}{hc}\right)^3\right]^N .$$

Study the thermodynamics of this system, checking in particular that

$$PV = \frac{1}{3}U, \quad \frac{U}{N} = 3T, \quad \text{and} \quad \frac{C_P}{C_V} = \frac{4}{3}.$$

Compare to the previous question. (3.15)

3. (25 points) Show that for a system in the canonical ensemble

$$\left\langle(\Delta E)^3\right\rangle = T^4\left(\frac{\partial C_V}{\partial T}\right)_V + 2T^3 C_V.$$

Verify that for an ideal gas

$$\left\langle\left(\frac{\Delta E}{U}\right)^2\right\rangle = \frac{2}{3N} \quad \text{and} \quad \left\langle\left(\frac{\Delta E}{U}\right)^3\right\rangle = \frac{8}{9N^2} .$$

(3.18)

4. (25 points) Show that in the relativistic case the equipartition theorem takes the form

$$\left\langle\frac{m_0 u^2}{\sqrt{1-u^2/c^2}}\right\rangle = 3T ,$$

where m_0 is the rest mass of the particle and u its speed. Check that in the extreme relativistic case the mean thermal energy per particle is twice its value in the non-relativistic case (3.24).