Sterile Neutrino Theory

- Sterile neutrinos
- Necessary ingredients
- Models

Reference: *The Standard Model and Beyond*, CRC Press
Sterile Neutrinos

- Most $m_\nu$ models involve sterile neutrinos (quark-lepton symmetry)

- Mass anywhere from sub-eV to $M_P \sim 10^{19}$ GeV

- LSND/MiniBooNE: possible oscillation between active and sterile
  - Mixing between active and sterile of same helicity
  - Need two types of small (eV-scale) masses (usually Dirac/Majorana)

- Other topics
  - Alternative explanations for Solar, atmospheric, LBL, LSND/MiniBooNE
    (non-standard interactions, MSW effects with $Z'$ and/or sterile, RSFP, non-orthogonal, decay, CPT, LIV, EP, decoherence)
  - Warm dark matter (e.g., keV), pulsar kicks, supernovae, collider implications
Neutrino Preliminaries

- **Weyl fermion**
  - Minimal (two-component) fermionic degree of freedom
  - $\psi_L \leftrightarrow \psi_R^c$ by $CP$ ($\psi_R^c \sim \psi_L^\dagger$)

- **Active Neutrino** (a.k.a. ordinary, doublet)
  - in $SU(2)$ doublet with charged lepton $\rightarrow$ normal weak interactions
  - $\nu_L \leftrightarrow \nu_R^c$ by $CP$

- **Sterile Neutrino** (a.k.a. singlet, right-handed)
  - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
  - $\nu_R \leftrightarrow \nu_L^c$ by $CP$
  - Almost always present: Are they light? Do they mix?
- **Fermion Mass**

  - Transition between right and left Weyl spinors:
    \[ m\bar{\psi}_L\psi_R + m^*\bar{\psi}_R\psi_L \rightarrow m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \]
    ( \( m \geq 0 \) by \( \psi_{L,R} \) phase changes)

- **Dirac Mass**

  - Connects two distinct Weyl spinors
    (usually active to sterile):
    \[ m_D(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) = m_D\bar{\nu}_D\nu_D \]
  - Dirac field: \( \nu_D \equiv \nu_L + \nu_R \)
  - 4 components, \( \Delta L = 0 \)
  - \( \Delta t^3_L = \pm \frac{1}{2} \) → Higgs doublet
  - Why small? (Large dimensions? Higher-dimensional operators? String instantons?)

\[ m_D = h_\nu\nu = \sqrt{2}h_\nu\langle \phi^0 \rangle \]
- **Majorana Mass**

  - Connects Weyl spinor with itself:
    
    \[
    \frac{m_T}{2} (\bar{\nu}_L \nu_R^c + \bar{\nu}_R^c \nu_L) = \frac{m_T}{2} \bar{\nu}_M \nu_M \quad \text{(active)}
    \]

    \[
    \frac{m_S}{2} (\bar{\nu}_L^c \nu_R + \bar{\nu}_R \nu_L^c) = \frac{m_S}{2} \bar{\nu}_{M_S} \nu_{M_S} \quad \text{(sterile)}
    \]

  - Majorana fields:
    
    \[
    \nu_M \equiv \nu_L + \nu_R^c = \nu_M^c
    \]

    \[
    \nu_{M_S} \equiv \nu_L^c + \nu_R = \nu_{M_S}^c
    \]

  - 2 components, \( \Delta L = \pm 2 \), self-conjugate

  - Active: \( \Delta t^3_L = \pm 1 \) (triplet or higher-dimensional operator)

  - Sterile: \( \Delta t^3_L = 0 \) (singlet or bare mass)

  - Phase of \( \nu_L \) or \( \nu_L^c \) fixed by \( m_{T,S} \geq 0 \)
Sterile Neutrinos at the eV scale

- LSND/MiniBooNE data suggest
  \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) oscillations \((\Delta m^2 \sim 1 \text{eV}^2)\)

- Invisible \(Z\)-width:
  \( N_\nu = 2.984 \pm 0.009 \) light active neutrinos \(\Rightarrow\) sterile \(\nu\)

- \(\nu_\mu \leftrightarrow \nu_e\) suggests CP violation \(\Rightarrow\)
  \(\geq 2\) sterile \(\nu\) (or alternative)

- Global 1 + 3 and 2 + 3 fits, including reactor, KARMEN, NOMAD
  (Kopp, Maltoni, Swetz, 1103.4570; Giunti, Laveder, 1107.1452, 1109.4033)
  - Reactor flux reanalysis (Mueller et al, 1101.2663) improves consistency
  - 2 + 3 gives rough but not perfect fit
Giunti, Laveder, 1107.1452

Kopp, Maltoni, Swetz, 1103.4570

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Cosmology, $\beta$ Decay, $\beta\beta_{0\nu}$

- CMB (WMAP-7) and other: allow or weakly favor $N_s = 1 - 2$ sterile neutrinos with $m \lesssim 0.2 - 0.4$ eV
  - However, $m \sim$ eV disfavored in $\Lambda$CDM ($\Sigma$)
  - Also, BBN: $N_s \lesssim 1.3$
  - Bounds weakened by additional radiation, $w < -1$, $\nu$ degeneracy (Hamann et al, 1108.4136)

- Light Dirac: limits not applicable

Hamann et al, 1006.5276
FIG. 2: Same as Fig. 1, for the 2+3 (top) and 3+2 (bottom) cases.

$$\Sigma = \sum_i |m_i|$$ (cosmology)

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 |m_i|^2}$$ (β decay)

$$m_{\beta\beta} = \sum_i U_{ei}^2 |m_i|$$ (ββ0ν)

Barry, Rodejohann, Zhang, 1105.3911

$\Sigma \gtrsim 1.6 \gtrsim 3.4 \gtrsim 4$

exp: (0.5 – 1?) → (0.05 – 0.1)

$\Sigma \gtrsim 0.16 \gtrsim 0.69 \gtrsim 0.93$

exp: (1 – 2) → 0.2

$\Sigma \gtrsim 0 – 0.08 \gtrsim 0.2 – 0.7 \gtrsim 0.2 – 0.9$

exp: (0.2 – 0.7) → 0.02

Barry, Rodejohann, Zhang, 1105.3911

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Mixed Models

- Can have simultaneous Majorana and Dirac mass terms

\[-L = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^0 & \bar{\nu}_L^{0c} \end{pmatrix}_{\text{weak eigenstates}} \begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix} \begin{pmatrix} \nu_R^{0c} \\ \nu_R^0 \end{pmatrix} + h.c.\]

- \( m_T \): \( |\Delta L| = 2, \quad |\Delta t_L^3| = 1 \) \hspace{1cm} \text{(Majorana)}
- \( m_D \): \( |\Delta L| = 0, \quad |\Delta t_L^3| = \frac{1}{2} \) \hspace{1cm} \text{(Dirac)}
- \( m_S \): \( |\Delta L| = 2, \quad |\Delta t_L^3| = 0 \) \hspace{1cm} \text{(Majorana)}
\begin{itemize}

\item **Mass eigenvalues:**

\[
A_L^{\nu^\dagger}\begin{pmatrix}
m_T & m_D \\
m_D & m_S \\
\end{pmatrix}A_R^{\nu} = \begin{pmatrix}
m_1 & 0 \\
0 & m_2 \\
\end{pmatrix}
\]

\item **(Majorana) mass eigenstates:** \(\nu_{iM} = \nu_{iL} + \nu_{iR}^c = \nu_{iM}^c, \quad i = 1, 2\)

\[
\begin{pmatrix}
\nu_{1L} \\
\nu_{2L} \\
\end{pmatrix} = A_L^{\nu^\dagger}\begin{pmatrix}
\nu_{L}^0 \\
\nu_{L}^c \\
\end{pmatrix}, \quad \begin{pmatrix}
\nu_{1R}^c \\
\nu_{2R}^c \\
\end{pmatrix} = A_R^{\nu^\dagger}\begin{pmatrix}
\nu_{R}^0 \\
\nu_{R}^c \\
\end{pmatrix},
\]

\end{itemize}

\( M = M^T \) (unlike Dirac mass matrix) \(\Rightarrow A_L^{\nu} = A_R^{\nu^*}\)

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Special Cases

\[-\mathcal{L} = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^0 & \bar{\nu}_L^{0c} \end{pmatrix} \begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix} \begin{pmatrix} \nu_R^0 \\ \nu_R^{0c} \end{pmatrix} + h.c.\]

- Majorana ($m_D = 0$):

\[m_1 = m_T : \quad \nu_{1L} = \nu_L^0, \quad \nu_{1R}^c = \nu_R^0 \]
\[m_2 = m_S : \quad \nu_{2L} = \nu_L^{0c}, \quad \nu_{2R}^c = \nu_R^0 \]

\[-\Delta L = \pm 2, \text{ but no } \nu_L^0 - \nu_L^{0c} \text{ or } \nu_R^{0c} - \nu_R^0 \text{ mixing}\]
• **Dirac** ($m_T = m_S = 0$):

\[ m_1 = +m_D : \quad \nu_{1L} = \frac{1}{\sqrt{2}}(\nu^0_L + \nu^{0c}_L), \quad \nu^c_{1R} = \frac{1}{\sqrt{2}}(\nu^0_R + \nu^0_R) \]

\[ m_2 = -m_D : \quad \nu_{2L} = \frac{1}{\sqrt{2}}(\nu^0_L - \nu^{0c}_L), \quad \nu^c_{2R} = \frac{1}{\sqrt{2}}(\nu^0_R - \nu^0_R) \]

- **Dirac $\nu$ (4 components)** equivalent to two degenerate ($|m_1| = |m_2|$)
  - Majorana $\nu$'s with $m_1 = -m_2$ and $45^\circ$ mixing (cancel in $\beta\beta_{0\nu}$)
- Useful description for Dirac limit of general case
- Recover usual Dirac expression

\[ -\mathcal{L} = \frac{m_D}{2}(\bar{\nu}_{1L}\nu^{c}_{1R} - \bar{\nu}_{2L}\nu^{c}_{2R}) + h.c. = m_D(\bar{\nu}^0_L\nu^0_R + \bar{\nu}^0_R\nu^0_L) \]

- $L$ conserved $\Rightarrow$ no $\nu^0_L - \nu^{0c}_L$ or $\nu^{0c}_R - \nu^0_R$ mixing
**Seesaw** (a.k.a. minimal or Type I seesaw) \((m_S \gg m_{D,T})\):

\[
\begin{aligned}
m_1 &\sim m_T - \frac{m_D^2}{m_S} : \quad \nu_{1L} \sim \nu^0_L - \frac{m_D}{m_S} \nu^{0c}_L \sim \nu^0_L, \\
m_2 &\sim m_S : \quad \nu_{2L} \sim \frac{m_D}{m_S} \nu^0_L + \nu^{0c}_L \sim \nu^{0c}_L
\end{aligned}
\]

- E.g., \(m_T = 0, m_D = \mathcal{O}(m_t)\), \(m_S = \mathcal{O}(M_X \sim 10^{14} \text{ GeV})\):
  \[|m_1| \sim \frac{m_D^2}{m_S} \ll m_D\]
  \[\nu_{1M} \sim \nu^0_L + \nu^{0c}_R \text{ (active)}\]

- Lower \(m_S\) possible (even TeV)
- Heavy \((\sim\text{sterile})\) \(\nu_{2M}\) decouples at low energy
- \(\nu_{2M}\) decays \(\Rightarrow\) leptogenesis
- Light sector essentially pure active
Active-Sterile ($\nu^0_L - \nu^0_L^c$) Mixing (LSND/MiniBooNE)

- No active-sterile mixing for Majorana, Dirac, or seesaw
- $m_D$ and $m_S$ (and/or $m_T$) both small and comparable (mechanism?)
  (or small active-sterile and sterile-sterile Dirac)
- Pseudo-Dirac ($m_T, m_S \ll m_D$):
  - Small mass splitting, small $L$ violation, e.g.,
    
    \[ m_T = \epsilon, \quad m_S = 0 \quad \Rightarrow \quad |m_{1,2}| = m_D \pm \epsilon/2 \]
  - But small extra $\Delta m^2$ could affect Solar/supernova oscillations
  - Solar: need $m_{S,T} \lesssim 10^{-9}$ eV (de Gouvêa,Huang,Jenkins, 0906.1611)
- Reactor and accelerator disappearance limits (new flux calculation)
- Cosmological implications

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• **Mechanisms for very small masses** (Majorana, Dirac, or both)
  - Very small couplings
  - Loops
  - Geometric suppressions
  - Higher-dimensional operators (HDO)

• **Focus on correlated explanations for small Majorana and Dirac**
Apparent Fine-Tuning

- **Dirac:** \( m_D \sim h_\nu \nu \), \( \nu = 246 \text{ GeV} \Rightarrow h_\nu \sim 10^{-12} \) for \( m_D \sim 0.1 \text{ eV} \)

- **Sterile Majorana:**
  \( m_S \sim \Gamma_S M_P \), \( M_P \sim 2 \times 10^{18} \text{ GeV} \Rightarrow \Gamma_S \sim 10^{-27} \) for \( m_S \sim 1 \text{ eV} \)

- May be due to geometric suppression or HDO in underlying theory

- No predictions without further input
Loops

- Would need very high order (or very small couplings), and suppression mechanism for lower-order

- Often combined with other mechanisms
Geometric Suppressions

- Wave function overlaps in large (and/or warped) extra dimensions, with $\nu_R$ propagating in bulk (cf., gravity)

\[ m_D \sim \frac{\nu M_F}{M_P}, \quad M_F = \left( \frac{M_P^2}{V_\delta} \right)^{\frac{1}{\delta+2}} = \text{fundamental scale} \]

- $M_F \sim 100 \text{ TeV} \Rightarrow m_D \sim 0.01 \text{ eV}$

- Kaluza-Klein excitations (Dirac sterile) for $V_\delta = R^\delta$:
  \[ m_{KK}/n \sim 1/R \sim M_F(M_F/M_P)^{2/\delta} \quad \xrightarrow{M_F \sim 100 \text{ TeV, } \delta = 2} \quad 10 \text{ eV} \]

- Mixings too small in simplest versions
- Can enhance by additional small mass terms, unequal dimensions
- **Worldsheet instantons, e.g., intersecting D-brane** (Type IIA)
  - Closed strings (gravitons) and open strings ending on D-branes
  - D6-branes: fill ordinary space and 3 of the 6 extra dimensions

- Yukawa interactions $\sim \exp(-A_{ijk}) \rightarrow$ hierarchies
- $m_D$: $A_{L\nu L}^\nu H_u$ **not large enough** (at least in toroidal compactifications)
- $m_S$: no Majorana masses at perturbative level
- **D-brane instantons**

  - Anomalous $U(1)'$: $M_{Z'} \sim M_{str}$; acts like perturbative global symmetry (may forbid $\mu$, $R_P$ violation, $\nu_L^c\nu_L^c$, $L\nu_L^c H_u$, $QU^c H_u$, ...)

  - Field theory instantons: nonperturbative $e^{-1/g^2}$ effects from topologically non-trivial classical field configurations (e.g., $B + L$ violation in SM)

  - D instantons: nonperturbative violation of global symmetries

\[
\exp(-S_{\text{inst}}) \sim \exp \left( -\frac{2\pi}{\alpha_{GUT}} \frac{V_{E2}}{V_{D6}} f(\text{winding}) \right)
\]

  - Examples of small Dirac, small or intermediate $M_S$ (ordinary seesaw), stringy Weinberg operator ($LH_u L H_u / M$)

  - No known reason for correlated $m_D, m_S$
Higher-Dimensional Operators (HDO)

- Let $\mathcal{O}$ be an operator, such as $L\nu^c_L$ (Dirac mass), $\nu^c_L\nu^c_L$ (sterile Majorana), or $LL$ (active Majorana)
  \[(L \equiv \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L, \text{Dirac and } SU(2) \text{ indices suppressed})\]

- $L\nu^c_L$ and $LL$ forbidden by $SU(2)$; $\nu^c_L\nu^c_L$ by new physics (usually) $\Rightarrow$
  \[\mathcal{L} \sim h_\nu LH_u \nu^c_L, \quad h_S S \nu^c_L \nu^c_L, \quad \frac{C}{\mathcal{M}} \overbrace{LH_u LH_u}^{\text{Weinberg op}}\]

  $H_u = \text{Higgs doublet, } S = \text{singlet, } \mathcal{M} = \text{new physics scale (HDO)}$

- **Seesaw:** $h_S\langle 0|S|0 \rangle \sim 10^{14}$ GeV ($\ll M_P$), $h_\nu \sim 1$ $\Rightarrow$
  Weinberg operator, $\mathcal{M}/C \sim h_S\langle 0|S|0 \rangle$, $m_\nu \sim 0.1$ eV
  (or $h_S\langle 0|S|0 \rangle \sim 10$ TeV for $h_\nu \sim 10^{-5} \sim m_e/\nu$)
  - $\mathcal{M}$ may also be induced by heavy triplet, neutralinos, string excitations, KK or winding modes, moduli, \ldots

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• Additional symmetries (gauge, global, discrete) or string constraints (heterotic or type II) may further suppress coefficients

\[ \mathcal{L} \sim \frac{S_p}{M_p} L H_u \nu_L^c, \quad \frac{S^{q+1}}{M_q} S \nu_L^c \nu_L^c, \quad \frac{S^{r-1}}{M_r} L H_u L H_u \]

- \( \frac{S_p}{M_p} \rightarrow \frac{S_1 \cdots S_p}{M_1 \cdots M_p} \)
- May also be loop factors
- Can extend to flavor structure (Froggatt-Nielsen)
- Various modified/extended/inverted seesaws via HDO

• Small Dirac for \( p > 0 \)
  - \( m_D \sim 0.1 \text{ eV} \) for \( S/M \sim 10^{-12} \), e.g., \( M = M_P, S \sim 10^3 \text{ TeV} \)
  - Majorana masses may be forbidden (⇒ pure Dirac)
  - Alternative: pseudo-Dirac with \( q \geq 2, r \geq 2 \) (\( m_{S,T} < 10^{-9} \text{ eV} \))
\[ m_D \sim \frac{S^p \nu}{\mathcal{M}^p} \sim 0.1 \text{ eV}, \quad m_S \sim \frac{S^{q+1}}{\mathcal{M}^q}, \quad m_T \sim \frac{S^{r-1} \nu^2}{\mathcal{M}^r} \]
• Both \( m_S \) and \( M_D \) suppressed by symmetries, e.g., \( p = q = r = 1 \)

\[
m_D \sim \Gamma_D \frac{S \nu_u}{\mathcal{M}}, \quad m_S \sim \Gamma_S \frac{S^2}{\mathcal{M}} \sim 1 \text{ eV} , \quad m_T \sim \Gamma_T \frac{\nu^2}{\mathcal{M}}
\]

– Example: mini-seesaw \((S \gg \nu)\) with \( \Gamma_D = \Gamma_S = 1, \Gamma_T = 0 \) \(\Rightarrow\)

\[
m_1 \sim -\frac{(\nu S/\mathcal{M})^2}{S^2/\mathcal{M}} = -\frac{\nu^2}{\mathcal{M}}, \quad m_2 \sim \frac{S^2}{\mathcal{M}}
\]

\[
|\theta| \sim \frac{\nu}{S} \sim \sqrt{\frac{|m_1|}{m_2}}, \quad \mathcal{M} \sim 10^{15} \text{ GeV}
\]

– \( m_T \) comparable for \( \Gamma_T \sim 1 \)
Fan, PL

\[ \mathcal{M} = \begin{pmatrix} 0 & \frac{\Gamma_D \nu S}{\mathcal{M}} \\ \frac{\Gamma_D \nu S}{\mathcal{M}} & \frac{\Gamma_S S^2}{\mathcal{M}} \end{pmatrix} \]

\[ \Gamma_D = \Gamma_S = 1 \]

\[ \mathcal{M} = 1.2 \times 10^{15} \text{ GeV} \]

\[ \nu = 246 \text{ GeV} \]
• **Can extend to** $2 + 3$ \([1 + 3]\)
  
  – Inverted [normal] hierarchy favored for $\Gamma_T \sim 0$
  – Can incorporate flavor structures, e.g., tri-bimaximal

• **Examples from** $U(1)'$, mirror worlds, TC/ETC (including loops)

• **Hybrids with other schemes** (e.g., ordinary high-scale seesaw) **possible**

• **Alternative:** heavy active ($\sim 1$ eV) **with** $\Gamma_D = \Gamma_T = 1, \Gamma_S = 0$, $S < \nu$
Fan, PL

\[
\mathcal{M} = \begin{pmatrix}
\Gamma T \frac{\nu^2}{\mathcal{M}} & \Gamma D \frac{\nu S}{\mathcal{M}} \\
\Gamma D \frac{\nu S}{\mathcal{M}} & 0
\end{pmatrix}
\]

\[\Gamma_D = \Gamma_T = 1\]

\[\mathcal{M} = 6 \times 10^{13} \text{ GeV}\]

\[\nu = 246 \text{ GeV}\]
Conclusions

- Sterile neutrinos present in most theories

- LSND/MiniBooNE: need (small) active-sterile mixing (same helicity)
  - *Not* pure Majorana or Dirac, pseudo-Dirac, high-scale seesaw
  - The three miracles!

- Need mechanism for two types of small masses
  (usually Dirac and Majorana)

- Small masses from HDO, string instantons, large/warped extra dimensions

- Low-scale seesaw: Dirac and Majorana masses both suppressed by symmetries (mass-mixing relation)