Neutrinos and Strings

- Introduction
- Neutrino preliminaries
- Models
- String embeddings
  - Intersecting brane
  - The $Z_3$ heterotic orbifold
  - Embedding the Higgs triplet
- Outlook
Neutrino mass

- Nonzero mass may be first break with standard model

- Enormous theoretical effort: GUT, family symmetries, bottom up
  - Majorana masses may be favored because not forbidden by SM gauge symmetries
  - GUT seesaw (heavy Majorana singlet). Usually ordinary hierarchy.
  - Higgs triplets ("type II seesaw"), often assuming GUT, Left-Right relations
The minimal seesaw

- Active (sterile) neutrinos $\nu_L$ ($N_R$) (3 flavors each)

$$L = \frac{1}{2} \left( \bar{\nu}_L \, \bar{N}_L^c \right) \begin{pmatrix} m_T & m_D \\ m_D^T & m_S \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + hc$$

- $m_T = m_T^T$ = triplet Majorana mass matrix (Higgs triplet)
- $m_D$ = Dirac mass matrix (Higgs doublet)
- $m_S = m_S^T$ = singlet Majorana mass matrix (Higgs singlet)
• Ordinary (type I) seesaw: \( m_T = 0 \) and (eigenvalues) \( m_S \gg m_D \):

\[
m^\text{eff}_\nu = -m_D m_S^{-1} m_D^T
\]

diagonalized by \( U_\nu \), with

\[
U_{PMNS} = U^\dagger_e U_\nu
\]

• To achieve large mixings, most models assume either
  
  – \( U_e \sim I \) in basis with manifest symmetries for \( m_D, S \Rightarrow \) need large mixings in \( U_\nu \) (requires clever \( m_D, m_S \) collaboration)
  
  – Large \( U_e \) mixings from lopsided \( m_e \) in basis with \( m_D, S \sim \) diagonal (harder to achieve in SO(10) than SU(5))

• \( SO(10) \) models, combined with family symmetries, often large Higgs representations (e.g., 126-plet); typically, \( m_S \sim 10^{14} \) GeV
Extended (TeV) Seesaw

• $m_\nu \sim m_{p+1}/m_S^p$, $p > 1$ (e.g., $m \sim 100$ MeV, $m_S \sim 1$ TeV for $p = 2$)

• $\nu_L$, $N_R$, $N_R'$ (3 flavors each)

$$L = \frac{1}{2} (\bar{\nu}_L \ N_L^c \ \bar{N}_L^c) \begin{pmatrix} 0 & m_D & m_{D'} \\ m^T_D & 0 & m_{SS'} \\ m^T_{D'} & m^T_{SS'} & 0 \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \\ N_R' \end{pmatrix} + hc$$

or

$$L = \frac{1}{2} (\bar{\nu}_L \ N_L^c \ \bar{N}_L^c) \begin{pmatrix} 0 & m_D & 0 \\ m^T_D & 0 & m_{SS'} \\ 0 & m^T_{SS'} & m_S' \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \\ N_R' \end{pmatrix} + hc$$
Triplet models

- Introduce Higgs triplet $T = (T^{++} \ T^+ \ T^0)^T$ with weak hypercharge $Y = 1$

- Majorana masses $m_T$ generated from $L_\nu = \lambda_{ij}^T L_i T L_j$ if $\langle T^0 \rangle \neq 0$

- Old Gelmini-Roncadelli model: $\langle T^0 \rangle \ll$ EW scale with spontaneous $L$ violation
  - Excluded by $Z \rightarrow$ Majoron + scalar (equivalent to $\Delta N_\nu = 2$)

- Modern triplet models (type II seesaw) break $L$ explicitly by $THH$ couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)

- Often considered in $SO(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.
• Can achieve small Dirac masses (neutrino or other) by higher dimensional operators

\[ L_\nu \sim \left( \frac{S}{M_{Pl}} \right)^p L N_L^c H_2, \quad \langle S \rangle \ll M_{Pl} \]

\[ \Rightarrow m_D \sim \left( \frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle \]

• Large \( p \Rightarrow \langle S \rangle \) close to \( M_{Pl} \) (e.g., anomalous \( U(1)_A \))

• Small \( p \Rightarrow \) intermediate scale \( \ll M_{Pl} \)

• Similar HDO may give light steriles and ordinary/sterile mixing
Other Models

- Large extra dimensions (suppressed Dirac Yukawa couplings)
- $R$-parity violation in supersymmetry
- TeV scale loops with new ad hoc scalars
- Ad hoc flavor symmetries, textures, anarchic models
- Anthropic considerations (string landscape)
Neutrino Mass in Strings

• Very little work from string constructions, even though may be Planck scale effect

• Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane)
  (Due to string symmetries or constraints, not simplicity or elegance)

  – “Right-handed” neutrinos may not be gauge singlets
  – Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
  – GUT Yukawa relations broken
  – String symmetries/constraints severely restrict couplings, e.g., Majorana masses, or simultaneous Dirac and Majorana masses
Quasi-realistic string constructions

- **Two classes of quasi-realistic**: intersecting D-brane, heterotic

- **Intersecting D-brane** (review: R. Blumenhagen, M. Cvetic, P.L., G. Shiu, hep-th/0502005)
  - Closed strings (gravitons) and open strings ending on D-branes
  - D6-branes: fill ordinary space and 3 of the 6 extra dimensions
  - Stringy implementation of “brane world” ideas
- Gauge interactions from strings beginning/ending on stack of parallel branes (one for each group factor)
- Chiral matter: strings at intersection of branes, e.g., $SU(N) \times SU(M) \rightarrow$ bifundamental $(N, \bar{M})$
– Family replication from multiple intersections on compactified geometry
– Yukawa interactions $\sim \exp(-A_{ijk}) \rightarrow$ hierarchies
– Existing models: conserved $L$; no diagonal (Majorana) triangles
– However, no realistic model with large enough $A$ for small Dirac neutrino masses (more generic geometries?)
The $E_8 \times E_8$ Heterotic String

- Dirac masses
  - Can achieve small Dirac masses (neutrino or other) by higher dimensional operators

$$L_\nu \sim \left( \frac{S}{M_{Pl}} \right)^p LN_L^c H_2, \quad \langle S \rangle \ll M_{Pl}$$

$$\Rightarrow m_D \sim \left( \frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle$$

- Recent variant: $N_L^c$ is a modulus (Bouchard, Cvetič, Donagi, hep-th/0602096)
• Majorana masses

- Can one generate large effective $m_S$ from

$$W_\nu \sim c_{ij} \frac{S^{q+1}}{M_{Pl}^q} N_i N_j \Rightarrow (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{q+1}}{M_{Pl}^q},$$

consistent with $D$ and $F$ flatness?

- Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints?

- Are bottom-up model assumptions for relations to quark, charged lepton masses maintained?
The $Z_3$ Heterotic Orbifold (J. Giedt, G. Kane, P.L., B. Nelson, hep-th/0502032)

- Systematically studied large class of vacua
  * Is minimal seesaw common?
  * If rare, possibly guidance to model building
  * Clues to textures, etc.

- Several models from each of 20 patterns; $W$ through degree 9; huge number of $D$ flat directions reduced greatly by $F$-flatness

- Only two patterns had Majorana mass operators $\langle S_1 \cdots S_{n-2} \rangle_{NN}/M_{PL}^{n-3}$

- None had simultaneous Dirac operators $\langle S'_1 \cdots S'_{d-3} \rangle_{NLH_u}/M_{PL}^{d-3}$ leading to $\Delta m^2 > 10^{-10}$ eV$^2$ (one apparent model ruined by off-diagonal Majorana)

- Feature of $Z_3$ orbifold? Or more general?
• **Systematic searches in other constructions important** (Is seesaw generic? Rare? Alternatives?)

• **Consider alternatives seriously**
  
  – **Small Dirac masses from high degree terms** (very common in constructions) (could also give light sterile \( \nu \)'s and mixing)
  
  – **Extended seesaws**, \( m_\nu \sim m_D^{2+k}/M^{1+k} \), with \( k \geq 1 \) and low (e.g., TeV) scale \( M \)
  
  – **Higgs triplet models**: non-trivial to embed in strings (higher level), but very predictive (e.g., inverted hierarchy with nearly bi-maximal mixing) (B. Nelson, PL, hep-ph/0507063)
Triplet models

• Introduce Higgs triplet $T = (T^+ \ T^0)^T$ with weak $Y = 1$

• Majorana masses $m_T$ generated from $L_\nu = \lambda^{T}_{i j} L_i T L_j$ if $\langle T^0 \rangle \neq 0$

• General SUSY case

$$W_\nu = \lambda^{T}_{i j} L_i T L_j + \lambda_1 H_1 T H_1 + \lambda_2 H_2 \bar{T} H_2$$

$$+ M_T T \bar{T} + \mu H_1 H_2$$

$T, \bar{T}$ are triplets with $Y = \pm 1$, $M_T \sim 10^{12} - 10^{14}$ GeV. Typically,

$$\langle T^0 \rangle \sim -\lambda_2 \langle H_2^0 \rangle^2 / M_T \quad \Rightarrow m^\nu_{ij} = -\lambda^T_{i j} \lambda_2 \frac{v_2^2}{M_T}$$

• Most previous models: GUT/LR symmetry, ordinary hierarchy
• Expect $\lambda^{T}_{ij} = 0$ for $i = j$ (off-diagonal) $\Rightarrow m_{ii} = 0$

• Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal

• Partial explanation: $SU(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $SU(2) \subset SU(2) \times SU(2)$ (Have $Z_3$ constructions with some but not all of the features, B. Nelson, PL, hep-ph/0507063)

$$W \sim \lambda^T_{1j} L_1(2, 1) T(2, 2) L_j(1, 2), \ j = 2, 3$$

yields

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

• Typical string case: $|a| = |b|$
• HDO (or $SU(2) \subset SU(2) \times SU(2) \times SU(2)$) can give $m_{23}^\nu \neq 0$

• For

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

can take $a, b, c$ real w.l.o.g. by redefinition of fields (not true for general $m^\nu$)

• $\text{Tr } m^\nu = 0$ and $m^\nu = m^{\nu \dagger} \Rightarrow m_1 + m_2 + m_3 = 0$
• $|\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3} \text{ eV}^2$, $\Delta m^2_\odot \sim 8 \times 10^{-5} \text{ eV}^2 \implies \text{two solutions}$
  
  - For $\Delta m^2_\odot = 0$
    
    (a) $m_i \propto 1, -\frac{1}{2}, -\frac{1}{2}$ (ordinary, with shifted masses)
    (b) $m_i \propto 1, -1, 0$ (inverted)
  
  - With $\Delta m^2_\odot \neq 0$
    
    (a) $m_i = 0.054, -0.026, -0.026 \text{ eV} \ (\sum |m_i| = 0.107 \text{ eV (cosmology)})$
    (b) $m_i = 0.046, -0.045, -0.001 \text{ eV} \ (\sum |m_i| = 0.092 \text{ eV (cosmology)})$

\[
  m^\nu_a \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad m^\nu_b \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}
\]

- (a) leads to unrealistic mixing matrix $\implies$ consider (b)
A special texture

- The $L_e - L_\mu - L_\tau$ conserving texture

\[ m' \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \]

has been considered phenomenologically by many authors (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)
\[ m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \]

- **New aspects**
  - Strong string motivation
  - Motivation for special case \(|a| = |b|\)
  - Can perturb by HOT
  - No reason for \(U_e = I\) in this basis

- Yields inverted hierarchy, with eigenvalues 0, \(\pm \sqrt{|a|^2 + |b|^2}\)

- **Diagonalization:** \(\tan \theta_{\text{Atm}} = b/a \Rightarrow\) need \(|b| = |a|\) for maximal
• If $U_e = I$: $\theta_\odot = \frac{\pi}{4}$ (maximal) (experiment: $\frac{\pi}{4} - \theta_\odot = 0.19^{+0.05}_{-0.06}$, $2\sigma$)
  
  – Comparable to Cabibbo angle, $\theta_C \sim 0.23$

• Perturbations on $m^\nu$ cannot give both $\Delta m^2_\odot$ and $\frac{\pi}{4} - \theta_\odot \sim 0.19$ (cf $\theta_C \sim 0.23$) without fine-tuning between terms, e.g.,

\[
\frac{\Delta m^2_\odot}{\sqrt{2} |\Delta m^2_{\text{Atm}}|} \sim (m^\nu_{23} + m^\nu_{11}) \sim \frac{1}{43}
\]

\[
\frac{\pi}{4} - \theta_\odot \sim \frac{1}{4} (m^\nu_{23} - m^\nu_{11}) \sim 0.19
\]
However, \( U_e \neq I \) with small angles (comparable to CKM) can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

\[
U_e^* \sim \begin{pmatrix} 1 & -s_{12}^e & 0 \\ s_{12}^e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

yields

\[
\frac{\pi}{4} - \theta_\odot \sim \frac{s_{12}^e}{\sqrt{2}} \quad \Rightarrow \quad s_{12}^e \sim 0.27^{+0.07}_{-0.08}
\]

\[
|U_{e3}|^2 \sim \frac{(s_{12}^e)^2}{2} \sim (0.017 - 0.059), \quad 2\sigma \ (\text{exp} : < 0.032)
\]

\[
m_{\beta\beta} \sim m_2(\cos^2 \theta_\odot - \sin^2 \theta_\odot) \sim 0.018 \text{ eV}
\]
Outlook

- Neutrino mass likely due to large or Planck scale effects, but little work in string context

- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for $m_\nu$

- No examples of minimal seesaw in large class of heterotic $Z_3$ orbifold vacua

- Small Dirac, extended seesaw, Higgs triplet (inverted hierarchy in string context) should be seriously considered
Outlook

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Happy Birthday Gary!
Neutrino Preliminaries

- **Weyl fermion**
  - Minimal (two-component) fermionic degree of freedom
  - $\psi_L \leftrightarrow \psi^c_R$ by CPT

- **Active Neutrino (a.k.a. ordinary, doublet)**
  - in $SU(2)$ doublet with charged lepton $\rightarrow$ normal weak interactions
  - $\nu_L \leftrightarrow \nu^c_R$ by CPT

- **Sterile Neutrino (a.k.a. singlet, right-handed)**
  - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
  - $N_R \leftrightarrow N^c_L$ by CPT
  - Almost always present: Are they light? Do they mix?
• Dirac Mass

- Connects distinct Weyl spinors (usually active to sterile):
  \[(m_D \bar{\nu}_L N_R + h.c.)\]
- 4 components, \(\Delta L = 0\)
- \(\Delta I = \frac{1}{2} \rightarrow\) Higgs doublet
- Why small? HDO? LED?
- Variant: couple active to anti-active, e.g., \(m_D \bar{\nu}_e L \nu^c_{\mu R} \Rightarrow L_e - L_\mu\) conserved; \(\Delta I = 1\)
• **Majorana Mass**

  – Connects Weyl spinor with itself:
    \[
    \frac{1}{2}(m_T \bar{\nu}_L \nu^c_R + h.c.) \text{ (active);} \\
    \frac{1}{2}(m_S \bar{\nu}^c_L \bar{\nu}_R + h.c.) \text{ (sterile)}
    \]
  
  – 2 components, \( \Delta L = \pm 2 \)

  – Active: \( \Delta I = 1 \rightarrow \) triplet or seesaw

  – Sterile: \( \Delta I = 0 \rightarrow \) singlet or bare mass

• **Mixed Masses**

  – Majorana and Dirac mass terms

  – Seesaw for \( m_S \gg m_D \)

  – Ordinary-sterile mixing for \( m_S \) and \( m_D \) both small and comparable (or \( m_S \ll m_d \) (pseudo-Dirac))