Neutrino Masses from the Top Down

- Introduction
- Neutrino preliminaries
- The GUT seesaw
- Neutrinos in string constructions
- The triplet model

(Work in progress, in collaboration with J. Giedt, G. Kane, B. Nelson.)
Nonzero mass may be first break with standard model

Enormous theoretical effort: GUT, family symmetries, bottom up

- Majorana masses may be favored because not forbidden by SM gauge symmetries
- GUT seesaw (heavy Majorana singlet). Usually ordinary hierarchy.
- Higgs triplets (“type II seesaw”), often assuming GUT, Left-Right relations
• Very little work from string constructions, even though probably Planck scale


- J. Giedt, G. Kane, PL, B. Nelson, in progress. (Heterotic $Z_3$ orbifolds. So far, no Majorana.)
- Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane)
  (Due to string symmetries or constraints, not simplicity or elegance)

- “Right-handed” neutrinos may not be gauge singlets
- Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
- GUT Yukawa relations broken
- String symmetries/constraints forbid couplings, e.g., Majorana masses, or diagonal (same family or same flavor) Majorana
- Very nonstandard triplet or singlet seesaws (inverted hierarchy for triplet), extended seesaw, or small Dirac masses from HDO.

(Work in progress, in collaboration with J. Giedt, G. Kane, B. Nelson.)
• **Weyl fermion**
  - Minimal (two-component) fermionic degree of freedom
  - $\psi_L \leftrightarrow \psi^c_R$ by CPT

• **Active Neutrino (a.k.a. ordinary, doublet)**
  - in $SU(2)$ doublet with charged lepton $\rightarrow$ normal weak interactions
  - $\nu_L \leftrightarrow \nu^c_R$ by CPT

• **Sterile Neutrino (a.k.a. singlet, right-handed)**
  - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
  - $N_R \leftrightarrow N^c_L$ by CPT
  - Almost always present: Are they light? Do they mix?
• Dirac Mass

- Connects distinct Weyl spinors (usually active to sterile):
  \[(m_D \bar{\nu}_L N_R + \text{h.c.})\]

- 4 components, \(\Delta L = 0\)

- \(\Delta I = \frac{1}{2} \rightarrow \) Higgs doublet

- Why small? LED? HDO?

- Variant: couple active to anti-active, e.g., \(m_D \bar{\nu}_e L \nu^c_{\mu R} \Rightarrow L_e - L_\mu \) conserved; \(\Delta I = 1\)
Majorana Mass

- Connects Weyl spinor with itself:
  $$\frac{1}{2}(m_T \nu_L \nu_R^c + h.c.) \text{ (active);}$$
  $$\frac{1}{2}(m_S \bar{\nu}_L^c \nu_R + h.c.) \text{ (sterile)}$$
- 2 components, $\Delta L = \pm 2$
- Active: $\Delta I = 1 \rightarrow$ triplet or seesaw
- Sterile: $\Delta I = 0 \rightarrow$ singlet or bare mass

Mixed Masses

- Majorana and Dirac mass terms
- Seesaw for $m_S \gg m_D$
- Ordinary-sterile mixing for $m_S$ and $m_D$ both small and comparable (or $m_S \ll m_d$ (pseudo-Dirac))
3 $\nu$ Patterns

- Solar: LMA (SNO, Kamland)

- $\Delta m^2_{\odot} \sim 8 \times 10^{-5}$ eV$^2$, nonmaximal

- Atmospheric:
  $|\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3}$ eV$^2$, near-maximal mixing

- Reactor: $U_{e3}$ small
− Mixings: let $\nu_{\pm} \equiv \frac{1}{\sqrt{2}} (\nu_\mu \pm \nu_\tau)$:

$$
\begin{align*}
\nu_3 & \sim \nu_+ \\
\nu_2 & \sim \cos \theta \odot \nu_- - \sin \theta \odot \nu_e \\
\nu_1 & \sim \sin \theta \odot \nu_- + \cos \theta \odot \nu_e
\end{align*}
$$

3 _______ 2 _______ 1 _______

2 _______ 1 _______ 3 _______

− Hierarchical pattern

* Analogous to quarks, charged leptons
* $\beta\beta_{0\nu}$ rate very small

− Inverted quasi-degenerate pattern

* $\beta\beta_{0\nu}$ if Majorana
* SN1987A energetics (if $U_{e3} \neq 0$)?
* May be radiative unstable
– Degenerate patterns
  * Motivated by CHDM (no longer needed)
  * Strong cancellations needed for $\beta\beta_{0\nu}$ if Majorana
  * May be radiative unstable
• 4 $\nu$ Patterns

  – LSND: $\Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2$
  – $Z$ lineshape: 2.986(7) active $\nu$'s lighter than $M_Z/2 \rightarrow$ fourth sterile $\nu_S$

  \[ \begin{array}{cc}
    \text{2 + 2 patterns} \\
    \text{3 + 1 patterns}
  \end{array} \]

\[ \begin{array}{cc}
    \text{2 + 2} \\
    \text{3 + 1}
  \end{array} \]

• Pure ($\nu_\mu - \nu_s$) excluded for atmospheric by SuperK, MACRO

• Pure ($\nu_e - \nu_s$) excluded for solar by SNO, SuperK

• More general admixtures possible, but very poor global fits
The GUT Seesaw

• Elegant mechanism for small Majorana masses

• Leptogenesis

• Expect small mixings in simplest versions (can evade by lopsided $e/d$, Majorana textures, etc.)

• Large Majorana often forbidden, e.g., by extra $U(1)$’s

• Direct Majorana masses and large scales forbidden in some string constructions

• GUTs, adjoint Higgs, large Higgs hard to accommodate in simplest heterotic constructions
• **LSND**: active-sterile difficult in simple versions

• Therefore, explore alternatives, e.g., with small Dirac and/or Majorana masses
  
  – Small Majorana from loops, $R_p$ violation, TeV seesaw, or triplet
  
  – Small Dirac from large extra dimension or by higher dimensional operators in intermediate scale models (e.g. $U(1)'$)
  
  – Variant ordinary and triplet seesaws motivated by string constructions
Neutrinos in string constructions

Key ingredients of most GUT/bottom up models forbidden or different in known constructions (heterotic or intersecting brane)

- Bifundamentals, singlets, or adjoints; not large representations
- $L$ may be conserved, or extra $U(1)'$ charge for $N_R$
- String constraints may forbid couplings allowed by 4d symmetries
- Superpotential terms leading to Majorana masses, or diagonal (same family or same flavor) Majorana usually absent
- GUT Yukawa relations broken
- Non-zero superpotential terms may be equal (gauge couplings)
- Hierarchies from HDO (heterotic), intersection triangles (intersecting brane)
Can achieve small Dirac masses (neutrino or other) by higher dimensional operators or by large intersection areas

\[ L_\nu \sim \left( \frac{S}{M_{Pl}} \right)^p L N^c_L H_2, \quad \langle S \rangle \ll M_{Pl} \]

\[ \Rightarrow m_D \sim \left( \frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle \]

- Large \( p \Rightarrow \langle S \rangle \) close to \( M_{Pl} \) (e.g., anomalous \( U(1)' \))

- Small \( p \Rightarrow \) intermediate scale \( \ll M_{Pl} \)
Intermediate scale in (non-anomalous) $U(1)'$ from $D$ and (almost) $F$ flat direction:

Two SM singlets charged under $U(1)'$. If no $F$ terms,

$$V(S_1, S_2) = m_1^2|S_1|^2 + m_2^2|S_2|^2 + \frac{g'^2Q'^2}{2}(|S_1|^2 - |S_2|^2)^2$$

Break at EW scale for $m_1^2 + m_2^2 > 0$, at intermediate scale for $m_1^2 + m_2^2 < 0$ (stabilized by loops or HDO)
The ordinary seesaw

- Active (sterile) neutrinos $\nu_L (N_R)$ (3 flavors each)

$$L = \frac{1}{2} (\bar{\nu}_L \; \bar{N}^c_L) \begin{pmatrix} m_T & m_D \\ m_T^T & m_S \end{pmatrix} \begin{pmatrix} \nu^c_R \\ N_R \end{pmatrix} + h c$$

- $m_T = m_T^T =$ triplet Majorana mass matrix (Higgs triplet)
- $m_D =$ Dirac mass matrix (Higgs doublet)
- $m_S = m_S^T =$ singlet Majorana mass matrix (Higgs singlet); eg, 126 of $SO(10)$
• Ordinary (type I) seesaw: \( m_T = 0 \) and (eigenvalues) \( m_S \gg m_D \):

\[
m_\nu^{\text{eff}} = -m_D m_S^{-1} m_T^T
\]

with

\[
U_{PMNS} = U_e^\dagger U_\nu
\]

• Most models assume either
  
  – \( U_e \sim I \) in basis with manifest symmetries for \( m_D, S \Rightarrow \) large mixings in \( U_\nu \)
  
  – Large \( U_e \) mixings from lopsided \( m_e \) in basis with \( m_D, S \sim \) diagonal (harder to achieve in \( SO(10) \) than \( SU(5) \))

• \( SO(10) \) models usually yield ordinary hierarchy
• **String constructions**
  
  – Can one generate large effective $m_S$ from

  \[ W_\nu \sim c_{ij} \frac{S^{q+1}}{M_{Pl}^q} N_i N_j \Rightarrow (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{q+1}}{M_{Pl}^q}, \]

  consistent with $D$ and $F$ flatness?

  – Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints?

  – Is $c_{ii} = 0$? (Diagonal superpotential rare. Would be very nonstandard.)

  – Are bottom-up model assumptions for relations to quark, charged lepton masses maintained?

• **Under investigation for $Z_3$ orbifold. So far, no examples.**

• **Flipped $SU(5)$ example?** (Non GUT-like) (Ellis, Leontaris, Lola, Nanopoulos)
- Extended (TeV) Seesaw?
  
  \[- m_\nu \sim m^{p+1}/m_S^p, \quad p > 1 \]  
  (e.g., \( m \sim 100 \text{ MeV}, m_S \sim 1 \text{ TeV} \) for \( p = 2 \))

- \( \nu_L, N_R, N'_R \) (3 flavors each)

\[
L = \frac{1}{2} (\bar{\nu}_L \quad \tilde{N}^c_L \quad \tilde{N'}^c_L) \begin{pmatrix}
0 & m_D & m_D' \\
\frac{m_T^T}{m_D'} & 0 & m_{SS'} \\
m_D' & m_{SS'} & 0
\end{pmatrix} \begin{pmatrix}
\nu^c_R \\
N^c_R \\
N'^c_R
\end{pmatrix} + hc
\]

or

\[
L = \frac{1}{2} (\bar{\nu}_L \quad \tilde{N}^c_L \quad \tilde{N'}^c_L) \begin{pmatrix}
0 & m_D & 0 \\
\frac{m_T^T}{m_D'} & 0 & m_{SS'} \\
0 & m_{SS'} & m_S
\end{pmatrix} \begin{pmatrix}
\nu^c_R \\
N^c_R \\
N'^c_R
\end{pmatrix} + hc
\]

(Faraggi et al.: may occur in specific heterotic model, with dynamical assumptions.)
Triplet models

- Introduce Higgs triplet $T = (T^{++} \ T^+ \ T^0)^T$ with weak hypercharge $Y = 1$

- Majorana masses $m_T$ generated from $L_\nu = \lambda_{ij}^{T} L_i T L_j$ if $\langle T^0 \rangle \neq 0$

- Old Gelmini-Roncadelli model: $\langle T^0 \rangle \ll$ EW scale with spontaneous $L$ violation
  - Excluded by $Z \rightarrow$ Majoron + scalar (equivalent to $\Delta N_\nu = 2$)

- Modern triplet models (type II seesaw) break $L$ explicitly by $THH$ couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)

- Often considered in $SO(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.
• General SUSY case

\[ W_\nu = \lambda_{ij}^T L_i T L_j + \lambda_1 H_1 T H_1 + \lambda_2 H_2 \bar{T} H_2 \]
\[ + M_T T \bar{T} + \mu H_1 H_2 \]

\( T, \bar{T} \) are triplets with \( Y = \pm 1, M_T \sim 10^{12} - 10^{14} \) GeV. Typically,

\[ \langle T^0 \rangle \sim -\lambda \langle H_2^0 \rangle^2 / m_T \quad \Rightarrow \]

\[ m_{ij}^\nu = -\lambda_{ij}^T \lambda_2 \frac{v_2^2}{M_T} \]
String constructions

- Expect $\lambda_{ij}^T = 0$ for $i = j$ (off-diagonal) $\Rightarrow m^\nu_{ii} = 0$

- Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal

- Partial explanation: $SU(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $SU(2) \subset SU(2) \times SU(2)$ (Have $Z_3$ constructions with some but not all of the features.)

$$W \sim \lambda_{1j}^T L_1(2, 1) T(2, 2) L_j(1, 2), \ j = 2, 3$$

yields

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- Typical string case: $|a| = |b|$
HDO (or \( SU(2) \subset SU(2) \times SU(2) \times SU(2) \)) can give \( m_{23}^{\nu} \neq 0 \)

For

\[
m^{\nu} = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}
\]

can take \( a, b, c \) real w.l.o.g. by redefinition of fields (not true for general \( m^{\nu} \))

\[ \text{Tr} \ m^{\nu} = 0 \text{ and } m^{\nu} = m^{\nu \dagger} \Rightarrow m_1 + m_2 + m_3 = 0 \]
• $|\Delta m_{\text{Atm}}^2| \sim 2 \times 10^{-3} \text{ eV}^2$, $\Delta m_{\odot}^2 \sim 8 \times 10^{-5} \text{ eV}^2 \Rightarrow \text{two solutions}$
  
  - For $\Delta m_{\odot}^2 = 0$
    
    (a) $m_i \propto 1, -\frac{1}{2}, -\frac{1}{2}$ (ordinary, with shifted masses)
    (b) $m_i \propto 1, -1, 0$ (inverted)
  
  - With $\Delta m_{\odot}^2 \neq 0$
    
    (a) $m_i = 0.054, -0.026, -0.026 \text{ eV}$ ($\sum |m_i| = 0.107 \text{ eV}$ (cosmology))
    (b) $m_i = 0.046, -0.045, -0.001 \text{ eV}$ ($\sum |m_i| = 0.092 \text{ eV}$ (cosmology))

  $m_{\nu}^a \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$  $m_{\nu}^b \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$

  - (a) leads to unrealistic mixing matrix $\Rightarrow$ consider (b)
• The $L_e - L_\mu - L_\tau$ conserving texture

$$m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

has been considered phenomenologically by many authors (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)
\[ m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \]

- **New aspects**
  - Strong string motivation
  - Motivation for special case \( |a| = |b| \)
  - Most likely perturbation in 23 element from HOT

- **Diagonalization:** \( \tan \theta_{\text{Atm}} = b/a \Rightarrow \text{need} \ |b| = |a| \text{ for maximal} \)

- \( \tan^2 \theta_\odot = 1 \text{ (maximal)} \) (experiment \( \tan^2 \theta_\odot = 0.40^{+0.09}_{-0.07} \))
• Majorana mass matrix

\[ m^\nu \sim \begin{pmatrix}
0 & 1 & -1 \\
1 & 0 & 0 \\
-1 & 0 & 0 \\
\end{pmatrix} \]

• Inverted hierarchy

• Bimaximal mixing for \( U_e = I \):

\[ U_\nu \sim \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\
\end{pmatrix} \]
• Perturbations on $m^\nu$ cannot give both $\Delta m^2_\odot$ and $\frac{\pi}{4} - \theta_\odot \sim \theta_C \sim 0.23$ without fine-tuning between terms, e.g.,

$$\frac{1}{4\sqrt{2}} \frac{\Delta m^2_\odot}{\Delta m^2_{\text{Atm}}} = -\frac{\epsilon_{23}}{4} \sim 0.007 \neq \frac{\pi}{4} - \theta_\odot \sim 0.23$$
• However, $U_e \neq I$ with small angles (comparable to CKM) can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

$$U_e^\dagger \sim \begin{pmatrix}
1 & -s_{12} & 0 \\
s_{12} & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}$$

yields

$$\theta_\odot \sim \frac{\pi}{4} - \frac{s_{12}}{\sqrt{2}} = 0.56^{+0.05}_{-0.04}$$

$$|U_{e3}|^2 \sim \frac{(s_{12})^2}{2} \sim (0.023 - 0.081), \ 90\% \ (\text{exp} : < 0.03)$$

$$m_{\beta\beta} \sim m_2 (\cos^2 \theta_\odot - \sin^2 \theta_\odot) \sim 0.020 \text{ eV}$$
• Detailed $Z_3$ constructions for higher level embeddings (triplets) and for heavy Majorana neutrinos

• Implications for $m_e, m_q$

• Implications of additional Higgs

• RGE effects

• Leptogenesis
Conclusions

- Neutrino mass likely due to large or Planck scale effects, but little work in string context

- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for $m_\nu$

- Preliminary conclusion: inverted hierarchy (pseudo Dirac), extended seesaw, or small Dirac favored

- Inverted hierarchy (e.g., from triplet) very predictive