An Example of $F$-Term Breaking (O’Raifeartaigh) models

$$W = m\Phi_2\Phi_3 + h\Phi_1 \left(\Phi_3^2 - \mu^2\right),$$

$m$, $h$, $\mu^2$ real and positive.

$$F_1^* = -h \left(\phi_3^2 - \mu^2\right), \quad F_2^* = -m\phi_3, \quad F_3^* = -m\phi_2 - 2h\phi_1\phi_3$$

$$V_F = \sum_i |F_i|^2 > 0 \text{ at the minimum.}$$

$$V_F = h^2\mu^4 + h^2|\phi_3|^4 - h^2\mu^2 \Re \phi_3^2 + m^2|\phi_3|^2 + m^2|\phi_2|^2$$

$$+ 4h^2|\phi_1|^2|\phi_3|^2 - 2hm \Re \phi_2^\dagger\phi_1\phi_3$$

Assume $m^2 > 2h^2\mu^2 \Rightarrow \langle \phi_2 \rangle = \langle \phi_3 \rangle = 0$, $\langle \phi_1 \rangle$ undetermined.

For $\langle \phi_1 \rangle = 0$,

$$m_1^2 = 0, \quad m_2^2 = m^2, \quad m_{3R}^2 = m^2 - h^2\mu^2, \quad m_{3I}^2 = m^2 + h^2\mu^2,$$

where $\phi_3 = (\phi_{3R} + i\phi_{3I})/\sqrt{2}$
\[ \mathcal{L}_f = -m \xi_2 \xi_3 + h.c. + \text{Yukawa terms} \]

\(\xi_2\) and \(\xi_3\) combine to form a Dirac fermion with mass \(m\), while \(\xi_1\) remains massless.

\(\xi_1\) is the massless Goldstino.

**Sum rule**

\[ m_B^2 = 2m_1^2 + 2m_2^2 + m_{3R}^2 + m_{3I}^2 = 4m^2 \]
Abelian $U(1)$, single chiral field, $q = 1 \Rightarrow W = 0$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\lambda}\sigma^{\mu}\partial_{\mu}\lambda + \left| (\partial_{\mu} + igA_{\mu})\phi \right|^2$$

$$+ i\bar{\xi}\sigma^{\mu} (\partial_{\mu} + igA_{\mu})\xi - \sqrt{2}g(\bar{\xi}\lambda\phi + \phi^\dagger\lambda\xi) - V(\phi)$$

$$V(\phi) = \frac{1}{2} |D|^2 = \frac{1}{2} |\kappa + g\phi^\dagger\phi|^2$$

$$(V = \theta\sigma^{\mu}\bar{\theta}A_{\mu} + \theta\theta\bar{\theta}\bar{\lambda} + \bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D)$$

$\kappa = \text{Fayet-Iliopoulos term; absent for non-abelian}$
\[ V(\phi) = \frac{1}{2} |D|^2 = \frac{1}{2} |\kappa + g\phi^\dagger \phi|^2 \]

For \( \kappa/g \geq 0 \), \( \phi = 0 \) at minimum. Gauge symmetry unbroken. Massless gauge boson/gaugino.

However, \( D \neq 0 \) so supersymmetry broken; massive scalar and massless Goldstino.

\[ m^2_{\phi} = gD_{\min} = g\kappa, \quad m_\xi = 0 \]

For \( \kappa/g < 0 \), \( \langle \phi \rangle \neq 0 \), gauge symmetry broken, but \( D = 0 \) so supersymmetry preserved.

Write \( \kappa = -g\nu^2/2 \rightarrow \langle \phi \rangle = \nu/\sqrt{2} \) and \( \phi = (\nu + h)/\sqrt{2} \).

Massive scalar \( h \), Dirac fermion (combining \( \xi \) and \( \lambda \)) and massive \( A_\mu \) all have mass \( g\nu \) (massive vector supermultiplet)