Electroweak Theory: 3

- Introduction
- QED
- The Fermi theory
- The standard model
- Precision tests
- CP violation; $K$ and $B$ systems
- Higgs physics
- Prospectus
References

- Slides at [http://www.sns.ias.edu/~pgl/talks/](http://www.sns.ias.edu/~pgl/talks/) (subject to revision)
- **P. Langacker**, *The Standard Model and Beyond*, (CRC Press, 2010) (especially chapters 6, 7)
- **E.D. Commins and P.H. Bucksbaum**, *Weak Interactions of Leptons and Quarks*, (Cambridge, 1983)
- **P. Renton**, *Electroweak Interactions*, (Cambridge, 1990)
- *Precision Tests of the Standard Electroweak Model*, ed. **P. Langacker** (World, 1995) (especially Fetscher and Gerber; Herczog; Deutsch and Quin)
The Weak Interactions of Hadrons

- Semi-Leptonic Processes in the Fermi Theory
- $\pi$ and $K$ Decays, and the Strong Interactions
- $\beta$ Decay and Related Processes
- Charm ($c$) Quark and Third Family
- CKM Universality
This Lecture

- **The Standard Electroweak Model**
  - Beyond the Fermi Theory
  - Gauge Theories and the Standard Model
  - The Standard Electroweak Model
  - Spontaneous Symmetry Breaking and the Higgs Mechanism
  - The Lagrangian after Symmetry Breaking
Beyond the Fermi Theory

- Fermi theory successfully describes large number of WCC processes
- However, not renormalizable
  - Radiative corrections divergent (but needed)
  - Unitarity violated at high energy \( (E_{CM} \sim 1 \text{ TeV}) \)
  - Loop processes such as \( K^0 - \bar{K}^0 \) mixing meaningless/incorrect
- More complete theory needed

\[
\begin{align*}
\text{Diagram 1: } & e^- \rightarrow J^\mu \rightarrow J^\dagger_\mu \rightarrow \nu_e \rightarrow e^- \\
\text{Diagram 2: } & \nu_e \rightarrow e^- \rightarrow \nu_e \rightarrow e^- \\
\text{Diagram 3: } & s \rightarrow u \rightarrow u \rightarrow s \\
\text{Diagram 4: } & d \rightarrow \bar{s} \rightarrow \bar{s} \rightarrow d
\end{align*}
\]
Gauge Theories

Standard Model is remarkably successful gauge theory of the microscopic interactions

- Gauge symmetry $\Rightarrow$ (apparently) massless spin-1 (vector, gauge) bosons
- Interactions $\Leftrightarrow$ group, representations, gauge coupling
- Like QED ($U(1)$), but gauge self interactions for non-abelian
- Application to strong (short range) $\Rightarrow$ confinement
- Application to weak (short range) $\Rightarrow$ spontaneous symmetry breaking (Higgs or dynamical)
- Unique renormalizable field theory for spin-1
Non-Abelian

- $n$ non-interacting fermions of same mass $m$:

$$\left( i\gamma^{\mu} \frac{\partial}{\partial x^{\mu}} - m \right) \psi_a = 0, \quad a = 1 \cdots n,$$

invariant under (global) $SU(n)$ group,

$$\left( \begin{array}{c} \psi_1 \\ \vdots \\ \psi_n \end{array} \right) \rightarrow \exp\left( i \sum_{i=1}^{N} \beta^i L^i \right) \left( \begin{array}{c} \psi_1 \\ \vdots \\ \psi_n \end{array} \right).$$

$L^i$ are $n \times n$ generator matrices ($N = n^2 - 1$); $\beta^i$ are real parameters

$$[L^i, L^j] = i c_{ijk} L^k$$

($c_{ijk}$ are structure constants)
• **Gauge (local) transformation:** $\beta^i \rightarrow \beta^i(x) \Rightarrow$

$$\left( i \gamma^\mu \frac{\partial}{\partial x^\mu} \delta_{ab} - g \sum_{i=1}^N \gamma^\mu A^i_\mu L^i_{ab} - m \delta_{ab} \right) \psi_b = 0$$

• Invariant under

$$\Phi \equiv \left( \begin{array}{c} \psi_1 \\ \vdots \\ \psi_n \end{array} \right) \rightarrow \Phi' \equiv U \Phi$$

$$\vec{A}_\mu \cdot \vec{L} \rightarrow \vec{A}'_\mu \cdot \vec{L} \equiv U \vec{A}_\mu \cdot \vec{L} U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1}$$

$$U \equiv e^{i \vec{\beta} \cdot \vec{L}}$$

(1)
• Gauge invariance implies:
  
  - $N$ (apparently) massless gauge bosons $A^i_\mu$
  - Specified interactions (up to gauge coupling $g$, group, representations), including self interactions

  \[
  \psi_a \quad \leftrightarrow \quad -i g L^i_{a b} \gamma^\mu \\
  A^i_\mu \quad \leftrightarrow \quad g \quad \leftrightarrow \quad g^2
  \]

• Generalize to other groups, representations, chiral ($L \neq R$)
  
  - Chiral Projections: $\psi_{L(R)} \equiv \frac{1}{2} (1 \mp \gamma_5) \psi$ (independent fields)
    (Chirality = helicity up to $O(m/E)$)
The Standard Model

- **Gauge group** $SU(3) \times SU(2) \times U(1)$; gauge couplings $g_s, g, g'$

  \[
  \begin{pmatrix}
  u \\
  d
  \end{pmatrix}_L \quad \begin{pmatrix}
  u \\
  d
  \end{pmatrix}_L \quad \begin{pmatrix}
  u \\
  d
  \end{pmatrix}_L \quad \begin{pmatrix}
  \nu_e \\
  e^-
  \end{pmatrix}_L
  \]

  \[
  u_R \quad u_R \quad u_R \quad \nu_{eR}(?)
  \]

  \[
  d_R \quad d_R \quad d_R \quad e_R^-
  \]

  ($L =$ left-handed, $R =$ right-handed)

- **$SU(3)$**: $u \leftrightarrow u \leftrightarrow u, \quad d \leftrightarrow d \leftrightarrow d$ (8 gluons)

- **$SU(2)$**: $u_L \leftrightarrow d_L, \quad \nu_{eL} \leftrightarrow e_L^-$ ($W^\pm$); phases ($W^0$)

- **$U(1)$**: phases ($B$)

- **Heavy families** $(c, s, \nu_\mu, \mu^-), (t, b, \nu_\tau, \tau^-)$
The Standard Electroweak Model

\[ \mathcal{L}_{SU(2) \times U(1)} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{\text{Yukawa}} \]

Gauge part

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^i_{\mu \nu} F^{i\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} \]

Field strength tensors

\[
\begin{align*}
B_{\mu \nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
F^i_{\mu \nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g \epsilon_{ijk} W^j_\mu W^k_\nu, \quad i = 1 \cdots 3
\end{align*}
\]

\(g(g')\) is \(SU(2)\) (\(U(1)\)) gauge coupling; \(\epsilon_{ijk}\) is totally antisymmetric symbol

Three and four-point self-interactions for the \(W_i\)

\(B\) and \(W_3\) mix to form \(\gamma, \ Z\)
$U(1)$: $\Phi_j \rightarrow \exp(i g' y_j \beta) \Phi_j$, $y_j = q_j - t_j^3 = \text{weak hypercharge}$

Scalar part

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger D_\mu \phi - V(\phi)$$

where $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is the (complex) Higgs doublet with $y_\phi = 1/2$

Gauge covariant derivative:

$$D_\mu \phi = \left( \partial_\mu + ig \frac{\tau^i}{2} W^i_\mu + ig' \frac{1}{2} B_\mu \right) \phi$$

where $\tau^i$ are the Pauli matrices

Three and four-point interactions between gauge and scalar fields
Higgs potential

\[ V(\phi) = +\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \]

\[ \phi^\dagger \phi = \phi^+ + \phi^0 \]

Allowed by renormalizability and gauge invariance

Spontaneous symmetry breaking for \( \mu^2 < 0 \)

Vacuum stability: \( \lambda > 0 \)

Quartic self-interactions
Fermion part

\[ \mathcal{L}_f = \sum_{m=1}^{F} (\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL}) + (\bar{u}^0_{mR} i \not{D} u^0_{mR} + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR} + \bar{\nu}^0_{mR} i \not{D} \nu^0_{mR}) \]

*L-doublets*

\[ q^0_{mL} = \begin{pmatrix} u^0_m \\ d^0_m \end{pmatrix}_L \quad l^0_{mL} = \begin{pmatrix} \nu^0_m \\ e^{-0}_m \end{pmatrix}_L \]

*R-singlets*

\[ u^0_{mR}, \ d^0_{mR}, \ e^{-0}_m, \ \nu^0_{mR}(?) \]

\( F \geq 3 \) families; \( m = 1 \cdots F \) = family index;
\( ^0 \) = weak eigenstates (definite \( SU(2) \) rep.), mixtures of mass eigenstates (flavors);
quark color indices \( \alpha = r, \ g, \ b \) suppressed (e.g., \( u^0_{m\alpha L} \)).

Can add gauge singlet \( \nu^0_{mR} \) for Dirac neutrino mass term
Different (chiral) $L$ and $R$ representations lead to parity and charge conjugation violation (maximal for $SU(2)$)

Fermion mass terms forbidden by chiral symmetry

Triangle anomalies absent for chosen hypercharges and 3 colors
(includes quark-lepton cancellations) (exercize)
Gauge covariant derivatives

\[
D_{\mu}q^0_{mL} = \left( \partial_{\mu} + \frac{ig}{2}\tau^iW^i_{\mu} + \frac{g'}{6}B_{\mu} \right) q^0_{mL}
\]

\[
D_{\mu}l^0_{mL} = \left( \partial_{\mu} + \frac{ig}{2}\tau^iW^i_{\mu} - \frac{g'}{2}B_{\mu} \right) l^0_{mL}
\]

\[
D_{\mu}u^0_{mR} = \left( \partial_{\mu} + \frac{g'}{3}B_{\mu} \right) u^0_{mR}
\]

\[
D_{\mu}d^0_{mR} = \left( \partial_{\mu} - \frac{ig'}{3}B_{\mu} \right) d^0_{mR}
\]

\[
D_{\mu}e^0_{mR} = \left( \partial_{\mu} - ig'B_{\mu} \right) e^0_{mR}
\]

\[
D_{\mu}\nu^0_{mR} = \left( \partial_{\mu} \right) \nu^0_{mR}
\]

Read off \( W \) and \( B \) couplings to fermions

\[
-\frac{ig}{2}\tau^i\gamma_{\mu} \left( \frac{1-\gamma_5}{2} \right)
\]

\[
-ig'y\gamma_{\mu} \left( \frac{1+\gamma_5}{2} \right)
\]
Spontaneous Symmetry Breaking (Higgs mechanism)

Gauge invariance implies massless gauge bosons and fermions

Weak interactions short ranged \( \Rightarrow \) spontaneous symmetry breaking for mass; also for fermions

Allow classical (ground state) expectation value for Higgs field

\[ v = \langle 0 | \phi | 0 \rangle = \text{constant}, \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \]

\( \partial_\mu v \neq 0 \) increases energy, but important for monopoles, strings, domain walls, phase transitions (e.g., EWPT, baryogenesis)

Minimize \( V(v) \) to find \( v \) and quantize \( \phi' = \phi - v \)
$SU(2) \times U(1)$: introduce Hermitian basis

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix}$$

where $\phi_i = \phi_i^\dagger$.

$$V(\phi) = \frac{1}{2}\mu^2 \left( \sum_{i=1}^{4} \phi_i^2 \right) + \frac{1}{4}\lambda \left( \sum_{i=1}^{4} \phi_i^2 \right)^2$$

is $O(4)$ invariant.

w.l.o.g. choose $\langle 0|\phi_i|0 \rangle = 0$, $i = 1, 2, 4$ and $\langle 0|\phi_3|0 \rangle = \nu$

$$V(\phi) \rightarrow V(\nu) = \frac{1}{2}\mu^2\nu^2 + \frac{1}{4}\lambda\nu^4$$
For $\mu^2 < 0$, minimum at

$$V'(\nu) = \nu(\mu^2 + \lambda \nu^2) = 0$$

$$\Rightarrow \nu = (-\mu^2/\lambda)^{1/2}$$

SSB for $\mu^2 = 0$ also; must consider loop corrections

$$\phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \equiv v \Rightarrow \text{the generators } L^1, L^2, \text{ and } L^3 - Y \text{ spontaneously broken, } L^1 v \neq 0, \text{ etc (} L^i = \tau^i_2, \ Y = \frac{1}{2} I \)$$

$$Qv = (L^3 + Y)v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} v = 0 \Rightarrow U(1)_Q \text{ unbroken } \Rightarrow SU(2) \times U(1)_Y \rightarrow U(1)_Q$$
Rewrite Lagrangian in New Vacuum

Physical Higgs scalar (oscillations around minimum): \( M_H = \sqrt{2\lambda \nu} \)

Higgs covariant kinetic energy terms:

\[
(D_\mu \phi)^\dagger D^\mu \phi = \frac{1}{2}(0 \nu) \left[ \frac{g}{2} \tau^i W^i_\mu + \frac{g'}{2} B_\mu \right]^2 \begin{pmatrix} 0 \\ \nu \end{pmatrix} + H \text{ terms}
\]

\[
\rightarrow M_W^2 W^{+\mu} W_\mu^- + \frac{M_Z^2}{2} Z^\mu Z_\mu
\]

\[\text{+ } H \text{ kinetic energy and gauge interaction terms}\]

\[M_W = \frac{g\nu}{2}\]

\[m_e = \frac{h_e \nu}{\sqrt{2}}\]
Mass eigenstate bosons: \( W, Z, \) and \( A \) (photon)

\[
W^{\pm} = \frac{1}{\sqrt{2}} (W^1 \mp iW^2) \\
Z = -\sin \theta_W B + \cos \theta_W W^3 \\
A = \cos \theta_W B + \sin \theta_W W^3
\]

Weak angle: \( \tan \theta_W \equiv g'/g \)

Masses:

\[
M_W = \frac{g\nu}{2}, \quad M_Z = \sqrt{g^2 + g'^2} \frac{\nu}{2} = \frac{M_W}{\cos \theta_W}, \quad M_A = 0
\]

(Goldstone scalars “eaten” \( \rightarrow \) longitudinal components of \( W^{\pm}, Z \))
Will show: Fermi constant $G_F/\sqrt{2} \sim g^2/8M_W^2$

$(G_F = 1.166364(5) \times 10^{-5} \text{ GeV}^{-2} \text{ from muon lifetime})$

Electroweak scale:

$$\nu = 2M_W/g \sim (\sqrt{2}G_F)^{-1/2} \sim 246 \text{ GeV}$$

Will show: $g = e/\sin \theta_W$ ($\alpha = e^2/4\pi \sim 1/137.036$) \Rightarrow

$$M_W = M_Z \cos \theta_W = \frac{g\nu}{2} \sim \frac{(\pi\alpha/\sqrt{2}G_F)^{1/2}}{\sin \theta_W}$$

Weak neutral current: $\sin^2 \theta_W \sim 0.23 \Rightarrow M_W \sim 78 \text{ GeV}$, and $M_Z \sim 89 \text{ GeV}$ (increased by $\sim 2$ GeV by loop corrections)

Discovered at CERN: UA1 and UA2, 1983
**Fermion Masses and Mixings**

- **Yukawa (Higgs-fermion) interaction** (+ analogous $u, e, \nu$ terms)

\[
-\mathcal{L}_{Yuk} = \sum_{m,n=1}^{F} \Gamma_{mn}^{d} \bar{q}_{mL}^{0} \phi d_{nR}^{0} + h.c.
\]

\[
= \sum_{m,n=1}^{F} \Gamma_{mn}^{d} \left[ \bar{u}_{mL}^{0} \phi^{+} d_{nR}^{0} + \bar{d}_{mL}^{0} \phi^{0} d_{nR}^{0} \right] + h.c.
\]
• For $\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$ (unitary gauge)

$$-\mathcal{L}_{Yuk} \Rightarrow \sum_{m,n=1}^{F} M_{mn}^{d} \bar{d}_{mn}^{L}d_{mn}^{R}\left(1 + \frac{H}{\nu}\right) + h.c.$$ 

$$= \sum_{i} m_{i} \bar{d}_{i}^{L}d_{i}^{R}\left(1 + \frac{H}{\nu}\right) + h.c. = \sum_{i} m_{i} \bar{d}_{i}^{L}d_{i}^{R}\left(1 + \frac{H}{\nu}\right)$$

with $M^{d} \equiv \Gamma^{d} \nu / \sqrt{2}$

• $d_{i} = d_{iL} + d_{iR}$ are mass eigenstates of mass $m_{i}$

• For $F = 3$

$$A_{L}^{d\dagger}M^{d}A_{R}^{d} = \begin{pmatrix} m_{d} & 0 & 0 \\ 0 & m_{s} & 0 \\ 0 & 0 & m_{b} \end{pmatrix}, \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} = A_{L,R}^{d,\text{unitary}} \begin{pmatrix} d_{1}^{0} \\ d_{2}^{0} \\ d_{3}^{0} \end{pmatrix}_{L,R}$$

• Higgs ($H$) couplings to fermions are diagonal in flavor and $\propto$ mass
Fermi Theory incorporated in SM and made renormalizable

\[ W \text{-fermion interaction} \]

\[ \mathcal{L} = -\frac{g}{2\sqrt{2}} \left( J_{W}^{\mu} W_{\mu}^{-} + J_{W}^{\mu \dagger} W_{\mu}^{+} \right) \]

**Charge-raising current** (ignoring \( \nu \) masses)

\[ J_{W}^{\mu \dagger} = \sum_{m=1}^{F} \left[ \bar{\nu}_{m}^{0} \gamma^{\mu} (1 - \gamma^{5}) e_{m}^{0} + \bar{u}_{m}^{0} \gamma^{\mu} (1 - \gamma^{5}) d_{m}^{0} \right] \]

\[ = (\bar{\nu}_{e} \bar{\nu}_{\mu} \bar{\nu}_{\tau}) \gamma^{\mu} (1 - \gamma^{5}) \begin{pmatrix} e^{-} \\ \mu^{-} \\ \tau^{-} \end{pmatrix} + (\bar{u} \bar{c} \bar{t}) \gamma^{\mu} (1 - \gamma^{5}) V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \]
Ignore $\nu$ masses for now

Pure $V - A \Rightarrow$ maximal $P$ and $C$ violation; $CP$ conserved except for phases in $V$

$V = A_L^{u\dagger} A_L^d$ is $F \times F$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix from mismatch between weak and Yukawa interactions

Cabibbo matrix for $F = 2$

$$V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}$$

$\sin \theta_c \approx 0.23 \equiv$ Cabibbo angle

Good zeroth-order description since third family almost decouples

General unitary $2 \times 2$: 1 angle and 3 (unobservable) $q_L$ phases
CKM matrix for $F = 3$ involves 3 angles and 1 $CP$-violating phase (after removing unobservable $q_L$ phases) (new interactions involving $q_R$ could make observable)

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{td} & V_{td}
\end{pmatrix}
\]

Extensive studies, especially in $B$ decays, to test unitarity of $V$ as probe of new physics and test origin of $CP$ violation

Need additional source of $CP$ breaking for baryogenesis
Effective zero-range 4-fermi interaction (Fermi theory)

For $|Q| \ll M_W$, neglect $Q^2$ in $W$ propagator

$$-L_{cc}^{\text{eff}} = \left( \frac{g}{2\sqrt{2}} \right)^2 J^\mu_W \left( \frac{-g_{\mu\nu}}{Q^2 - M_W^2} \right) J^{\dagger \nu}_W \sim \frac{g^2}{8M_W^2} J^\mu_W J^{\dagger \mu}_W$$

Fermi constant: $\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{8M_W^2} = \frac{1}{2\nu^2}$

Muon lifetime: $\tau^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} \Rightarrow G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$

Weak scale: $\nu = \sqrt{2}\langle 0|\phi^0|0\rangle \simeq 246 \text{ GeV}$

Excellent description of $\beta$, $K$, hyperon, heavy quark, $\mu$, and $\tau$ decays, $\nu_\mu e \rightarrow \mu^- \nu_e$, $\nu_\mu n \rightarrow \mu^- p$, $\nu_\mu N \rightarrow \mu^- X$
Full theory probed:

\[ e^\pm p \rightarrow (\bar{\nu}_e X \text{ at high energy (HERA)}) \]

Electroweak radiative corrections (loop level)

(Very important. Only calculable in full theory.)

\[ M_{K_S} - M_{K_L}, \text{ kaon } CP \text{ violation, } B \leftrightarrow \bar{B} \text{ mixing (loop level)} \]

(CKMFITTER group:

http://ckmfitter.in2p3.fr/)
Quantum Electrodynamics (QED)

Incorporated into standard model

Lagrangian:

\[ \mathcal{L} = -\frac{gg'}{\sqrt{g^2 + g'^2}} J_Q^\mu (\cos \theta_W B_\mu + \sin \theta_W W^3_\mu) \]

Photon field:

\[ A_\mu = \cos \theta_W B_\mu + \sin \theta_W W^3_\mu \]

Positron electric charge: \( e = g \sin \theta_W \), where \( \tan \theta_W \equiv g'/g \)
Electromagnetic current:

\[ J_Q^\mu = \sum_{m=1}^{F} \left[ \frac{2}{3} \bar{u}_m^0 \gamma^\mu u_m^0 - \frac{1}{3} \bar{d}_m^0 \gamma^\mu d_m^0 - \bar{e}_m^0 \gamma^\mu e_m^0 \right] \]

Electric charge: \( Q = T^3 + Y \), where \( Y = \) weak hypercharge

(coefficient of \( ig'B_\mu \) in covariant derivatives)

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

Purely vector (parity conserving): \( L \) and \( R \) fields have same charge

\( q_i = t_i^3 + y_i \) is the same for \( L \) and \( R \) fields, even though \( t_i^3 \) and \( y_i \) are not
The Weak Neutral Current

Prediction of $SU(2) \times U(1)$

$$\mathcal{L} = -\frac{\sqrt{g^2 + g'^2}}{2} J_Z^\mu \left( -\sin \theta_W B_\mu + \cos \theta_W W^3_\mu \right)$$

$$= -\frac{g}{2 \cos \theta_W} J_Z^\mu Z_\mu$$

Neutral current process and effective 4-fermi interaction for $|Q| \ll M_Z$
Neutral current:

\[ J^\mu_Z = \sum_m \left[ \bar{u}^0_{mL} \gamma^\mu u^0_{mL} - \bar{d}^0_{mL} \gamma^\mu d^0_{mL} + \bar{\nu}^0_{mL} \gamma^\mu \nu^0_{mL} - \bar{e}^0_{mL} \gamma^\mu e^0_{mL} \right] \]

\[ -2 \sin^2 \theta_W J^\mu_Q \]

\[ = \sum_m \left[ \bar{u}_{mL} \gamma^\mu u_{mL} - \bar{d}_{mL} \gamma^\mu d_{mL} + \bar{\nu}_{mL} \gamma^\mu \nu_{mL} - \bar{e}_{mL} \gamma^\mu e_{mL} \right] \]

\[ -2 \sin^2 \theta_W J^\mu_Q \]

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

GIM mechanism: \( c \) quark predicted so that \( s_L \) could be in doublet to avoid unwanted flavor changing neutral currents (FCNC) at tree and loop level

Parity and charge conjugation violated but not maximally: first term is pure \( V - A \), second is \( V \)
Effective 4-fermi interaction for $|Q^2| \ll M_Z^2$:

$$-\mathcal{L}_{NC}^{eff} = \frac{G_F}{\sqrt{2}} J_{Z}^{\mu} J_{Z\mu}$$

Coefficient same as WCC because

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{g^2 + g'^2}{8 M_Z^2}$$
10. Electroweak model and constraints on new physics

Figure 10.1: Scale dependence of the weak mixing angle defined in the MS scheme [129] (for the scale dependence of the weak mixing angle defined in a mass-dependent renormalization scheme, see Ref. 126). The minimum of the curve corresponds to $Q = M_W$, below which we switch to an effective theory with the $W^\pm$ bosons integrated out, and where the β-function for the weak mixing angle changes sign. At the location of the $W^\pm$ boson mass and each fermion mass, there are also discontinuities arising from scheme dependent matching terms which are necessary to ensure that the various effective field theories within a given loop order describe the same physics. However, in the MS scheme these are very small numerically and barely visible in the figure provided one decouples quarks at $Q = \hat{m}_q(\hat{m}_q)$. The width of the curve reflects the theory uncertainty from strong interaction effects which at low energies is at the level of $\pm 7 \times 10^{-5}$ [129]. Following the estimate [130] of the typical momentum transfer for parity violation experiments in Cs, the location of the APV data point is given by $\mu = 2.4 \text{ MeV}$. For $\nu$-DIS we chose $\mu = 20 \text{ GeV}$ which is about half-way between the averages of $\sqrt{Q^2}$ for $\nu$ and $\bar{\nu}$ interactions at NuTeV. The Tevatron measurements are strongly dominated by invariant masses of the final state dilepton pair of $O(M_Z)$ and can thus be considered as additional $Z$ pole data points, yielding $\bar{s}_{2Z} = 0.2316 \pm 0.0018$. However, for clarity we displayed the point horizontally to the right. E.g., $Q_W(\text{Cs})$ is extracted by measuring experimentally the ratio of the parity violating amplitude, $E_{\text{PNC}}$, to the Stark vector transition polarizability, $\beta$, and by calculating theoretically $E_{\text{PNC}}$ in terms of $Q_W$. One can then write, $Q_W = N \left( \text{Im} E_{\text{PNC}} \beta \right) \exp \left( |e| a_B \text{Im} E_{\text{PNC}} Q_W N \right)$ th + $\exp \left( a^2_B |e| \right)$ th.
The $Z$, the $W$, and the Weak Neutral Current

- Primary prediction and test of electroweak unification
- WNC discovered 1973 (Gargamelle at CERN, HPW at FNAL)
- 70’s, 80’s: weak neutral current experiments (few %)
  - Pure weak: $\nu N$, $\nu e$ scattering
  - Weak-elm interference in $eD$, $e^+e^-$, atomic parity violation
  - Model independent analyses ($\nu e$, $\nu q$, $eq$)
  - $SU(2) \times U(1)$ group/representations; $t$ and $\nu_\tau$ exist; $m_t$ limit; hint for SUSY unification; limits on TeV scale physics
- $W$, $Z$ discovered directly 1983 (UA1, UA2)
- 90's: $Z$ pole (LEP, SLD), 0.1%; lineshape, modes, asymmetries
- LEP 2: $M_W$, Higgs search, gauge self-interactions
- Tevatron: $m_t$, $M_W$, Higgs search
- 4th generation weak neutral current experiments (atomic parity (Boulder); $\nu_e; \nu N$ (NuTeV); polarized Møller asymmetry (SLAC))
• SM correct and unique to zeroth approx. (gauge principle, group, representations)

• SM correct at loop level (renorm gauge theory; $m_t, \alpha_s, M_H$)

• TeV physics severely constrained (unification vs compositeness)

• Consistent with light elementary Higgs

• Precise gauge couplings (SUSY gauge unification)