Massive Neutrinos and (Heterotic) String Theory

- Introduction
- Neutrino preliminaries
- The GUT seesaw
- A string primer
- The $Z_3$ heterotic orbifold

(In collaboration with J. Giedt, G. Kane, B. Nelson.)
Neutrino mass

- Nonzero mass may be first break with standard model
- Enormous theoretical effort: GUT, family symmetries, bottom up
  - Majorana masses may be favored because not forbidden by SM gauge symmetries
  - GUT seesaw (heavy Majorana singlet). Usually ordinary hierarchy.
  - Higgs triplets ("type II seesaw"), often assuming GUT, Left-Right relations
• Very little work from string constructions, even though probably Planck scale


- J. Giedt, G. Kane, PL, B. Nelson, this work. (Systematic study of heterotic \( Z_3 \) orbifolds.)
• Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane) (Due to string symmetries or constraints, not simplicity or elegance)

  – “Right-handed” neutrinos may not be gauge singlets
  – Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
  – GUT Yukawa relations broken
  – String symmetries/constraints severely restrict couplings, e.g., Majorana masses, or simultaneous Dirac and Majorana masses
  – Small Dirac masses from HDO, extended (TeV-scale) seesaw, or triplet seesaw (with inverted hierarchy) may be more likely
• Weyl fermion
  - Minimal (two-component) fermionic degree of freedom
  - $\psi_L \leftrightarrow \psi_R^c$ by CPT

• Active Neutrino (a.k.a. ordinary, doublet)
  - in $SU(2)$ doublet with charged lepton $\rightarrow$ normal weak interactions
  - $\nu_L \leftrightarrow \nu_R^c$ by CPT

• Sterile Neutrino (a.k.a. singlet, right-handed)
  - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
  - $N_R \leftrightarrow N_L^c$ by CPT
  - Almost always present: Are they light? Do they mix?
- **Dirac Mass**

  - Connects distinct Weyl spinors (usually active to sterile): 
    \[ (m_D \tilde{\nu}_L N_R + h.c.) \]
  - 4 components, \( \Delta L = 0 \)
  - \( \Delta I = \frac{1}{2} \rightarrow \) Higgs doublet
  - Why small? HDO? LED?
  - Variant: couple active to anti-active, e.g., \( m_D \tilde{\nu}_e L \nu^c_{\mu R} \Rightarrow L_e - L_\mu \) conserved; \( \Delta I = 1 \)

\[ \nu_L \begin{array}{c} v = \langle \phi \rangle \end{array} \begin{array}{c} h \end{array} \begin{array}{c} \ldots \ldots \circ \end{array} \begin{array}{c} N_R \end{array} \begin{array}{c} m_D = hv \end{array} \]
• **Majorana Mass**

  - Connects Weyl spinor with itself:
    \[ \frac{1}{2}(m_T \bar{\nu}_L \nu^c_R + h.c.) \text{ (active)}; \]
    \[ \frac{1}{2}(m_S \bar{N}^c_L N_R + h.c.) \text{ (sterile)} \]
  - 2 components, \( \Delta L = \pm 2 \)
  - Active: \( \Delta I = 1 \rightarrow \text{triplet or seesaw} \)
  - Sterile: \( \Delta I = 0 \rightarrow \text{singlet or bare mass} \)

• **Mixed Masses**

  - Majorana and Dirac mass terms
  - Seesaw for \( m_S \gg m_D \)
  - Ordinary-sterile mixing for \( m_S \) and \( m_D \) both small and comparable (or \( m_S \ll m_d \) (pseudo-Dirac))
3 $\nu$ Patterns

- **Solar:** LMA (SNO, Kamland)

- $\Delta m^2_{\odot} \sim 8 \times 10^{-5}$ eV$^2$, nonmaximal

- **Atmospheric:**
  \[ |\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3} \text{ eV}^2, \text{ near-maximal mixing} \]

- **Reactor:** $U_{e3}$ small
− Mixings: let $\nu_{\pm} \equiv \frac{1}{\sqrt{2}} (\nu_\mu \pm \nu_\tau)$:

\[
\begin{align*}
\nu_3 & \sim \nu_+ \\
\nu_2 & \sim \cos \theta \nu_- - \sin \theta \nu_e \\
\nu_1 & \sim \sin \theta \nu_- + \cos \theta \nu_e
\end{align*}
\]

− Hierarchical pattern

* Analogous to quarks, charged leptons

* $\beta\beta_{0\nu}$ rate very small

− Inverted quasi-degenerate pattern

* $\beta\beta_{0\nu}$ if Majorana

* SN1987A energetics (if $U_{e3} \neq 0$)?

* May be radiative unstable
– Degenerate patterns
  * Motivated by CHDM (no longer needed)
  * Strong cancellations needed for $\beta\beta_{0\nu}$ if Majorana
  * May be radiative unstable
4 $\nu$ Patterns

- LSND: $\Delta m^2_{\text{LSND}} \sim 1 \text{ eV}^2$
- $Z$ lineshape: $2.986(7)$ active $\nu$'s lighter than $M_Z/2 \rightarrow$ fourth sterile $\nu_S$

- $2 + 2$ patterns
- $3 + 1$ patterns

$2 + 2 \quad 3 + 1$

- Pure $(\nu_\mu - \nu_s)$ excluded for atmospheric by SuperK, MACRO
- Pure $(\nu_e - \nu_s)$ excluded for solar by SNO, SuperK
- More general admixtures possible, but very poor global fits
- Additional sterile (e.g., $3 + 2$) fit better but may have cosmological difficulties
The minimal seesaw

- Active (sterile) neutrinos $\nu_L (\bar{N}_R)$ (3 flavors each)

\[
L = \frac{1}{2} (\bar{\nu}_L \bar{N}_{L}^c) \begin{pmatrix} m_T & m_D \\ m_D^* & m_S \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \bar{N}_R \end{pmatrix} + \text{hc}
\]

- $m_T = m_{T}^T$ = triplet Majorana mass matrix (Higgs triplet)
- $m_D$ = Dirac mass matrix (Higgs doublet)
- $m_S = m_{S}^T$ = singlet Majorana mass matrix (Higgs singlet)
• Ordinary (type I) seesaw: $m_T = 0$ and (eigenvalues) $m_S \gg m_D$:

$$m_{\nu}^{\text{eff}} = -m_D m_S^{-1} m_D$$

with

$$U_{PMNS} = U_e^\dagger U_\nu$$
Implementation in GUTs

- Elegant mechanism for small Majorana masses
- Leptogenesis
- Expect small mixings in simplest versions (can evade by lopsided $e/d$, Majorana textures, etc.)
- Most models require large Higgs representations, e.g., 126 of $SO(10)$ (alternative: higher dimensional operators)
- Large Majorana often forbidden, e.g., by extra $U(1)$’s
- LSND: active-sterile difficult in simple versions
Neutrinos in string constructions

Key ingredients of most GUT/bottom up models forbidden or different in known constructions (heterotic or intersecting brane)

- Bifundamentals, singlets, or adjoints; not large representations
- $L$ may be conserved, or extra $U(1)'$ charge for $N_R$
- String constraints may forbid couplings allowed by 4d symmetries
- Superpotential terms leading to Majorana masses, or diagonal (same family or same flavor) Majorana usually absent
- GUT Yukawa relations broken
- Non-zero superpotential terms may be equal (gauge couplings)
- Hierarchies from HDO (heterotic), intersection triangles (intersecting brane)
Dirac masses

- Can achieve small Dirac masses (neutrino or other) by higher dimensional operators or by large intersection areas

\[ L_\nu \sim \left( \frac{S}{M_{Pl}} \right)^p LN^c_L H_2, \quad \langle S \rangle \ll M_{Pl} \]

\[ \Rightarrow m_D \sim \left( \frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle \]

- Large \( p \Rightarrow \langle S \rangle \) close to \( M_{Pl} \) (e.g., anomalous \( U(1)_A \))

- Small \( p \Rightarrow \) intermediate scale \( \ll M_{Pl} \)

- Similar HDO may give light steriles and ordinary/sterile mixing
Can one generate large effective $m_S$ from

$$W_\nu \sim c_{ij} \frac{S^{q+1}}{M_{Pl}^q} N_i N_j \Rightarrow (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{q+1}}{M_{Pl}^q},$$

consistent with $D$ and $F$ flatness?

Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints?

Are bottom-up model assumptions for relations to quark, charged lepton masses maintained?
No completely realistic constructions

Existing constructions usually focus on quark sector
- Neutrino masses rarely considered, and then as afterthought
- No construction has yielded GUT-like seesaw

Analyze $Z_3$ heterotic orbifolds (semi-realistic 3-family models) in detail, focussing on neutrino sector (Joel Giedt, G. Kane, PL, Brent Nelson)

Large number of possible vacua:
- Is the minimal seesaw generic?
- Is some subclass of vacua favored?
- Any clue about hierarchies, mixings, etc?
A String Primer

- Two classes of quasi-realistic: intersecting D-brane, heterotic

- Intersecting D-brane
  - Closed strings (gravitons) and open strings ending on D-branes
  - D6-branes: fill ordinary space and 3 of the 6 extra dimensions
  - Stringy implementation of “brane world” ideas
– Gauge interactions from strings beginning/ending on stack of parallel branes (one for each group factor)
– Chiral matter: strings at intersection of branes, e.g., $SU(N) \times SU(M) \rightarrow$ bifundamental $(N, \bar{M})$
– Family replication from multiple intersections on compactified geometry
– Yukawa interactions \(\sim \exp(-A_{ijk})\) → hierarchies
– Existing models: conserved \(L\); no diagonal (Majorana) triangles
The $Z_3$ Heterotic Orbifold

- $E_8 \times E_8$ closed strings
  $SU(3) \times SU(2) \times U(1)^5$ hidden

- Three families automatic

- Tremendous symmetry, stringy selection rules $\rightarrow$ restricted couplings

- Chiral fermions, $N = 1$ supersymmetry $\rightarrow$ orbifold

- Anomalous $U(1)_A$; $F$ and $D$ flatness; vacuum, restabilization

Madison (March 11, 2005)  Paul Langacker (Penn)
The $\mathbb{Z}_3$ Orbifold

- Compactify on three two-tori $T^2$
• $n = 0$ strings closed on $R^2$; winding states $n = 1, 2$ closed on $T^2$ (not $R^2$)

• $Z_2$ orbifold: identify $\vec{x}$ and $-\vec{x} \rightarrow$ two fixed points; twisted states closed on orbifold
- $Z_3$ orbifold: identify $2\pi/3$ rotations $\rightarrow$ three fixed points
Anomalous $U(1)_A$; $F$ and $D$ flatness; vacuum restabilization

- One linear combination of the $U(1)^5$ may be anomalous
- Green-Schwarz mechanism cancels anomaly in 4d

$$\begin{align*}
\text{SU}(N) 
\uparrow & \text{SU}(N) \\
\text{U}(1) & \text{SU}(N) \\
\text{SU}(N) & \text{U}(1) \\
\uparrow & \text{SU}(N) \\
\text{SU}(N) & \text{SU}(N) \\
\text{B}_2 & = 0
\end{align*}$$

- Fayet-Iliopoulos term added to the $D$-term of $U(1)_A$

$$\xi_{FI} = \frac{g^2_{\text{STR}} \text{Tr} \ Q^A}{192\pi^2} M_{\text{PL}}^2$$

Madison (March 11, 2005) 
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Supersymmetry is restored when certain scalar fields acquire VEV’s such that $D$- and $F$ flatness conditions are satisfied:

$$D_A \equiv \sum_i Q_i^{(A)} |S_i|^2 + \xi_{FI} = 0$$

$$D_a \equiv \sum_i Q_i^{(a)} |S_i|^2 = 0$$

$$F_i \equiv \frac{\partial W}{\partial S_i} = 0; \quad W = 0$$

VEVs $|S_i|$ reduce gauge symmetries, give masses (restabilization)

Other $S_i$ VEVs can acquire intermediate scale masses by radiative breaking
Search for Minimal Seesaw

- Look for structure in $Z_3$ heterotic orbifold:

$$W_{\text{eff}} = (\nu_i, N_i) \begin{pmatrix} 0 & (m_D)_{ij} \\ (m_D)_{ji} & (m_M)_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j \end{pmatrix}$$

- Require simultaneous Majorana and Dirac couplings, and appropriate hypercharge

- *Don’t* insist on realistic quark sector

- Majorana mass from $\langle S_1 \cdots S_{n-2} \rangle N N / M_{\text{PL}}^{n-3}$

- Dirac mass from $\langle S'_1 \cdots S'_{d-3} \rangle N L H_u / M_{\text{PL}}^{d-3}$

- Only 5 embeddings into $E_8 \times E_8$, 4 realistic hidden sector groups

  $\rightarrow$ 175 models in 20 patterns with same $\xi_{\text{FI}}$ (Giedt)
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• Classified superpotential terms of degree \( \leq 9 \)

• Large number \((O(50))\) fields in each, \(\sim\) half are SM singlets

• None are singlets under all \(U(1)\)’s

• Huge number of terms, but small wrt number of fields due to symmetries/selection rules

• \( r_{FI} = \sqrt{|\xi_{FI}|}/M_{PL} \sim \langle S_i \rangle/M_{PL} \)
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• Require $F$ and $D$ flatness

• Examined 3 models from each pattern (conjecture: all models in pattern equivalent)

• Studied subset of flat directions with 1d $D$ flatness and minimal $F$-flatness (more general directions very complicated)

• Huge number of $D$-flat directions, reduced drastically by $F$-flatness
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<tr>
<td>4.7</td>
<td>5285</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>4.8</td>
<td>49</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
• For each surviving direction, looked for candidate Majorana mass terms \( \langle S_1 \cdots S_{n-2} \rangle_{NN} \), where the \( \langle S_i \rangle \neq 0 \) for that direction

• Only two patterns out of 20 (2.6 and 1.1) have candidate Majorana mass terms

• Must still check:
  – Is there a surviving hypercharge \( Y \) with \( Y_N = 0 \)?
  – Are there candidate Dirac couplings \( \langle S'_1 \cdots S'_{d-3} \rangle_{NLH_u} \) at low enough order?
  – Do \( L, H \), and quark candidates have correct \( Y \)?
Six directions have Majorana mass terms of form

\[
\begin{align*}
\text{I - monomial:} & \quad (4, 4, 6, 7, 18, 35, 43, 43), \\
\text{Eff. Maj. mass:} & \quad (4, 5, 5)
\end{align*}
\]

- Numbers refer to a classification of the chiral matter superfields
- I-monomial lists $S_i$ fields with VEVs (of order $r_{\text{FI}} M_{\text{PL}} \sim 0.1 M_{\text{PL}}$)
- Underlined fields are the $S_i$, others ($N_5$) are Majorana neutrinos
- Family indices suppressed

Madison (March 11, 2005)            Paul Langacker (Penn)
• However, no Dirac couplings involving $N_5$ through degree $d \leq 6$, i.e., none through order $S'^{d-3}N_5LH_u$

• Light seesaw masses would be of order

$$m_\nu \sim \left( \frac{r_{FI}^{d-3}v_u}{r_{FI}M_{PL}} \right)^2 \sim r_{FI}^{2d-7} \times 10^{-5} \text{ eV} \underbrace{\Rightarrow}_{d>6} < 10^{-10} \text{ eV}$$

• Also eight directions of form

$$\begin{align*}
I \text{ – monomial} &: (4, 4, 7, 18, 19, 27, 43, 43), \\
\text{Eff. Maj. mass} &: (7, 7, 19, 27, 43, 43, 43, 34, 34)
\end{align*}$$

• However, no Dirac couplings of degree $< 9 \Rightarrow m_\nu \leq 10^{-10} \text{ eV}
Pattern 1.1

- No anomalous $U(1)_A$; VEVs may still be determined, e.g., by radiative breaking of non-anomalous, typically at intermediate scale

- Two classes of directions with Majorana masses, but first has no Dirac couplings through (needed) degree 6. Second class promising:

  \[ I \text{ – monomial} : \quad (3, 3, 8, 21, 22, 29, 46, 72), \]
  \[ \text{Eff. Maj. mass} : \quad (8, 22, 46, 72, 9, 9) \]

- There is also a candidate Dirac mass: $N_9L_{36}L_{64}$, where $L_{36}$, $L_{64}$ are two $SU(2)$ doublets
• Can define appropriate hypercharge for all fields \( \rightarrow L_{36} = L, L_{64} = H_u \) (family indices suppressed)
  – A second set of Majorana and Dirac couplings of higher degree also present (not shown)
  – No realistic quark Yukawas (and no GUT-type relations)
  – Undesired doubling of leptons and Higgs

• Apparently, we have found an example of a seesaw, even if not fully realistic!

• We were about to study family structure (scale, hierarchy, mixings)
The Fatal Flaw

- The same direction has degree 3 mass terms coupling \( N_9 \) to other fields \( \tilde{N} \):

\[
W_{\text{mix}} = \lambda S_8 N_9 \tilde{N}_{14} + \lambda S_{22} N_9 \tilde{N}_{27} + \lambda S_{72} N_9 \tilde{N}_{50} + \lambda S_{46} N_9 \tilde{N}_{81}
\]

\[
L = (\nu_L \quad \tilde{N} \quad N) \begin{pmatrix} 0 & 0 & A \\ 0 & 0 & B \\ A & B & C \end{pmatrix} \begin{pmatrix} \nu_L \\ \tilde{N} \\ N \end{pmatrix},
\]

with \( B \gg C \gg A \)

- Three massless and six supermassive neutrinos! (no additional terms generated to needed order)

- This could also occur for other apparent seesaws
Outlook

- Neutrino mass likely due to large or Planck scale effects, but little previous work in string context

- No viable examples of minimal seesaw in huge class of $Z_3$ orbifold vacua
  - Could consider more general vacua (two independent VEVs, cancellations of $F$ terms)
  - Other types of orbifolds and heterotic constructions? Will also have strong gauge and stringy constraints. ($L$ conserved in existing intersecting brane)

- Even if a few examples are found, they don’t appear generic
Consider alternatives seriously

- Small Dirac masses from high degree terms (very common in constructions) (could also give light sterile $\nu$'s and mixing)

- Extended seesaws, $m_\nu \sim m_D^{2+k}/M^{1+k}$, with $k \geq 1$ and low (e.g., TeV) scale $M$

- Higgs triplet models: non-trivial to embed in strings (higher level), but very predictive (e.g., inverted hierarchy with nearly bi-maximal mixing) (B. Nelson, PL)
Extended (TeV) Seesaw?

- \( m_\nu \sim m_p^{p+1}/m_p^p, \quad p > 1 \) (e.g., \( m \sim 100 \) MeV, \( m_S \sim 1 \) TeV for \( p = 2 \))

- \( \nu_L, N_R, N'_R \) (3 flavors each)

\[
L = \frac{1}{2} (\bar{\nu}_L \, \bar{N}^c_L \, \bar{N}'^c_L) \begin{pmatrix}
  0 & m_D & m_{D'} \\
  m_T^D & 0 & m_{S_{SS}'} \\
  m_T^{D'} & m_{S_{SS}'} & 0
\end{pmatrix} \begin{pmatrix}
  \nu^c_R \\
  N_R \\
  N'_R
\end{pmatrix} + hc
\]

or

\[
L = \frac{1}{2} (\bar{\nu}_L \, \bar{N}^c_L \, \bar{N}'^c_L) \begin{pmatrix}
  0 & m_D & 0 \\
  m_T^D & 0 & m_{S_{SS}'} \\
  0 & m_{S_{SS}'} & m'_S
\end{pmatrix} \begin{pmatrix}
  \nu^c_R \\
  N_R \\
  N'_R
\end{pmatrix} + hc
\]

(Faraggi et al.: may occur in specific heterotic model, with dynamical assumptions.)
Triplet models

- Introduce Higgs triplet $T = (T^{++} \ T^+ \ T^0)^T$ with weak hypercharge $Y = 1$

- Majorana masses $m_T$ generated from $L_\nu = \lambda_{ij}^T L_i T L_j$ if $\langle T^0 \rangle \neq 0$

- Old Gelmini-Roncadelli model: $\langle T^0 \rangle \ll$ EW scale with spontaneous $L$ violation
  - Excluded by $Z \rightarrow$ Majoron + scalar (equivalent to $\Delta N_\nu = 2$)

- Modern triplet models (type II seesaw) break $L$ explicity by $THH$ couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)

- Often considered in $SO(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.
- General SUSY case

\[
W_\nu = \lambda_{ij}^T L_i T L_j + \lambda_1 H_1 T H_1 + \lambda_2 H_2 \bar{T} H_2 \\
+ M_T T \bar{T} + \mu H_1 H_2
\]

\(T, \bar{T}\) are triplets with \(Y = \pm 1\), \(M_T \sim 10^{12} - 10^{14}\) GeV. Typically,

\[
\langle T^0 \rangle \sim -\lambda \langle H_2^0 \rangle^2 / m_T \quad \Rightarrow
\]

\[
m_{ij}^{\nu} = -\lambda_{ij}^T \lambda_2 \frac{v_2^2}{M_T}
\]
String constructions

- Expect $\lambda_{ij}^T = 0$ for $i = j$ (off-diagonal) $\Rightarrow m_{ii}^\nu = 0$

- Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal

- Partial explanation: $SU(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $SU(2) \subset SU(2) \times SU(2)$ (Have $Z_3$ constructions with some but not all of the features.)

\[ W \sim \lambda_{1j}^T L_1(2, 1) T(2, 2) L_j(1, 2), \ j = 2, 3 \]

yields

\[ m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \]

- Typical string case: $|a| = |b|$
• HDO (or $SU(2) \subset SU(2) \times SU(2) \times SU(2)$) can give $m_{23}^\nu \neq 0$

• For

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

can take $a, b, c$ real w.l.o.g. by redefinition of fields (not true for general $m^\nu$)

• Tr $m^\nu = 0$ and $m^\nu = m^\nu^\dagger \Rightarrow m_1 + m_2 + m_3 = 0$
\[ |\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3} \text{ eV}^2, \Delta m^2_\odot \sim 8 \times 10^{-5} \text{ eV}^2 \Rightarrow \text{two solutions} \]

- For \( \Delta m^2_\odot = 0 \)
  
  (a) \( m_i \propto 1, -\frac{1}{2}, -\frac{1}{2} \) (ordinary, with shifted masses)
  
  (b) \( m_i \propto 1, -1, 0 \) (inverted)

- With \( \Delta m^2_\odot \neq 0 \)
  
  (a) \( m_i = 0.054, -0.026, -0.026 \) eV (\( \sum |m_i| = 0.107 \) eV (cosmology))
  
  (b) \( m_i = 0.046, -0.045, -0.001 \) eV (\( \sum |m_i| = 0.092 \) eV (cosmology))

\[
m^\nu_a \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad m^\nu_b \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}
\]

- (a) leads to unrealistic mixing matrix \( \Rightarrow \) consider (b)
A special texture

- The $L_e - L_\mu - L_\tau$ conserving texture

\[ m^{\nu} \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \]

has been considered phenomenologically by many authors (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)
\[ m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \]

- **New aspects**
  - Strong string motivation
  - Motivation for special case \(|a| = |b|\)
  - Most likely perturbation in 23 element from HOT

- **Diagonalization:** \(\tan \theta_{\text{Atm}} = b/a \Rightarrow \text{need } |b| = |a| \text{ for maximal}\)

- \(\tan^2 \theta_\odot = 1 \text{ (maximal)}\) (experiment \(\tan^2 \theta_\odot = 0.40^{+0.09}_{-0.07}\))
• **Majorana mass matrix**

\[ m^\nu \sim \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \]

• **Inverted hierarchy**

• **Bimaximal mixing for** \( U_e = I \):

\[ U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{2} & \frac{\sqrt{2}}{2} & \frac{1}{\sqrt{2}} \\ \frac{-1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \]
Perturbations on $m^\nu$ cannot give both $\Delta m^2_\odot$ and $\frac{\pi}{4} - \theta_\odot \sim \theta_C \sim 0.23$ without fine-tuning between terms, e.g.,

$$\frac{1}{4\sqrt{2}} \frac{\Delta m^2_\odot}{\Delta m^2_{Atm}} = -\frac{\epsilon_{23}}{4} \sim 0.007 \neq \frac{\pi}{4} - \theta_\odot \sim 0.23$$
• However, $U_e \neq I$ with small angles (comparable to CKM) can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

$$U_e^\dagger \sim \begin{pmatrix} 1 & -s_{12} & 0 \\ s_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

yields

$$\theta_\odot \sim \frac{\pi}{4} - \frac{s_{12}}{\sqrt{2}} = 0.56^{+0.05}_{-0.04}$$

$$|U_{e3}|^2 \sim \frac{(s_{12})^2}{2} \sim (0.023 - 0.081), \ 90\% \ (\text{exp}: < 0.03)$$

$$m_{\beta\beta} \sim m_2 (\cos^2 \theta_\odot - \sin^2 \theta_\odot) \sim 0.020 \ \text{eV}$$
Conclusions

- Neutrino mass likely due to large or Planck scale effects, but little work in string context

- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for $m_\nu$

- No examples of minimal seesaw in large class of heterotic $Z_3$ orbifold vacua

- Small Dirac, extended seesaw, Higgs triplet (inverted hierarchy in string context) may be more likely