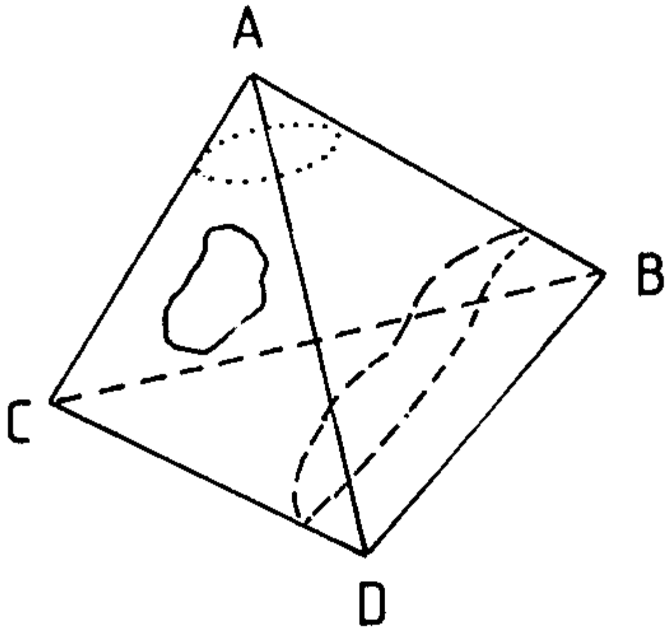
Neutrino Mass in Strings



- Introduction
- Neutrino preliminaries
- Models
- String embeddings
  - Intersecting brane
  - The Z_3 heterotic orbifold
  - Embedding the Higgs triplet
- Outlook

Neutrino mass

- Nonzero mass may be first break with standard model
- Enormous theoretical effort: GUT, family symmetries, bottom up
  - Majorana masses may be favored because not forbidden by SM gauge symmetries
  - GUT seesaw (heavy Majorana singlet). Usually ordinary hierarchy.
  - Higgs triplets ("type II seesaw"), often assuming GUT, Left-Right relations

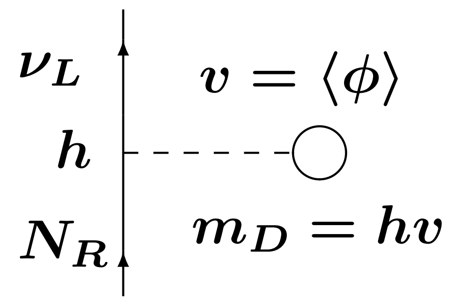
Neutrino Preliminaries

- **Weyl fermion**
  - Minimal (two-component) fermionic degree of freedom
  - $\psi_L \leftrightarrow \psi_R^c$ by CPT

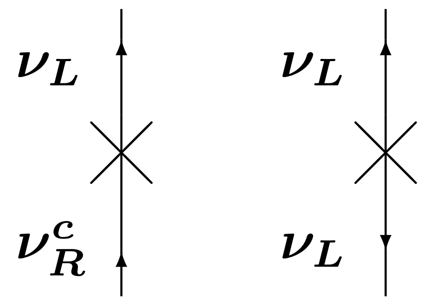
- **Active Neutrino (a.k.a. ordinary, doublet)**
  - in $SU(2)$ doublet with charged lepton $\longrightarrow$ normal weak interactions
  - $\nu_L \leftrightarrow \nu_R^c$ by CPT

- **Sterile Neutrino (a.k.a. singlet, right-handed)**
  - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
  - $N_R \leftrightarrow N_L^c$ by CPT
  - Almost always present: Are they light? Do they mix?

- Dirac Mass
  - Connects distinct Weyl spinors (usually active to sterile): $(m_D \bar{\nu}_L N_R + h.c.)$
  - 4 components, $\Delta L = 0$
  - $\Delta I = \frac{1}{2} \rightarrow$ Higgs doublet
  - Why small? HDO? LED?
  - Variant: couple active to anti-active, e.g., $m_D \bar{\nu}_{eL} \nu^c_{\mu R} \Rightarrow L_e - L_\mu$ conserved; $\Delta I = 1$



- **Majorana Mass**
  - Connects Weyl spinor with itself: $\frac{1}{2}(m_T \bar{\nu}_L \nu_R^c + h.c.)$ (active); $\frac{1}{2}(m_S \bar{N}_L^c N_R + h.c.)$ (sterile)
  - 2 components, $\Delta L = \pm 2$
  - Active: $\Delta I = 1 \rightarrow$ triplet or seesaw
  - Sterile: $\Delta I = 0 \rightarrow$ singlet or bare mass



- **Mixed Masses**
  - Majorana and Dirac mass terms
  - Seesaw for $m_S \gg m_D$
  - Ordinary-sterile mixing for m_S and m_D both small and comparable (or $m_S \ll m_d$ (pseudo-Dirac))

The minimal seesaw

- Active (sterile) neutrinos ν_L (N_R) (3 flavors each)

$$L = \frac{1}{2} \left(\bar{\nu}_L \;\; \bar{N}_L^c \right) \begin{pmatrix} m_T & m_D \\ m_D^T & m_S \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{hc}$$

  - $m_T = m_T^T =$ triplet Majorana mass matrix (Higgs triplet)
  - $m_D =$ Dirac mass matrix (Higgs doublet)
  - $m_S = m_S^T =$ singlet Majorana mass matrix (Higgs singlet)

- **Ordinary (type I) seesaw: $m_T = 0$ and (eigenvalues) $m_S \gg m_D$:**

$$m_\nu^{\text{eff}} = -m_D m_S^{-1} m_D^T$$

diagonalized by U_ν, with

$$U_{PMNS} = U_e^\dagger U_\nu$$

- **To achieve large mixings, most models assume either**
  - **$U_e \sim I$ in basis with manifest symmetries for $m_{D,S} \Rightarrow$ need large mixings in U_ν** (requires clever m_D, m_S collaboration)
  - **Large U_e mixings from lopsided m_e in basis with $m_{D,S} \sim$ diagonal** (harder to achieve in $SO(10)$ than $SU(5)$)

- **$SO(10)$ models, combined with family symmetries, often large Higgs representations** (e.g., 126-plet); **typically, $m_S \sim 10^{14}$ GeV**

Extended (TeV) Seesaw

- $m_\nu \sim m^{p+1}/m_S^p$, $\quad p > 1$ (e.g., $m \sim 100$ MeV, $m_S \sim 1$ TeV for $p = 2$)

- ν_L, N_R, N'_R (3 flavors each)

$$L = \frac{1}{2}\left(\bar{\nu}_L \;\; \bar{N}_L^c \;\; \bar{N'}_L^c\right) \begin{pmatrix} 0 & \mathrm{m}_D & \mathrm{m}_{D'} \\ \mathrm{m}_D^T & 0 & \mathrm{m}_{SS'} \\ \mathrm{m}_{D'}^T & \mathrm{m}_{SS'}^T & 0 \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \\ N'_R \end{pmatrix} + \mathrm{hc}$$

or

$$L = \frac{1}{2}\left(\bar{\nu}_L \;\; \bar{N}_L^c \;\; \bar{N'}_L^c\right) \begin{pmatrix} 0 & \mathrm{m}_D & 0 \\ \mathrm{m}_D^T & 0 & \mathrm{m}_{SS'} \\ 0 & \mathrm{m}_{SS'}^T & m_{S'} \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \\ N'_R \end{pmatrix} + \mathrm{hc}$$

Triplet models

- Introduce Higgs triplet $T = (T^{++}\, T^{+}\, T^{0})^T$ with weak hypercharge $Y = 1$
- Majorana masses m_T generated from $L_\nu = \lambda^T_{ij} L_i T L_j$ if $\langle T^0 \rangle \neq 0$
- Old Gelmini-Roncadelli model: $\langle T^0 \rangle \ll$ EW scale with spontaneous L violation
  - Excluded by $Z \rightarrow$ Majoron $+$ scalar (equivalent to $\Delta N_\nu = 2$)
- Modern triplet models (type II seesaw) break L explicity by THH couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)
- Often considered in $SO(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.

Dirac Masses

- Can achieve small Dirac masses (neutrino or other) by higher dimensional operators

$$L_\nu \sim \left(\frac{S}{M_{Pl}}\right)^p L N_L^c H_2, \quad \langle S \rangle \ll M_{Pl}$$

$$\Rightarrow m_D \sim \left(\frac{\langle S \rangle}{M_{Pl}}\right)^p \langle H_2 \rangle$$

- Large $p \Rightarrow \langle S \rangle$ close to M_{Pl} (e.g., anomalous $U(1)_A$)

- Small $p \Rightarrow$ intermediate scale $\ll M_{Pl}$

- Similar HDO may give light steriles and ordinary/sterile mixing

Other Models

- Large extra dimensions (suppressed Dirac Yukawa couplings)
- R-parity violation in supersymmetry
- TeV scale loops with new ad hoc scalars
- Ad hoc flavor symmetries, textures, anarchic models
- Anthropic considerations (string landscape)

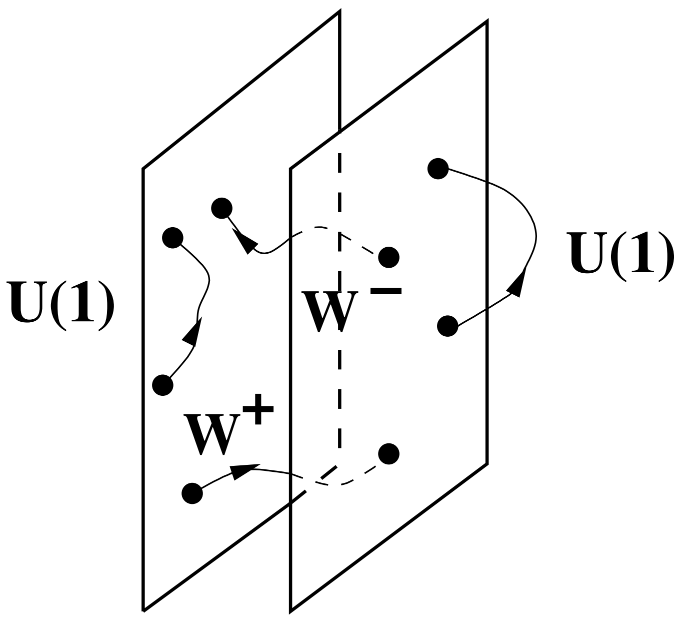
Neutrino Mass in Strings

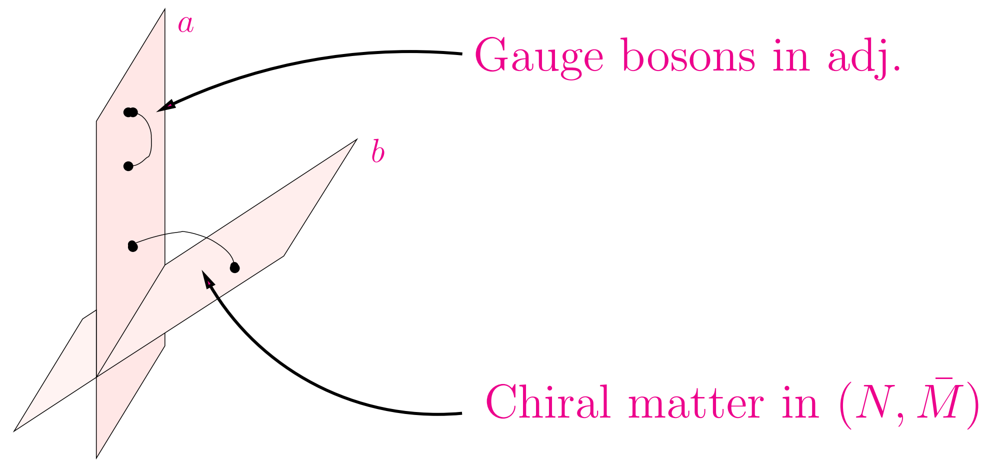
- Very little work from string constructions, even though may be Planck scale effect
- Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane)
  (Due to string symmetries or constraints, not simplicity or elegance)
  - "Right-handed" neutrinos may not be gauge singlets
  - Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
  - GUT Yukawa relations broken
  - String symmetries/constraints severely restrict couplings, e.g., Majorana masses, or simultaneous Dirac and Majorana masses

Quasi-realistic string constructions

- Two classes of quasi-realistic: intersecting D-brane, heterotic
- Intersecting D-brane (review: R. Blumenhagen, M. Cvetic, P.L., G. Shiu, hep-th/0502005)
  - Closed strings (gravitons) and open strings ending on D-branes
  - D6-branes: fill ordinary space and 3 of the 6 extra dimensions
  - Stringy implementation of “brane world” ideas

- **Gauge interactions from strings beginning/ending on stack of parallel branes (one for each group factor)**
- **Chiral matter: strings at intersection of branes, e.g., $SU(N) \times SU(M) \rightarrow$ bifundamental $(N, \bar{M})$**



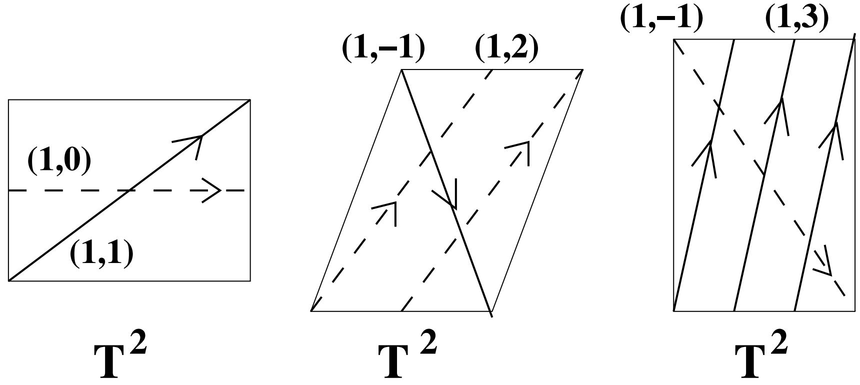


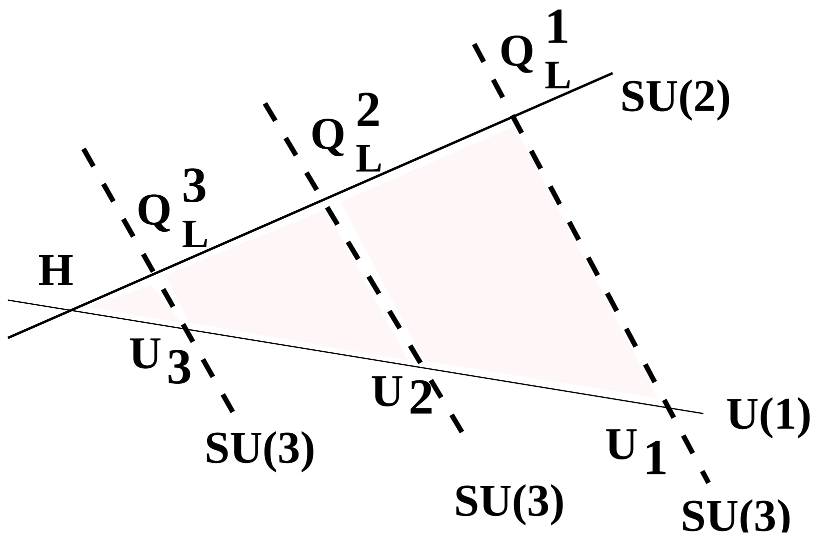
– Family replication from multiple intersections on compactified geometry

– Yukawa interactions $\sim \exp(-A_{ijk}) \longrightarrow$ hierarchies

– Existing models: conserved L; no diagonal (Majorana) triangles

– However, no realistic model with large enough A for small Dirac neutrino masses (more generic geometries?)





The $E_8 \times E_8$ Heterotic String

- **Dirac masses**
  - Can achieve small Dirac masses (neutrino or other) by higher dimensional operators

$$L_\nu \sim \left(\frac{S}{M_{Pl}}\right)^p L N_L^c H_2, \quad \langle S \rangle \ll M_{Pl}$$

$$\Rightarrow m_D \sim \left(\frac{\langle S \rangle}{M_{Pl}}\right)^p \langle H_2 \rangle$$

  - Recent variant: N_L^c is a modulus (Bouchard, Cvetič, Donagi, hep-th/0602096)

- **Majorana masses**
  - Can one generate large effective m_S from

$$W_\nu \sim c_{ij}\frac{S^{q+1}}{M_{Pl}^q}N_iN_j \quad \Rightarrow (m_S)_{ij} \sim c_{ij}\frac{\langle S\rangle^{q+1}}{M_{Pl}^q},$$

    consistent with D and F flatness?
  - Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints?
  - Are bottom-up model assumptions for relations to quark, charged lepton masses maintained?

- **The Z_3 Heterotic Orbifold** (J. Giedt, G. Kane, P.L., B. Nelson, hep-th/0502032)
  - Systematically studied large class of vacua
    - Is minimal seesaw common?
    - If rare, possibly guidance to model building
    - Clues to textures, etc.
  - Several models from each of 20 patterns; W through degree 9; huge number of D flat directions reduced greatly by F-flatness
  - Only two patterns had Majorana mass operators $\langle S_1 \cdots S_{n-2} \rangle NN/M_{\rm PL}^{n-3}$
  - *None* had simultaneous Dirac operators $\langle S_1' \cdots S_{d-3}' \rangle NLH_u/M_{\rm PL}^{d-3}$ leading to $\Delta m^2 > 10^{-10}$ eV^2 (one apparent model ruined by off-diagonal Majorana)
  - Feature of Z_3 orbifold? Or more general?

- *Systematic* searches in other constructions important (Is seesaw generic? Rare? Alternatives?)

- Consider alternatives seriously
  - Small Dirac masses from high degree terms (very common in constructions) (could also give light sterile ν's and mixing)
  - Extended seesaws, $m_\nu \sim m_D^{2+k}/M^{1+k}$, with $k \geq 1$ and low (e.g., TeV) scale M
  - Higgs triplet models: non-trivial to embed in strings (higher level), but very predictive (e.g., inverted hierarchy with nearly bi-maximal mixing) (B. Nelson, PL, hep-ph/0507063)

Triplet models

- Introduce Higgs triplet $T = (T^{++}\ T^{+}\ T^{0})^{T}$ with weak $Y = 1$
- Majorana masses m_T generated from $L_\nu = \lambda^T_{ij} L_i T L_j$ if $\langle T^0 \rangle \neq 0$
- General SUSY case

$$W_\nu = \lambda^T_{ij} L_i T L_j + \underbrace{\lambda_1 H_1 T H_1 + \lambda_2 H_2 \bar{T} H_2}_{\text{needed to avoid Majoron}}$$

$$+ M_T T \bar{T} + \mu H_1 H_2$$

T, $\bar{T}$ are triplets with $Y = \pm 1$, $M_T \sim 10^{12} - 10^{14}$ GeV. Typically,

$$\langle T^0 \rangle \sim -\lambda_2 \langle H_2^0 \rangle^2 / M_T \quad \Rightarrow m^\nu_{ij} = -\lambda^T_{ij} \lambda_2 \frac{v_2^2}{M_T}$$

- Most previous models: GUT/LR symmetry, ordinary hierarchy

String constructions

- Expect $\lambda^T_{ij} = 0$ for $i = j$ (off-diagonal) $\Rightarrow m^\nu_{ii} = 0$
- Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal
- Partial explanation: $SU(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $SU(2) \subset SU(2) \times SU(2)$ (Have Z_3 constructions with some but not all of the features, B. Nelson, PL, hep-ph/0507063)

$$W \sim \lambda^T_{1j} L_1(2,1) T(2,2) L_j(1,2), \; j = 2,3$$

yields

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- Typical string case: $|a| = |b|$

- **HDO (or $SU(2) \subset SU(2)\times SU(2)\times SU(2)$) can give $m^{\nu}_{23} \neq 0$**

- **For**

$$m^{\nu} = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

can take a, b, c real w.l.o.g. by redefinition of fields (not true for general m^{ν})

- $\mathrm{Tr}\ m^{\nu} = 0$ **and** $m^{\nu} = m^{\nu\dagger} \Rightarrow m_1 + m_2 + m_3 = 0$

- $|\Delta m^2_{\rm Atm}| \sim 2\times 10^{-3}$ eV^2, $\Delta m^2_\odot \sim 8\times 10^{-5}$ eV^2 $\Rightarrow$ two solutions
  - For $\Delta m^2_\odot$ =0

    (a) $m_i \propto 1,\ -\frac{1}{2},\ -\frac{1}{2}$ (ordinary, with shifted masses)

    (b) $m_i \propto 1,\ -1,\ 0$ (inverted)
  - With $\Delta m^2_\odot \neq 0$

    (a) $m_i = 0.054,\ -0.026,\ -0.026$ eV ($\sum |m_i| = 0.107$ eV (cosmology))

    (b) $m_i = 0.046,\ -0.045,\ -0.001$ eV ($\sum |m_i| = 0.092$ eV (cosmology))

$$m^\nu_a \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad m^\nu_b \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

  - (a) leads to unrealistic mixing matrix $\Rightarrow$ consider (b)

A special texture

- The $L_e - L_\mu - L_\tau$ conserving texture

$$m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

has been considered phenomenologically by *many* authors (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)

$$m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- New aspects
  - Strong string motivation
  - Motivation for special case $|a| = |b|$
  - Can perturb by HOT
  - No reason for $U_e = I$ in this basis
- Yields inverted hierarchy, with eigenvalues $0, \pm\sqrt{|a|^2 + |b|^2}$
- Diagonalization: $\tan\theta_{\rm Atm} = b/a \Rightarrow$ need $|b| = |a|$ for maximal

- If $U_e = I$: $\theta_\odot = \frac{\pi}{4}$ (maximal) (experiment: $\frac{\pi}{4} - \theta_\odot = 0.19^{+0.05}_{-0.06}$, 2σ)
  - Comparable to Cabibbo angle, $\theta_C \sim 0.23$

- Perturbations on m^ν cannot give both $\Delta m^2_\odot$ and $\frac{\pi}{4} - \theta_\odot \sim 0.19$ (cf $\theta_C \sim 0.23$) without fine-tuning between terms, e.g.,

$$\frac{\Delta m^2_\odot}{\sqrt{2}|\Delta m^2_{\rm Atm}|} \sim (m^\nu_{23} + m^\nu_{11}) \sim \frac{1}{43}$$

$$\frac{\pi}{4} - \theta_\odot \sim \frac{1}{4}(m^\nu_{23} - m^\nu_{11}) \sim 0.19$$

- However, $U_e \neq I$ with small angles (comparable to CKM) can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

$$U_e^\dagger \sim \begin{pmatrix} 1 & -s_{12}^e & 0 \\ s_{12}^e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

yields

$$\begin{aligned} \frac{\pi}{4} - \theta_\odot &\sim \frac{s_{12}^e}{\sqrt{2}} \quad \Rightarrow \quad s_{12}^e \sim 0.27^{+0.07}_{-0.08} \\ |U_{e3}|^2 &\sim \frac{(s_{12}^e)^2}{2} \sim (0.017 - 0.059),\ 2\sigma \ (\exp : < 0.032) \\ m_{\beta\beta} &\sim m_2(\cos^2\theta_\odot - \sin^2\theta_\odot) \sim 0.018 \text{ eV} \end{aligned}$$

Outlook

- Neutrino mass likely due to large or Planck scale effects, but little work in string context

- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for m_ν

- No examples of minimal seesaw in large class of heterotic Z_3 orbifold vacua

- Small Dirac, extended seesaw, Higgs triplet (inverted hierarchy in string context) should be seriously considered