Neutrino masses respecting string constraints

• Introduction
• Neutrino preliminaries
• The GUT seesaw
• Neutrinos in string constructions
• The triplet model

(Work in progress, in collaboration with J. Giedt, G. Kane, B. Nelson.)
Neutrino mass

- Nonzero mass may be first break with standard model
- Enormous theoretical effort: GUT, family symmetries, bottom up
  - Majorana masses may be favored because not forbidden by SM gauge symmetries
  - GUT seesaw (heavy Majorana singlet). Usually ordinary hierarchy.
  - Higgs triplets ("type II seesaw"), often assuming GUT, Left-Right relations
• Very little work from string constructions, even though probably Planck scale
  – Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane)
  – Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
  – String symmetries/constraints restrict couplings, e.g., diagonal Majorana masses
  – Very nonstandard triplet or singlet seesaws, favoring inverted hierarchy, extended seesaw, or small Dirac masses from HDO.

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• **Weyl fermion**
  - Minimal (two-component) fermionic degree of freedom
  - $\psi_L \leftrightarrow \psi_R^c$ by CPT

• **Active Neutrino (a.k.a. ordinary, doublet)**
  - in $SU(2)$ doublet with charged lepton $\rightarrow$ normal weak interactions
  - $\nu_L \leftrightarrow \nu_R^c$ by CPT

• **Sterile Neutrino (a.k.a. singlet, right-handed)**
  - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
  - $N_R \leftrightarrow N_L^c$ by CPT
  - Almost always present: Are they light? Do they mix?
- Dirac Mass

- Connects distinct Weyl spinors (usually active to sterile):
  \((m_D \bar{\nu}_L N_R + h.c.)\)

- 4 components, \(\Delta L = 0\)

- \(\Delta I = \frac{1}{2} \rightarrow \) Higgs doublet

- Why small? LED? HDO?

- Variant: couple active to anti-active, e.g., \(m_D \bar{\nu}_e L \nu^c_{\mu R} \Rightarrow L_e - L_\mu\) conserved; \(\Delta I = 1\)
• Majorana Mass

  - Connects Weyl spinor with itself:
    \[ \frac{1}{2}(m_T \bar{\nu}_L \nu^c_R + \text{h.c.}) \text{ (active);} \]
    \[ \frac{1}{2}(m_S \bar{N}_L^c N_R + \text{h.c.}) \text{ (sterile)} \]
  - 2 components, \( \Delta L = \pm 2 \)
  - Active: \( \Delta I = 1 \rightarrow \text{triplet or seesaw} \)
  - Sterile: \( \Delta I = 0 \rightarrow \text{singlet or bare mass} \)

• Mixed Masses

  - Majorana and Dirac mass terms
  - Seesaw for \( m_S \gg m_D \)
  - Ordinary-sterile mixing for \( m_S \) and \( m_D \) both small and comparable (or \( m_S \ll m_d \) (pseudo-Dirac))
3 $\nu$ Patterns

- **Solar:** LMA (SNO, Kamland)
  
  - $\Delta m^2_\odot \sim 8 \times 10^{-5}$ eV$^2$, nonmaximal

- **Atmospheric:**
  
  \[
  |\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3} \text{ eV}^2, \text{ near-maximal mixing}
  \]

- **Reactor:** $U_{e3}$ small
– Mixings: let $\nu_\pm \equiv \frac{1}{\sqrt{2}} (\nu_\mu \pm \nu_\tau)$:

\[
\begin{align*}
\nu_3 & \sim \nu_+ \\
\nu_2 & \sim \cos \theta_\odot \nu_- - \sin \theta_\odot \nu_e \\
\nu_1 & \sim \sin \theta_\odot \nu_- + \cos \theta_\odot \nu_e
\end{align*}
\]

3 _______ 2 _______ 1 _______

2 _______ 1 _______ 3 _______

– Hierarchical pattern

* Analogous to quarks, charged leptons

* $\beta\beta_{0\nu}$ if Majorana

* SN1987A energetics (if $U_{e3} \neq 0$)?

* May be radiative unstable

– Inverted quasi-degenerate pattern

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• Elegant mechanism for small Majorana masses

• Leptogenesis

• Expect small mixings in simplest versions (can evade by lopsided $e/d$, Majorana textures, etc.)

• Large Majorana often forbidden, e.g., by extra $U(1)$’s

• Direct Majorana masses and large scales forbidden in some string constructions

• GUTs, adjoint Higgs, large Higgs hard to accommodate in simplest heterotic constructions
• **LSND**: active-sterile difficult in simple versions

• **Therefore, explore alternatives, e.g., with small Dirac and/or Majorana masses**
  
  – Small Majorana from loops, $R_p$ violation, TeV seesaw, or triplet
  
  – Small Dirac from large extra dimension or by higher dimensional operators in intermediate scale models (e.g. $U(1)'$)
  
  – Variant ordinary and triplet seesaws motivated by string constructions
Neutrinos in string constructions

Key ingredients of most GUT/bottom up models forbidden or different in known constructions (heterotic or intersecting brane)

- Bifundamentals, singlets, or adjoints; not large representations
- String symmetries/constraints may forbid couplings allowed by 4d symmetries
- Diagonal superpotential terms (e.g., diagonal Majorana masses) usually absent
- GUT Yukawa relations broken
- Non-zero superpotential terms may be equal (gauge couplings)
- Hierarchies from HDO (heterotic), intersection triangles (intersecting brane)
• Can achieve small Dirac masses (neutrino or other) by higher dimensional operators or by large intersection areas

\[ L_\nu \sim \left( \frac{S}{M_{Pl}} \right)^p L N_L^c H_2, \quad \langle S \rangle \ll M_{Pl} \]

\[ \Rightarrow m_D \sim \left( \frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle \]

• Large \( p \Rightarrow \langle S \rangle \) close to \( M_{Pl} \) (e.g., anomalous \( U(1)' \))

• Small \( p \Rightarrow \) intermediate scale \( \ll M_{Pl} \)
Intermediate scale in (non-anomalous) $U(1)'$ from $D$ and (almost) $F$ flat direction:

Two SM singlets charged under $U(1)'$. If no $F$ terms,

$$V(S_1, S_2) = m_1^2|S_1|^2 + m_2^2|S_2|^2 + \frac{g'^2Q'^2}{2}(|S_1|^2 - |S_2|^2)^2$$

Break at EW scale for $m_1^2 + m_2^2 > 0$, at intermediate scale for $m_1^2 + m_2^2 < 0$ (stabilized by loops or HDO).
The ordinary seesaw

- Active neutrinos $\nu_L, N_R$ (3 flavors each)

$$L = \frac{1}{2} (\bar{\nu}_L \bar{N}_L^c) \begin{pmatrix} m_T & m_D \\ m_D^T & m_S \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{hc}$$

- $m_T = m_T^T =$ triplet Majorana mass matrix (Higgs triplet)
- $m_D =$ Dirac mass matrix (Higgs doublet)
- $m_S = m_S^T =$ singlet Majorana mass matrix (Higgs singlet); eg, 126 of $SO(10)$
• Ordinary (type I) seesaw: $m_T = 0$ and (eigenvalues) $m_S \gg m_D$:

$$m_{\nu}^{\text{eff}} = -m_D m_S^{-1} m_D^T$$

with

$$U_{PMNS} = U_e^\dagger U_\nu$$

• Most models assume either
  
  – $U_e \sim I$ in basis with manifest symmetries for $m_D, S \Rightarrow$ large mixings in $U_\nu$
  
  – Large $U_e$ mixings from lopsided $m_e$ in basis with $m_D, S \sim$ diagonal (harder to achieve in $SO(10)$ than $SU(5)$)

• $SO(10)$ models usually yield ordinary hierarchy
• String constructions: may be able to generate large effective $m_S$ from

$$W_\nu \sim c_{ij} \frac{S^{q+1}}{M_{Pl}^q} N_i N_j \Rightarrow (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{q+1}}{M_{Pl}^q}$$

• Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints? (Under investigation for $Z_3$ orbifold.)

• $c_{ii} = 0$ in all known examples $\Rightarrow$

$$m_S = \begin{pmatrix} 0 & m_{12} & m_{13} \\ m_{12} & 0 & m_{23} \\ m_{13} & m_{23} & 0 \end{pmatrix}$$
• Very different from standard seesaw textures

  – Case with three large eigenvalues requires complicated $m_D$
    and/or $m_e$

  – $2 \times 2$ case could resemble special pseudo-Dirac inverse
    hierarchy model found for triplets

  – Extended seesaw with greater than 3 $N$ fields? (Coriano, Faraggi;
    F., Thormeier)
Triplet models

• Introduce Higgs triplet $T = (T^+ T^+ T^0)^T$ with weak hypercharge $Y = 1$

• Majorana masses $m_T$ generated from $L_{\nu} = \lambda^T_{ij} L_i T L_j$ if $\langle T^0 \rangle \neq 0$

• Old Gelmini-Roncadelli model: $\langle T^0 \rangle \ll$ EW scale with spontaneous $L$ violation
  – Excluded by $Z \rightarrow$ Majoron + scalar (equivalent to $\Delta N_{\nu} = 2$)

• Modern triplet models (type II seesaw) break $L$ explicity by $T H H$ couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)

• Often considered in $SO(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.

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• General SUSY case

\[ W_\nu = \lambda^T_{ij} L_i T L_j + \lambda_1 H_1 T H_1 + \lambda_2 H_2 \bar{T} H_2 \]
\[ + M_T T \bar{T} + \mu H_1 H_2 \]

\( T, \bar{T} \) are triplets with \( Y = \pm 1 \), \( M_T \sim 10^{12} - 10^{14} \) GeV. Typically,

\[ \langle T^0 \rangle \sim -\lambda \langle H_2^0 \rangle^2 / m_T \Rightarrow \]

\[ m_{\nu_{ij}} = -\lambda^T_{ij} \lambda_2 \frac{v_2^2}{M_T} \]
String constructions

- Expect $\lambda^T_{i,j} = 0$ for $i = j$ (off-diagonal) $\Rightarrow m^\nu_{i,i} = 0$

- Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal

- Partial explanation: $SU(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $SU(2) \subset SU(2) \times SU(2)$ (Have $Z_3$ constructions with some but not all of the features.)

\[ W \sim \lambda^T_{1,j} L_{1}(2, 1) T(2, 2) L_{j}(1, 2), \; j = 2, 3 \]

yields

\[ m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \]

- Typical string case: $|a| = |b|
• HDO (or $SU(2) \subset SU(2) \times SU(2) \times SU(2)$) can give $m_{23}^\nu \neq 0$

• For

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

...can take $a, b, c$ real w.l.o.g. by redefinition of fields (not true for general $m^\nu$)

• $\text{Tr } m^\nu = 0$ and $m^\nu = m^{\nu\dagger}$ $\Rightarrow m_1 + m_2 + m_3 = 0$
• $|\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3}$ eV$^2$, $\Delta m^2_{\odot} \sim 8 \times 10^{-5}$ eV$^2$ ⇒ two solutions

- For $\Delta m^2_{\odot} = 0$
  
  (a) $m_i \propto 1, -\frac{1}{2}, -\frac{1}{2}$ (ordinary, with shifted masses)
  
  (b) $m_i \propto 1, -1, 0$ (inverted)

- With $\Delta m^2_{\odot} \neq 0$
  
  (a) $m_i = 0.054, -0.026, -0.026$ eV ($\sum |m_i| = 0.107$ eV (cosmology))
  
  (b) $m_i = 0.046, -0.045, -0.001$ eV ($\sum |m_i| = 0.092$ eV (cosmology))

\[
m^\nu_a \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad m^\nu_b \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}
\]

- (a) leads to unrealistic mixing matrix ⇒ consider (b)
A special texture

• The $L_e - L_\mu - L_\tau$ conserving texture

$$m^\nu \sim \begin{pmatrix}
0 & a & b \\
 a & 0 & 0 \\
b & 0 & 0
\end{pmatrix}$$

has been considered phenomenologically by many authors (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)
\( m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \)

- **New aspects**
  - Strong string motivation
  - Motivation for special case \(|a| = |b|\)
  - Most likely perturbation in 23 element from HOT

- **Diagonalization:** \( \tan \theta_{\text{Atm}} = b/a \Rightarrow \text{need } |b| = |a| \text{ for maximal} \)

- \( \tan^2 \theta_{\odot} = 1 \text{ (maximal)} \) (experiment \( \tan^2 \theta_{\odot} = 0.40^{+0.09}_{-0.07} \))
• Majorana mass matrix

\[ m^\nu \sim \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \]

• Inverted hierarchy

• Bimaximal mixing for \( U_e = I \):

\[ U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \]
Perturbations on $m^\nu$ cannot give both $\Delta m^2_\odot$ and $\frac{\pi}{4} - \theta_\odot \sim \theta_C \sim 0.23$ without fine-tuning between terms, e.g.,

$$\frac{1}{4\sqrt{2}} \frac{\Delta m^2_\odot}{\Delta m^2_{\text{Atm}}} = -\frac{\epsilon_{23}}{4} \sim 0.007 \neq \frac{\pi}{4} - \theta_\odot \sim 0.23$$
• However, $U_e \neq I$ with small angles (comparable to CKM) can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

$$U_e^\dagger \sim \begin{pmatrix}
1 & -s_{12}^e & 0 \\
&s_{12}^e & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}$$

yields

$$\theta_\odot \sim \frac{\pi}{4} - \frac{s_{12}^e}{\sqrt{2}} = 0.56^{+0.05}_{-0.04}$$

$$|U_{e3}|^2 \sim \frac{(s_{12})^2}{2} \sim (0.023 - 0.081), \ 90\% \ (\text{exp}: <0.03)$$

$$m_{\beta\beta} \sim m_2 (\cos^2 \theta_\odot - \sin^2 \theta_\odot) \sim 0.020 \text{ eV}$$
● Detailed $Z_3$ constructions for higher level embeddings (triplets) and for heavy Majorana neutrinos

● Implications for $m_e, m_q$

● Implications of additional Higgs

● RGE effects

● Leptogenesis
Conclusions

- Neutrino mass likely due to large or Planck scale effects, but little work in string context

- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for $m_\nu$

- Preliminary conclusion: inverted hierarchy (pseudo Dirac), extended seesaw, or small Dirac favored

- Inverted hierarchy (e.g., from triplet) very predictive