Who's Counting? Is it 10 or 11?
(dimensions, that is ---M Theory is making me Manic!)

Yes, it looks strange. The physicists that claim to have a beautiful and consistent theory of quantum gravity cannot agree on the number of dimensions!. The surprising answer is that it can be both, 10 *and* 11!. What happens is that the classical notion of spacetime looses its meaning in the quantum theory. It is replaced by a more general concept, a “quantum spacetime” where the dimension is not a well defined notion. We know a great deal about certain quantum spacetimes and they have many precise properties, but their dimension is not one of them. In fact, string theory leads to very surprising phenomena which clash our intuitive geometrical notions. Perhaps the simplest example arises when we have a circular dimension. Namely a dimension that is periodic, so that after traveling along a distance L along this dimension we come back to the point were we started. It turns out that in string theory a circle with size L is the same as a circle of size $L' = (2\pi l_s)^2/L$, where l_s is the typical size of a string, which is determined by its tension. How does this happen? Let us imagine how we would measure this dimension. When the circle is very large, then we can use just a metric tape. As it gets smaller, then we start seeing quantum effects. A particle moving along this circle has a quantized momentum. For simplicity, consider a massless particle. We can measure the size of the dimension by looking at the energy levels for this particle which are quantized with a characteristic spacing which is inversely proportional to the size. When we continue decreasing L these particles become more and more energetic. However, besides these particles we can have other excitations. For example, we can wrap a string on this circle. If the circle is very small these wound strings are very light. It turns out that these wound strings on a very small circle of size L are completely equivalent to massless particles moving along a circle of size $L' = (2\pi l_s)^2/L$.
In this simple example all that happens is that a very small circle becomes equivalent to a big circle. In this example the number of dimensions did not change, but there are other cases where it does. For example, a strings moving on a small three dimensional sphere which has the size of a string are equivalent to strings moving on a circle. The number of dimensions changes. It is three in one description while it is only one in the other. In addition, in string theory we can have spaces which have no geometric interpretation at all.

So if the number of dimensions is not well defined in string theory, why do people talk about it?. In string theory, what we know for sure is that the number of dimensions we need to describe the world we live in is greater than four. The cases when we have 10 or 11 are specially simple, and people have concentrated in studying these cases. They are simpler because in these cases the internal dimensions, namely the dimensions beyond the four we see, can be relatively large so that we can describe them using geometrical language. By relatively large I mean larger than the size of the string but smaller than any distance that we can experimentally see today. If the total number of dimensions is not 10 or 11, then necessarily some of the dimensions have string size and are therefore harder to describe.

What is the difference between 10 and 11?.
The simplest string theory is ten dimensional. Strings can interact with each other. If the interaction among strings is large, the theory is hard to describe. It turns out that when strings interact very, very strongly, something surprising happens. A new dimension opens up and we have a theory in eleven dimensions, the ten we started with plus an extra circle. In eleven dimensions we do not have strings, we have membranes. Membranes wrapped along the 11th dimension give rise to strings.

We do not know yet whether a description in terms of 10 or 11 dimensions is more appropriate for the universe where we live in. But these two possibilities are continuously connected. They are simply different possibilities for the internal geometry. Since the geometry of the internal space is quantum mechanical, asking what its dimension is might not be the right question.

One of the main lessons from recent developments is that the concept of spacetime itself will have to be replaced by some other concept at the quantum level. Let us just consider an analogy. Let us consider the surface of a lake. If we are classical physicists we would describe the waves that propagate on this surface, we will be able to say if we are above the surface or below the surface, etc. On the other hand, when we look at the lake with very high resolution we start seeing the individual water molecules. Then the surface of the lake becomes a lot less sharp. In fact, the surface is not a well defined concept at the atomic level. There are molecules constantly leaving the water into the air and vice-versa. We can only talk about the surface of the lake at macroscopic distances. The same happens with spacetime, we can use the standard geometric description only at long enough distances. Of course, in the case of spacetime we would need to go to distances smaller than the distances that are experimentally accessible today in order to see that the concept is breaking down. But string theory predicts, that just as in the case of the surface of the lake, spacetime will loose its meaning at very short distances. Of course this does not mean that we cannot describe the system. In the case of the water, we have a well defined description of the system in terms of the water molecules. Similarly, in the case of spacetime we have a good description of what is going on, at least in some special circumstances. But this description uses less intuitive variables, which we do not have the space to describe in more detail.

In summary, in a quantum spacetime the dimension might not be a well defined notion. When the space in question is small, it can interpolate continuously between different dimensions.

Prof. Juan Maldacena
Institute for Advanced Study