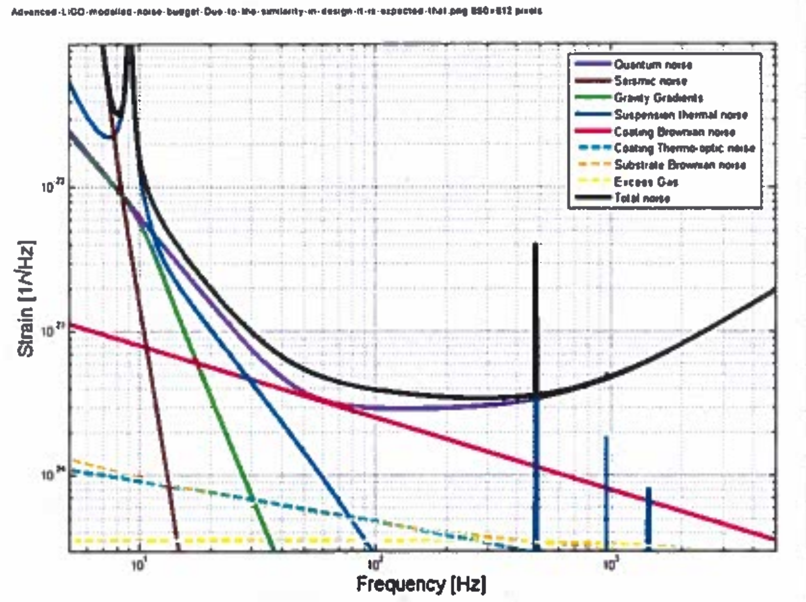
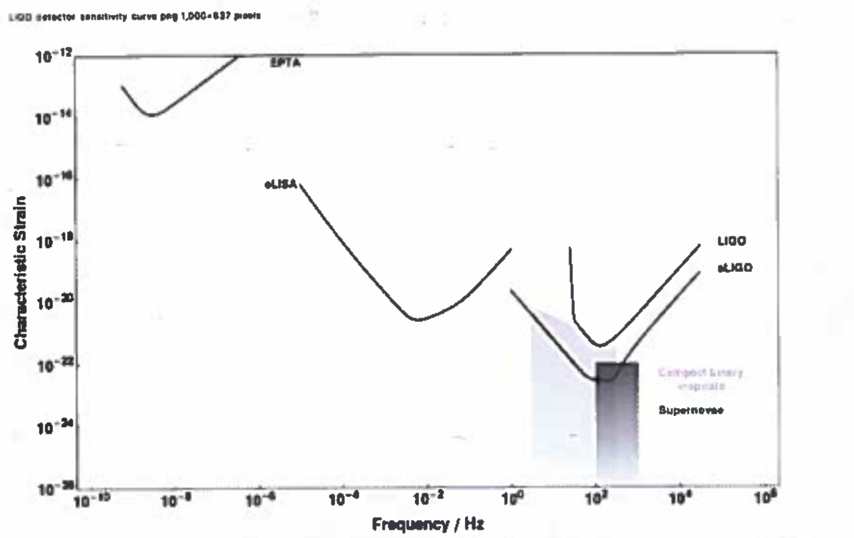
Lecture. (Lena)

Binary black holes.

LIGO





Matias told us that the sweetspot of LIGO is ~100Hz.

1. Let's try to determine what sort of events we will see.

Naively: $\nu_{gw} = 2\nu_{orb}$

$\nu_{orb} \sim \left(\frac{2\pi R_{sch}}{c}\right)^{-1}$

In conventional notations $\nu_{gw} = f_{gw}$

$$\frac{c}{2\pi\nu_{orb}} = \frac{c}{\pi\nu_{gw}} = R_{sch}$$

$$R_{sch} = \frac{3\times10^{10}}{\pi\times100} \simeq 10^8\,cm = 10^3\,km \;\circleddash$$

$R_{sch}(M_\odot) \simeq 3\,km$

$R_{sch} \sim M$

~ Intermediate mass BH / very heavy stellar mass BH

$\circleddash\; R_{sch}(3\times10^2 M_\odot)$

This is incorrect.

Peters 1964 (Phys Rev 136, 4B, #1224)

$$\dot\nu_{gw} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\nu_{gw}^{11/3}$$

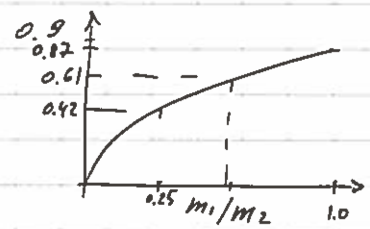
$$\dot\nu_{gw} = \frac{3\cdot2^5}{5}\pi^{1+\frac{5}{3}}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\nu_{gw}^{2+5/3},$$

where chirp mass

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

if $m_1 = m_2 = m$

$\mathcal{M} = m\cdot2^{-1/5} \simeq \frac{m}{1.15}$



$m_2 > m_1$

If we plug the numbers from above

$$\dot\nu_{gw} = \frac{3\cdot2^5}{5}\pi\left(\pi\frac{G\mathcal{M}}{c^2}\frac{1}{2}\nu_{gw}\right)^{5/3}\nu_{gw}^2 = \overbrace{\frac{3\cdot2^5}{5}\pi\times\frac{1}{2^{5/3}}}^{19}\nu_{gw}^2$$

$$\frac{T_{orb}}{4} = \frac{1}{4\nu_{orb}} = \frac{1}{2\nu_{gw}}$$

$$= 19\times\nu_{gw}^2 = 2\times10^5\,\frac{Hz}{sec}$$

We will never see this

Instead our estimations should be like this

$\nu_{gw} \sim 100\ Hz$

$\dot{\nu}_{gw} \sim 100 \frac{Hz}{sec}$ slow enough so we can actually see it, but not too slow

$$\frac{\dot{f}_{gw}}{f_{gw}^2} = \frac{3 \cdot 2^5}{5} \pi \frac{1}{4^{5/3}} \left(\frac{f_{gw}}{f_{orb}}\right)^{5/3}$$

$$\left(\frac{f_{gw}}{f_{orb}}\right)^{5/3} = \left[\frac{3 \cdot 2^5}{5} \pi \frac{1}{4^{5/3}}\right]^{-1} \frac{\dot{f}_{gw}}{f_{gw}^2}$$

$$\frac{f_{gw}}{f_{orb}} = \underbrace{\left[\underbrace{\frac{3 \cdot 2^5}{5} \pi \frac{1}{4^{5/3}}}_{\sim 6}\right]^{-3/5}}_{3^{-1}} \left(\frac{\dot{f}_{gw}}{f_{gw}^2}\right)^{3/5} = \underbrace{\frac{1}{3}\left(\frac{100}{100^2}\right)^{3/5}}_{0.02} = 0.02$$

$$\frac{1}{3}\left(\frac{1000}{100^2}\right)^{3/5} = 0.04$$

$$\frac{2\pi R_{sch}}{c} = 2 \times 10^{-4}$$

$$R_{sch} = \frac{3 \times 10^{10} \times 2 \times 10^{-4}}{2\pi} = 10^{+6} cm = 10 km$$

$R_{sch}(M_\odot) = 3 km \Rightarrow$ few $M_\odot$

We are interested in stellar size BH & NS

What are the separations?

$$t_H = \frac{1}{H_0} = 14 \times \text{billion yrs} = 14 \times 10^9 yrs$$

Lecture 1 notes:

$$t_{gw} = 3.3 \times 10^{17} yrs \frac{(a/1au)^4}{[M_1 M_2 (M_1 + M_2)/M_\odot^3]}$$

$M_1 = M_2 = M = 10 M_\odot$

$$= 1.6 \times 10^{14} yrs \left(\frac{a}{1au}\right)^4 < 14.4 \times 10^9 yrs \quad (1.4 \times 10^{10})$$

$$\left(\frac{a}{1au}\right)^4 - 10^{-4}$$

$a \simeq 0.1 au$

/ $R_\odot = 6.96 \times 10^{10} cm = 0.005 au$ /

$a = 20 R_\odot$

2\. BH formation. Masses. BH-BH systems. Making close binaries.

See short articles by
Mapelli Front. Astron. Space Sci; 9 July, 2020
Patricia Schmidt Front. Astron. Space Sci; 16 June, 2020
(and references inside)

Additions: Mapelli doesn't mention stalled shocks and failed SN. Just keep in mind it's an additional factor.

For reference and to get a better feel for the numbers.
LIGO horizon for NS-NS is 190 Mpc to be upgraded to 325 Mpc soon.
Amplitude of GW scales as $h \sim M_c^{5/3}$
$M_{NS} \sim 1 M_\odot \rightarrow M_{BH} \sim 30 M_\odot$ etc.

Date

(3) How does gravitational wave signal looks like.
See
Mroue, Scheel, Szilagyi, et al
"Catalog of 174 binary BH simulations for grav. wave astronomy"
Phys Rev Lett, 111, 241104 (2013)

Figure 3 for wave forms
Figure 2 for parameters.

4. What have we learned from observations?

Let's wait untill we have few 100 of GW events.

So far everything looks like the binaries are formed in isolation.
- masses are similar
- No NS-BH events
- So far we can't really constrain spin alignment.