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High energy neutrinos from astrophysical sources: An upper bound

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We show that cosmic-ray observations set a model-independent upper bound of $E_\nu^2\Phi_\nu<2\times10^{-8}$ GeV/cm^2 s sr to the intensity of high-energy neutrinos produced by photo-meson (or p-p) interactions in sources of size not much larger than the proton photo-meson (or p-p) mean-free-path. This bound applies, in particular, to neutrino production by either AGN jets or GRBs. The upper limit is two orders of magnitude below the intensity predicted in some popular AGN jet models and therefore contradicts the theory that the cosmic gamma-ray background is due to photo-pion interactions in AGN jets. The upper bound is consistent with our predictions from GRB models. The predicted intensity from GRBs is $E^2dN/dE\sim0.3\times10^{-8}$ GeV/cm^2 s sr for 10^{14} eV$<E<10^{16}$ eV; we also derive the expected intensity at higher energy. [S0556-2821(99)03902-8]

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I. INTRODUCTION

Large area, high-energy neutrino telescopes are being constructed to detect cosmologically distant neutrino sources (see [1] for a review). The main motivation of a search for cosmological high-energy neutrino sources derives from the fact that the cosmic-ray energy spectrum extends to $>10^{20}$ eV and is most likely dominated above $\sim3\times10^{18}$ eV by an extra-Galactic source of protons [2]. High-energy neutrino production is likely to be associated with the production of high-energy protons, through the decay of charged pions produced by photo-meson interaction of the high-energy protons with the radiation field of the source. Gamma-ray bursts (GRBs) [3] and active galactic nuclei (AGN) jets [4] have been suggested as possible sources of high-energy neutrinos that are associated with high-energy cosmic-rays. The predicted neutrino fluxes may be detectable with high-energy neutrino telescopes of effective area ~1 km^2.

We show here that high-energy cosmic-ray observations set a model-independent upper bound to the expected high-energy neutrino fluxes and are in conflict with the theory that the cosmic gamma-ray background is due to photo-pion interactions in AGN jets. The upper bound is stated in Eq. (3) and is illustrated in Fig. 1. The demonstration that the AGN jet models for the gamma-ray background are in conflict with the cosmic ray data is given in Sec. III. We also give a more detailed prediction than we gave in Ref. [3] for the expected GRB neutrino spectrum and discuss the compatibility of the detailed results with the general secondary-particle cooling constraints derived by Rachen and Mészáros [5].

It has been suggested that neutrinos may be produced in the cores of AGNs (rather than in the jets) by photo-meson interaction of protons accelerated to high energy in the AGN core; in this scenario neutrinos are produced very close to the central black-hole [6]. In this model, the proton photo-meson optical depth is very high, $\tau_{p\gamma}\sim100$, and high-energy nucleons do not escape the source. By construction, there can be no observational evidence except neutrinos for, or against, the hypothesized luminous AGN accelerator of high-energy protons. The hypothesized black-hole accelerators are ''neutrino-only'' factories. Therefore, cosmic-ray observations cannot set a limit to neutrino emission in this model. On the other hand, this model cannot explain, and is therefore not supported by, the existence of the extra-Galactic high-energy cosmic-ray flux.

This paper is organized as follows. In Sec. II we derive the general upper bound to neutrino fluxes from p-γ interactions for sources optically thin to p-γ reactions. We compare in this section the upper limit to the predictions from different models for neutrino sources. We also show that the upper bound cannot be avoided by cosmological evolutionary effects (Sec. II C) or by invoking magnetic fields (Sec. III). In Sec. IV we discuss the implications of the upper bound for AGN jet models of neutrino production. In Sec. V we discuss the implications for the GRB model of high energy neutrino production, derive a more detailed prediction (compared to our prediction in Ref. [3]) for the expected GRB neutrino spectrum, and compare our results with those of other authors. We discuss in Sec. VI our main conclusions.

II. UPPER BOUND TO THE NEUTRINO FLUX

We first derive in Sec. II A the upper bound to the high-energy neutrino flux from the sources at redshift $z<1$ that produce the observed cosmic-rays at energies greater than 10^{18} eV. We compare in Sec. II B the upper bound with the predictions of current models. In Sec. II C we discuss the modification of the upper bound by unobserved sources of cosmic rays at larger redshift.

A. Derivation of the upper bound

Cosmic-ray observations above 10^{17} eV indicate that an extra-Galactic source of protons dominates the cosmic-ray flux above $\sim3\times10^{18}$ eV [2], while the flux at lower energies is dominated by heavy ions, most likely of Galactic origin.

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The observed energy spectrum of the extra-Galactic component is consistent with that expected for a cosmological distribution of sources of protons, with injection spectrum $dN_{CR}/dE_{CR} \propto E_{CR}^{-2}$, as typically expected for Fermi acceleration [7]. The energy production rate of protons in the energy range 10^{19}–10^{21} eV is $\dot{\varepsilon}_{CR}^{[10^{19},10^{21}]} \sim 5\times 10^{44}$ erg Mpc^{-3} yr^{-1} [7], if the observed flux of ultra-high-energy cosmic-rays is produced by sources that are cosmologically distributed. The energy-dependent generation rate of cosmic-rays is therefore given by

$$E_{CR}^2 \frac{d\dot{N}_{CR}}{dE_{CR}} = \frac{\dot{\varepsilon}_{CR}^{[10^{19},10^{21}]}}{\ln(10^{21}/10^{19})} \approx 10^{44} \ \text{erg Mpc}^{-3}\,\text{yr}^{-1}. \quad (1)$$

If the high-energy protons produced by the extra-Galactic sources lose a fraction $\epsilon < 1$ of their energy through photo-meson production of pions before escaping the source, the resulting present-day energy density of muon neutrinos is $E_\nu^2 dN_\nu/dE_\nu \approx 0.25 \epsilon t_H E_{CR}^2 d\dot{N}_{CR}/dE_{CR}$, where $t_H \approx 10^{10}$ yr is the Hubble time. For energy independent ϵ the neutrino spectrum follows the proton generation spectrum, since the fraction of the proton energy carried by a neutrino produced through a photo-meson interaction, $E_\nu \approx 0.05 E_p$, is independent of the proton energy. The 0.25 factor arises because neutral pions, which do not produce neutrinos, are produced with roughly equal probability with charged pions, and because in the decay $\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_e + \overline{\nu}_\mu + \nu_\mu$ muon neutrinos carry approximately half the charged pion energy. Defining $I_{\max}$ as the muon neutrino intensity (ν_μ and $\overline{\nu}_\mu$ combined) obtained for $\epsilon = 1$,

$$I_{\max} \approx 0.25 \xi_Z t_H \frac{c}{4\pi} E_{CR}^2 \frac{d\dot{N}_{CR}}{dE_{CR}}$$
$$\approx 1.5\times 10^{-8} \xi_Z \ \text{GeV cm}^{-2}\,\text{s}^{-1}\,\text{sr}^{-1}, \quad (2)$$

the expected neutrino intensities are

$$E_\nu^2 \Phi_{\nu_\mu} \equiv \frac{c}{4\pi} E_\nu^2 \frac{dN_{\nu_\mu}}{dE_\nu} = \frac{1}{2} \epsilon I_{\max}, \quad \Phi_{\nu_e} \approx \Phi_{\overline{\nu}_\mu} \approx \Phi_{\nu_\mu}. \quad (3)$$

The quantity ξ_Z in Eq. (2) is of order unity and has been introduce here to describe the possible contribution of so far unobserved high redshift sources of high-energy cosmic rays and to include the effect of the redshift in neutrino energy. We estimate ξ_Z in Sec. II C.

B. Upper bound versus current models

Figure 1 compares the neutrino intensity predictions of GRB and AGN jet models with the intensity given by Eq. (2). The AGN core model predictions are shown for completeness. The intensities predicted by both AGN jet and core models exceed $I_{\max}$ by typically two orders of magnitude.

The intensity $I_{\max}$ is an upper bound to the intensity of high-energy neutrinos produced by photo-meson interaction in sources of size not much larger than the proton photo-meson mean-free-path. Higher neutrino intensities from such sources would imply proton fluxes higher than observed in cosmic-ray detectors. Clearly, higher neutrino intensities may be produced by sources where the proton photo-meson ''optical depth'' is much higher than unity, in which case only the neutrinos escape the source. However, the existence of such sources cannot be motivated by the observed high-energy cosmic-ray flux or by any observed electromagnetic radiation. We therefore refer in Fig. 1 to models with $\tau_{\gamma p} \gg 1$ as ''hidden core'' models.

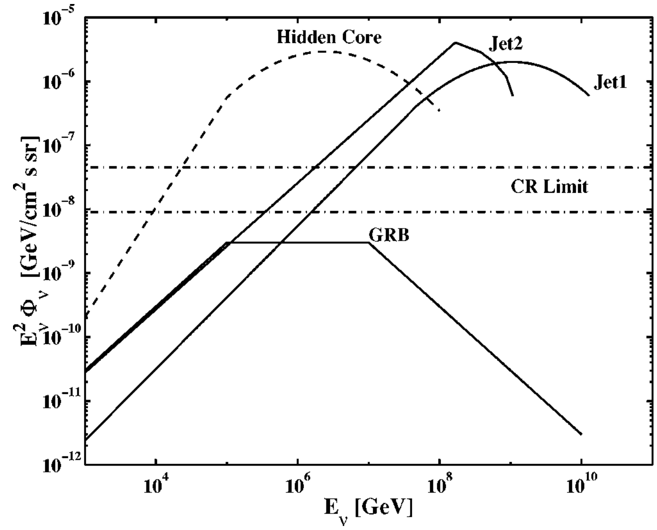

FIG. 1. Comparison of muon neutrino intensities (ν_μ and $\overline{\nu}_\mu$ combined) predicted by different models with the upper bound implied by cosmic ray observations. The dash-dotted lines give the upper bound, Eq. (2), corrected for neutrino energy loss due to redshift and for possible redshift evolution of the cosmic-ray generation rate. The lower line is obtained assuming no evolution, and the upper line assuming rapid evolution similar to the evolution of the quasi-stellar object (QSO) luminosity density. The AGN jet model predictions are taken from Ref. [4] (labeled ''Jet1'' and ''Jet2''). The GRB intensity is based on the estimate presented in this paper, following [3]. The AGN hidden-core conjecture, which produces only neutrinos and to which the upper bound does not apply, is taken from [6].

C. Evolution and redshift losses

In the derivation of Eq. (2) we have neglected the redshift energy loss of neutrinos produced at cosmic time $t < t_H$, and implicitly assumed that the cosmic-ray generation rate per unit (comoving) volume is independent of cosmic time. The generation rate may have been higher at earlier times, i.e. at high redshift. Cosmic rays above 10^{18} eV must originate from sources at $z < 1$. Energy loss due to redshift and pair production in interaction with the microwave background implies that in order to be observed with energy $E > 10^{18}$ eV, a proton should have been produced at $z = 1$ with energy exceeding the threshold for photo-meson production in interaction with the microwave background at that redshift. Photo-meson energy loss of protons produced above the threshold would reduce the proton energy to the threshold value in a short time, so that their observed energy (i.e. at

$z=0$) would be $\sim 10^{18}$ eV. Thus, the cosmic ray energy generation rate given in Eq. (1) is the present (i.e. low redshift, $z<1$) generation rate. An increase in the cosmic-ray energy generation rate per unit (comoving) volume above the value of Eq. (1) at large redshift, $z>1$, is consistent with observations, since it would not affect the observed flux above $\sim 10^{18}$ eV, and the contribution from $z>1$ sources to the observed flux below $\sim 10^{18}$ eV may be hard to detect due to the "background" of Galactic sources of heavy ion cosmic-rays which are most likely dominating the flux at this energy [2,8].

Let us consider the possible modification of Eq. (2) due to evolution and redshift losses. A neutrino with observed energy E must be produced at redshift z with energy $(1+z)E$. Thus, the present number density of neutrinos above energy E is given by

$$n_\nu(>E)=\int_0^{z_{\max}} dz \frac{dt}{dz}\, \dot{n}_\nu[>(1+z)E,z]$$

$$=\dot{n}_0(>E)\int_0^{z_{\max}} dz \frac{dt}{dz}(1+z)^{-1} f(z). \qquad (4)$$

Here we have used the fact that $\dot{n}_\nu(>E)\propto E^{-1}$ and denoted the ratio of (comoving) neutrino production rate at redshift z to the present rate, $\dot{n}_0$, by $f(z)$. Comparing Eqs. (2) and (4), and noting that $t_H\equiv\int_0^\infty dz(dt/dz)$, we find that the intensity $I_{\max}$ of Eq. (2) should be multiplied by a correction factor

$$\xi_Z=\frac{\int_0^{z_{\max}} dz g(z)(1+z)^{-7/2} f(z)}{\int_0^\infty dz g(z)(1+z)^{-5/2}}. \qquad (5)$$

Here, $g(z)\equiv -H_0(1+z)^{5/2}(dt/dz)$ is a weak function of redshift and cosmology; $g(z)\equiv 1$ for a flat universe with zero cosmological constant. Let us assume that the neutrino energy generation rate evolves rapidly with redshift, following the luminosity density evolution of QSOs [9], which may be described as $f(z)=(1+z)^\alpha$ with $\alpha\approx 3$ [10] at low redshift, $z<1.9$, $f(z)=$const for $1.9<z<2.7$, and an exponential decay at $z>2.7$ [11]. Using this functional form of $f(z)$, which is also similar [9] to that describing the evolution of star formation rate [12], we find that $\xi_Z\approx 3$ (with weak dependence on cosmology). For no evolution, $f(z)=$const, we have $\xi_Z\approx 0.6<1$ (with only a weak dependence on cosmology) due to a redshift energy loss of neutrinos.

III. CAN THE UPPER BOUND BE AVOIDED BY INVOKING MAGNETIC FIELDS?

One may try to invent arguments in order to avoid the upper bound on the neutrino flux by hypothesizing strong magnetic fields, which would affect the propagation of cosmic-ray protons, either in the neutrino source or in the inter-Galactic medium between the source and Earth. We show in this section that magnetic fields in the source cannot affect the upper bound, and that observational constraints on inter-Galactic magnetic fields imply that proposed field scenarios also do not affect the upper bound.

A. Magnetic fields in the neutrino source

One might try to argue that the upper bound $I_{\max}$ can be avoided even for sources with small photo-meson optical depth, if protons are prevented from escaping the source by magnetic confinement. However, a photo-meson interaction producing a charged pion also converts the proton to a neutron, which is not magnetically confined and will escape a source with small photo-meson optical depth before decaying to a proton. A neutron of high energy E propagates a distance $100(E/10^{19}\text{ eV})$ kpc prior to its decay. Thus, magnetic fields within the neutrino source cannot be used to evade the upper bound given in Eq. (2).

B. Uniformly distributed inter-galactic magnetic fields

The existence of uniformly distributed inter-galactic magnetic fields may limit the propagation distance of protons and therefore prevent their arrival at Earth from distant sources. However, imposing a limit on the propagation distance of cosmic-rays will not affect any of the arguments presented in this paper. Compare, e.g., the case where protons propagate on straight lines to the case where cosmic rays are confined by magnetic fields to their place of origin. In the former case, the present proton flux is obtained by integrating the contribution of distant sources over redshift, while in the latter case it is obtained by integrating the contribution from local sources over cosmic time. For a homogeneous universe both procedures yield the same result [e.g. the integration in Eq. (4) may be interpreted as integration over redshift or over time]. Thus, limiting the cosmic-ray propagation distance may affect the upper bound $I_{\max}$ only if propagation is limited to a distance d over which the cosmic ray production rate, averaged over a Hubble time, is inhomogeneous. In this case, if the cosmic-ray production rate in our local region of size d is lower than the universe average, the observed cosmic-ray flux would be lower than average and the neutrino flux, which is homogeneous throughout the universe, may exceed a bound based on the local cosmic-ray flux. However, the magnetic fields required to make such a scenario viable, i.e. to confine cosmic rays to small enough d to significantly affect $I_{\max}$, are large and are inconsistent with observations.

Consider a proton of energy E propagating through an intergalactic magnetic field of strength B and correlation length λ. Propagating a distance λ the proton is deflected by an angle $\sim\lambda/R_L$, where $R_L=E/eB$ is the Larmor radius. For the parameters of interest (see below) the deflection angle is small, and propagating a distance l the proton is deflected by an angle $(l/\lambda)^{1/2}\lambda/R_L$. Thus, we may define an effective mean free path, the propagation distance over which large deflection occurs, by $(l\lambda)^{1/2}/R_L=1$ and a diffusion coefficient for proton propagation, $D=lc/3$. For a propagation time t, protons are confined to a region of size $d\sim(Dt)^{1/2}=(ct/3\lambda)^{1/2}R_L$. For $t=t_H\approx 10^{10}$ yr, we have $d\sim 1(E/3\times 10^{19}\text{ eV})(B_{\rm nG}\lambda_{\rm Mpc}^{1/2})^{-1}$ Gpc. The propagation distance is determined by the product $B\lambda^{1/2}$. The upper limit on the intergalactic magnetic field implied by QSO Faraday rotation measurements, $B\lambda^{1/2}<1\text{ nG Mpc}^{1/2}$ [13,14], implies $d>1(E/3\times 10^{19}\text{ eV})$ Gpc. We conclude that the existence of a uniformly distributed inter-Galactic magnetic field would have no effect on $I_{\max}$.

C. Magnetic fields in large scale structures

The discussion in the previous subsections shows that the magnetic field in the source or a uniformly distributed inter-Galactic field would not affect the upper bound $I_{\max}$. Could magnetic fields associated with large scale galaxy structures, i.e. clusters, filaments and sheets, affect the upper bound?

Let us first consider galaxy clusters, where inter-galactic fields had been detected. The analysis of rotation measures of radio sources lying in the background of rich clusters implies the existence of strong fields between galaxies, $B \sim 1\ \mu$G with $\lambda \sim 10$ kpc, in the central 0.5 Mpc, cluster region [15]. However, confinement of high energy protons in the cores of rich clusters would have little effect on our results, unless most of the high energy neutrino sources reside in the centers of rich clusters, which is not the case for either the hypothesized AGN or GRB neutrino sources. Moreover, high-energy protons cannot be confined even in the central regions of a rich cluster, since for $B \sim 1\ \mu$G and $\lambda \sim 10$ kpc we have $d \sim 10(E/3\times 10^{19}\text{eV})$ Mpc over a Hubble time.

We next turn to large-scale filaments and sheets. Kulsrud *et al.* [16] have recently suggested that magnetic fields could be amplified by turbulence associated with the formation of large scale filaments and sheets to near equipartition with turbulent kinetic energy. For characteristic turbulent velocities of $v_t \sim 300$ km/s on ~ 1 Mpc scale, and characteristic filament and sheet densities of $n \sim 10^{-6}\ \text{cm}^{-3}$, this scenario predicts magnetic fields $B \sim 0.1(n/10^{-6}\ \text{cm}^{-3})^{1/2}$ $(v_t/300\ \text{km/s})\ \mu$G in the high density large scale filaments and sheets, with coherence length $L \sim 1$ Mpc, comparable to the filament (sheet) diameter (thickness). It is not clear whether the suggested scenario for an increase in the magnetic field strength and coherence length to equipartition with the largest turbulent eddies can be realized. Furthermore, even for a turbulent velocity of order several hundreds km per sec, the turnaround time of a ~ 1 Mpc eddy is longer than the Hubble time, and it is therefore not clear whether equipartition with the largest scale is achievable. Nevertheless, we consider this scenario here since it is consistent with the upper limit, $B < 1\ \mu$G, implied for a field coherent over ~ 1 Mpc inside high density large scale structures by QSO rotation measures [17]. Although the Larmor radius of a $\sim 10^{19}$ eV proton is smaller than $L \sim 1$ Mpc, confinement of particles would require a special field configuration. Even if such a configuration is produced by random turbulent motions, which seems unlikely, variation of the field over a scale L gives rise to particle drift velocity, $v_d \sim R_L c/L$, and therefore to the escape of particles on time scale $t_e \sim L^2/cR_L = 10^7\ (L/1\ \text{Mpc})^2\ (B/0.1\ \mu\text{G})\ (E/3\times 10^{19}\ \text{eV})$ yr. Since $t_e \ll t_H$, the hypothesized large-scale structure magnetic fields cannot affect the bound on neutrino flux.

Finally, we note that several authors have recently considered cosmic-ray proton propagation in a hypothesized large scale magnetic field, associated with our local super-cluster, of 0.1 μG strength and 10 Mpc coherence length corresponding to a hypothetical local turbulent eddy of comparable size [18,19]. For these parameters as well we have $t_e \ll t_H$, and therefore even this field structure would not affect the neutrino bound. Nevertheless, two points should be made. First, if cosmic-rays are confined to our local super-cluster, one would expect the local cosmic-ray flux to be higher than average (since the production rate averaged over Hubble time should be higher than average in overdense large scale regions), implying that the upper bound on the neutrino intensity is lower than $I_{\max}$ derived here. Second, it is hard to understand how the hypothesized magnetic field structure could have been formed. The overdensity in the local super-cluster is not large and an equipartition magnetic field of strength 0.1 μG therefore corresponds to a turbulent velocity $v_t \sim 10^3$ km/s. A turbulent eddy of this velocity coherent over tens of Mpc is inconsistent with local peculiar velocity measurements (See e.g. [20] for a recent review). Moreover, the corresponding eddy turnaround time is larger than the Hubble time.

IV. AGN JET MODELS

We consider in this section some popular models for neutrino production in which high-energy neutrinos are produced in the jets of active galactic nuclei [4]. In these models, the flux of high-energy neutrinos received at Earth is produced by ''blazars,'' AGN jets nearly aligned with our line of sight. Since the predicted neutrino intensities for these models exceed by typically two orders of magnitude the upper bound, Eq. (2), based on observed cosmic ray fluxes, it is important to verify that the models satisfy the assumption on which Eq. (2) is based, i.e. optical depth <1 to p-γ interaction.

The neutrino spectrum and flux are derived in AGN jet models on the basis of the following key considerations. It is assumed that protons are Fermi accelerated in the jet to high energy, with energy spectrum $dN_p/dE_p \propto E_p^{-2}$. For a photon spectrum $dN_\gamma/dE_\gamma \propto E_\gamma^{-2}$, as typically observed, the number of photons with energy above the threshold for pion production is proportional to the proton energy E_p (the threshold energy is inversely proportional to E_p). This implies that the proton photo-meson optical depth is proportional to E_p, and therefore, assuming that the optical depth is small, that the resulting neutrino spectrum is flatter than the proton spectrum, namely $dN_\nu/dE_\nu \propto E_\nu^{-1}$, as shown in Fig. 1. The spectrum extends to a neutrino energy which is $\approx 5\%$ of the maximum accelerated proton energy, which is typically 10^{19} eV in the models discussed.

The production of charged pions is accompanied by the production of neutral pions, whose decay leads to the emission of high-energy gamma-rays. It has been claimed [21] that the observed blazar emission extending to ~ 10 TeV [22] supports the hypothesis that the high-energy emission is due to neutral pion decay rather than to inverse Compton scattering by electrons. Thus, the normalization of the neutrino flux is determined by the assumption that neutral pion decay is the source of high-energy photon emission and that this emission from AGN jets produces the observed diffuse γ-ray background, $\Phi_\gamma(>100\ \text{MeV}) = 10^{-8}\ \text{erg/cm}^2\ \text{s sr}$ [23]. Under these assumptions the total neutrino energy flux is similar to the γ-ray background flux (see Fig. 1).

In the AGN jet models discussed above, the proton photo-meson optical depth $\tau_{p\gamma}$ at $E_p \leqslant 10^{19}$ eV is smaller than unity. This is evident from the neutrino energy spectrum shown in Fig. 1, which is flatter than the assumed proton spectrum at $E_p \leqslant 10^{19}$, as explained above. In fact, it is easy to see that these models are constrained to have $\tau_{p\gamma} \leqslant 10^{-3}$ at $E_p \sim 10^{19}$ eV. The threshold energy of photons for pair-production in interaction with a 1 TeV photon is similar to the photon energy required for resonant meson production in interaction with a proton of energy $E_p = 0.2\,\mathrm{GeV}^2/(0.5\,\mathrm{MeV})^2 \times 1\,\mathrm{TeV} = 10^{18}$ eV. Emission of ~ 1 TeV photons from blazars is now well established [24], and there is evidence that the high-energy photon spectrum extends as a power-law at least to ~ 10 TeV [22]. This is the main argument used [21] in support of the hypothesis that high-energy emission from blazars is due to pion decay rather than inverse Compton scattering. The observed high-energy emission implies that the pair-production optical depth for ~ 1 TeV photons is small, and that $\tau_{p\gamma} \leqslant 10^{-4}$ $(E_p/10^{18}\,\mathrm{eV})$, since the cross section for pair production is $\sim 10^4$ times larger than the cross section for photo-meson production. This result guarantees that the upper bound $I_{\max}$ on the neutrino intensity is valid for AGN jet models.

V. GAMMA-RAY BURSTS

In the GRB fireball model for high-energy neutrinos, the cosmic ray observations are naturally taken into account and the upper limit on high energy neutrino flux is automatically satisfied. In fact, it was the similarity between the energy density in cosmic ray sources implied by the cosmic ray flux observations and the GRB energy density in high energy protons that led to the initial suggestion that GRBs are the the source of high energy protons. Just as for AGN jets, the GRB fireballs are optically think to γ-p interactions that produce pions but—unlike the AGN jet models—the GRB model predicts a neutrino flux that satisfies the cosmic-ray upper bound discussed in Sec. II.

A. Neutrinos at energies $\sim 10^{14}$ eV

In the GRB fireball model [25], which has recently gained support from GRB afterglow observations [26], the observed gamma rays are produced by synchrotron emission of high-energy electrons accelerated in internal shocks of an expanding relativistic wind, with characteristic Lorentz factor $\Gamma \sim 300$ [27]. In this scenario, observed gamma-ray flux variability on time scale Δt is produced by internal collisions at radius $r_d \approx \Gamma^2 c \Delta t$ that arise from variability of the underlying source on the same time scale [28]. In the region where electrons are accelerated, protons are also expected to be shock accelerated, and their photo-meson interaction with observed burst photons will produce a burst of high-energy neutrinos accompanying the GRB [3]. If GRBs are the sources of ultra-high-energy cosmic-rays [29,30], then the expected GRB neutrino intensity is [3]

$$E_\nu^2 \Phi_{\nu_\mu} \approx E_\nu^2 \Phi_{\bar{\nu}_\mu} \approx E_\nu^2 \Phi_{\nu_e} \approx \frac{1}{2} f_\pi I_{\max} \approx 1.5 \times 10^{-9} \left(\frac{f_\pi}{0.2}\right) \min\{1, E_\nu / E_\nu^b\} \ \ \mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}, \quad E_\nu^b \approx 10^{14} \ \ \mathrm{eV}. \tag{6}$$

Here, f_π is the fraction of energy lost to pion production by high-energy protons. The derivation of f_π and E_ν^b and their dependence on GRB model parameters is given in the appendix [Eqs. (A2), (A1)]. The intensity given by Eq. (6) is ~ 5 times smaller than that given in Eq. (8) of Ref. [3], due to the fact that in Eq. (8) of Ref. [3] we neglected the logarithmic correction $\ln(100) = 5$ of Eq. (1).

The GRB neutrino intensity can be estimated directly from the observed gamma-ray fluence. The Burst and Transient Source Experiment (BATSE) measures the GRB fluence F_γ over a decade of photon energy, ~ 0.1 MeV to ~ 1 MeV, corresponding to half a decade of radiating electron energy (the electron synchrotron frequency is proportional to the square of the electron Lorentz factor). If electrons carry a fraction f_e of the energy carried by protons, then the muon neutrino fluence of a single burst is $E_\nu^2 dN_\nu/dE_\nu \approx 0.25 (f_\pi/f_e) F_\gamma/\ln(3)$. The average neutrino flux per unit time and solid angle is obtained by multiplying the single burst fluence with the GRB rate per solid angle, $\approx 10^3$ bursts per year over 4π sr. Using the average burst fluence $F_\gamma \approx 6 \times 10^{-6}$ erg/cm^2, we obtain a muon neutrino intensity $E_\nu^2 \Phi_\nu \approx 3 \times 10^{-9} (f_\pi/f_e)$ GeV/cm^2 s sr. Recent GRB afterglow observations typically imply $f_e \sim 0.1$ [26], and therefore $f_\pi/f_e \sim 1$. Thus, the neutrino intensity estimated directly from the gamma-ray fluence agrees with the estimate (6) based on the cosmic-ray production rate.

B. Neutrinos at high energy $>10^{16}$ eV

The neutrino spectrum (6) is modified at high energy, where neutrinos are produced by the decay of muons and pions whose lifetime $\tau_{\mu,\pi}$ exceeds the characteristic time for energy loss due to adiabatic expansion and synchrotron emission [3,5]. The synchrotron loss time is determined by the energy density of the magnetic field in the wind rest frame. For the characteristic parameters of a GRB wind, the muon energy for which the adiabatic energy loss time equals the muon lifetime, E_μ^a, is comparable to the energy E_μ^s at which the lifetime equals the synchrotron loss time, τ_μ^s. For pions, $E_\pi^a > E_\pi^s$. This and the fact that the adiabatic loss time is independent of energy and the synchrotron loss time is inversely proportional to energy imply that synchrotron losses are the dominant effect suppressing the flux at high energy. The energy above which synchrotron losses suppress the neutrino flux is

$$\frac{E^s_{\nu_\mu(\bar{\nu}_\mu,\nu_e)}}{E^b_\nu}\approx(\xi_B L_{\gamma,51}/\xi_e)^{-1/2}\Gamma^2_{300}\Delta t_{\rm ms}(E^b_\gamma/1\ \ \mathrm{MeV})\times\begin{cases}10 & \text{for } \bar{\nu}_\mu,\nu_e,\\ 100 & \text{for } \nu_\mu.\end{cases} \tag{7}$$

Here, $L_\gamma = 10^{51} L_{\gamma,51}$ erg/s is the observed gamma-ray luminosity, $\Delta t = 1\Delta t_{\rm ms}$ ms is the observed GRB variability time scale, $E^b_\gamma \sim 1$ MeV is the observed GRB photon break energy, $\Gamma = 300\Gamma_{300}$, and ξ_e and ξ_B are the fractions of GRB wind luminosity carried by electrons and magnetic fields. The observational constraints on these parameters are discussed in the Appendix. At neutrino energy $E_\nu \gg E^s_\nu$, the probability that a pion (muon) would decay before losing its energy is approximately given by the ratio of synchrotron cooling time to decay time $\tau^s_{\pi(\mu)}/\tau_{\pi(\mu)} = [E_\nu/E^s_{\nu_\mu(\bar{\nu}_\mu,\nu_e)}]^{-2}$, and the intensity of Eq. (6) is suppressed by a similar factor;

$$E^2_\nu\Phi_{\nu_\mu(\bar{\nu}_\mu,\nu_e)}\approx 1.5\times10^{-9}\left(\frac{f_\pi}{0.2}\right)\times\left[\frac{E_\nu}{E^s_{\nu_\mu(\bar{\nu}_\mu,\nu_e)}}\right]^{-2}\ \ \mathrm{GeV\ cm^{-2}\ s^{-1}\ sr^{-1}},\quad E_\nu\gg E^s_\nu. \tag{8}$$

Since the wind duration, i.e. the time over which energy is released from the source, is $T\sim 1$ s, internal shocks may occur due to variability on time scale δt larger than the source dynamical time, $\Delta t \sim 1\ \mathrm{ms} \leqslant \delta t \leqslant T \sim 1$ s. Collisions due to variability $\delta t > \Delta t$ are less efficient in producing neutrinos, $f_\pi \propto \delta t^{-1}$, since the radiation energy density is lower at larger collision radii, leading to a smaller probability for photo-meson interaction. However, at larger radii synchrotron losses cut off the spectrum at higher energy, $E^s(\delta t) \propto \delta t$. Collisions at large radii therefore result in extension of the neutrino spectrum of Eq. (6) to higher energy, beyond the cutoff energy Eq. (7), and therefore yield $E^2_\nu\Phi_\nu \propto E^{-1}_\nu$ for $E_\nu > E^s_\nu(\Delta t)$, since $f_\pi \propto \delta t^{-1} \propto [E^s(\delta t)]^{-1}$. This extension is shown in Fig. 1. We note that on a time scale $\Delta t \sim 1$ s the expanding wind is expected to interact with the surrounding medium, driving a relativistic shock into the ambient gas. Protons are expected to be accelerated to high energy in this shock, the ''external'' shock, as well. The neutrino intensity and spectrum produced in the external shock are given by Eqs. (A2), (6), (7) with $\Delta t \sim 1$ s. Because of the low efficiency of the external shock, $f_\pi \sim 10^{-4}$, its contribution to the neutrino flux is small. Note that as the external shock expands through a larger mass of the ambient gas it decelerates, and therefore on a time scale $\Delta t \gg 1$ s the shock Lorentz factor is not large enough to allow acceleration of protons to high energy.

C. Comparison with other authors

In agreement with Rachen and Mészáros [5], we find that the neutrino flux from GRBs is small above 10^{19} eV, and that a neutrino flux comparable to the γ-ray flux is expected only below $\sim 10^{17}$ eV. Our result is not in agreement, however, with that of Ref. [31], where a much higher flux at $\sim 10^{19}$ eV is obtained based on the equations of Ref. [3], which are the same equations as used here. There is a numerical error in the calculations of Ref. [31].[1] Finally, we note that the highest energy to which protons can be accelerated increases with the collision radius $E^{\max}_p \propto \delta t^{1/3}$ [29], and while $E^{\max}_p \sim 10^{20}$ eV for $\delta t \sim \Delta t \sim 1$ ms, collisions at larger radii, $\delta t \sim 0.03$ s, are required to allow acceleration to the highest observed energy, $\sim 3\times10^{20}$ eV. In agreement with Rachen and Mészáros [5], we find that at this radius the neutrino spectrum produced through photo-meson interactions extends to $\sim 10^{18}$ eV [see Eqs. (7), (A1)]. There is no contradiction, however, between production of high-energy protons above $\sim 3\times10^{20}$ eV and a break in the neutrino spectrum at $\sim 10^{16}$ eV, since the efficiency of neutrino production at collision radius corresponding to $\delta t \sim 0.03$ s is small and most of the flux is produced by collisions at smaller radii.

VI. DISCUSSION

We have shown that cosmic-ray observations set a model-independent upper bound of $E^2\Phi_\nu < 2\times10^{-8}$ GeV/cm^2 s sr to the intensity of high-energy neutrinos produced by photo-meson interaction in sources of size not much larger than the proton photo-meson mean-free-path, e.g. AGN jets and GRBs (see Fig. 1). This limit cannot be avoided by hypothesizing evolutionary effects of the sources (see discussion in Sec. II C) or by invoking magnetic field scenarios (see discussion in Sec. III).

Of possibly even greater interest to photon astronomers, we have shown that the cosmic ray measurements rule out the current version of theories in which the gamma-ray background is due to photo-meson interactions in AGN jets.

The neutrino flux predictions of AGN jet models are based on two key assumptions, namely that AGN jets produce the observed gamma-ray background and that high-energy photon emission from AGN jets is due to decay of neutral pions produced in photo-meson interactions of protons accelerated in the jet to high energy. Since the neutrino flux predicted by these assumptions is two orders of magnitude higher than the upper bound allowed by cosmic ray observations (see Fig. 1), at least one of the key assumptions is not valid, presumably the assumption that high-energy photon emission from AGN jets is due to photo-meson interactions. This conclusion is supported by multi-wavelength

[1]The parameters chosen in [31] are $L_\gamma = 10^{50}$ erg/s, $\Delta t = 10$ s, and $\Gamma = 100$. Using our equation (4) of Ref. [3], which is the same as Eq. (A2) of the present paper, we obtain for these parameters $f_\pi = 1.6\times10^{-4}$, while the author of [31] obtains, using the same equation, $f_\pi = 0.03$.

observations of the blazar Mkn 421, which show contemporaneous strong variability at TeV and x-ray energies with little evidence for GeV and optical variability [32,33]. This behavior suggests that the high-energy photon emission is due to inverse-Compton scattering by relativistic electrons [32].

Even if AGNs produce part of their high energy emission through γ-p interactions, the upper bound derived here implies an upper bound of ~ 1 detected AGN neutrino per year in a high-energy neutrino telescope with an effective area of 1 km^2 [1]. The discussion in this paper shows that there is no observational motivation that would lead one to expect that the flux of high-energy neutrinos from AGNs will be measurable in a km^2 telescope.

The upper bound Eq. (2) also applies to the intensity of high-energy neutrinos that may be produced through the decay of charged pions created by p-$p \rightarrow \pi^{\pm} X$ (rather than p-γ) interactions, as long as the p-p optical depth in the source is not large. At present, predictions of high-energy neutrino flux based on such models are not available in the literature.

The cosmic-ray flux below 3×10^{18} eV is steeper than at higher energy. This is most likely due [2] to a contribution to the cosmic-ray flux at lower energy from Galactic sources of heavy ions. This view has recently gained support from the detection in the Fly's Eye data of a small but statistically significant enhancement of the flux of cosmic-rays in the energy range of 2×10^{17} eV to 3×10^{18} eV along the Galactic plane [8]. Extra-galactic sources of cosmic-rays may therefore exist that produce cosmic-rays with energy $<3\times10^{18}$ eV at a rate higher than given by Eq. (1). While we have no observational evidence for the existence of such sources, we cannot rule them out based on cosmic-ray observations, and they may produce a flux of $<10^{17}$ eV neutrinos which is higher than the upper limit implied by Eq. (2). Note that this argument does not affect the validity of the upper bound (2) for AGN models, since the neutrino emission from these sources peaks at $\sim10^{18}$ eV.

The neutrino flux predicted by the GRB model is consistent with the upper bound derived here. The intensity estimate we give here, Eq. (6), is ~5 times smaller than that we gave in Ref. [3], where the logarithmic correction of Eq. (1) was neglected. The intensity calculated here implies a detection rate of ~20 neutrino induced muons per year for a 1 km^2 detector (over 4π sr). As discussed in [3], one may look for neutrino events in spatial and temporal coincidence (on a time scale of seconds) with GRBs.

The GRB neutrino spectrum is consistent with the secondary-particle cooling constraints derived by Rachen and Mészáros [5]. The neutrino flux above $\sim10^{16}$ eV is suppressed, but this is also consistent with the acceleration of protons to $>3\times10^{20}$ eV (see Sec. V C).

Finally, we note that the GRB neutrino flux discussed here is the flux produced *in situ*, i.e. within the source. The energy loss of high-energy protons, $>5\times10^{19}$ eV, through photo-meson production in interaction with microwave background photons would lead to a background neutrino intensity (which will not be temporally associated with GRBs) comparable to the upper bound shown in Fig. 1 at $E_\nu \sim10^{18}$ eV. This flux of high-energy neutrinos should exist regardless of the nature of the high-energy proton sources (assuming that these sources are indeed extragalactic: see [34]).

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APPENDIX: NEUTRINO PRODUCTION IN GRBS

GRBs are possible sources of high-energy cosmic-rays [29,30], which may account for the observed extra-Galactic high-energy proton flux [29,7]. In the GRB fireball model [25], which has recently gained support from GRB afterglow observations [26], the observed gamma rays are produced by synchrotron emission of high-energy electrons accelerated in internal shocks of an expanding relativistic wind. The hardness of the observed spectrum, which extends to ~100 MeV, requires wind Lorentz factors $\Gamma\sim300$ [27]. In this scenario, observed gamma-ray flux variability on a time scale Δt corresponds to internal collisions at a radius $r_d \approx \Gamma^2 c\Delta t$, which arise from variability of the underlying source on the same time scale [28]. Rapid variability time, ~1 ms, observed in some GRBs [35], and the fact that a significant fraction of bursts detected by the Burst and Transient Source Experiment (BATSE) show variability on the smallest resolved time scale, ~10 ms [36], imply that the sources are compact, with linear scale $r_0\sim10^7$ cm and characteristic dynamical time ~1 ms.

In the region where electrons are accelerated, protons are also expected to be shock accelerated, and their photo-meson interaction with observed burst photons will produce a burst of high-energy neutrinos accompanying the GRB [3]. The neutrino spectrum is determined in this model by the observed gamma-ray spectrum, which is well described by a broken power-law, $dN_\gamma/dE_\gamma \propto E_\gamma^{-\beta}$, with different values of β at low and high energy [37]. The observed break energy (where β changes) is typically $E_\gamma^b\sim1$ MeV, with $\beta\simeq1$ at energies below the break and $\beta\simeq2$ above the break. The interaction of protons accelerated to a power-law distribution, $dN_p/dE_p \propto E_p^{-2}$, with GRB photons results in a broken power law neutrino spectrum [3], $dN_\nu/dE_\nu \propto E_\nu^{-\beta}$, with $\beta=1$ for $E_\nu<E_\nu^b$ and $\beta=2$ for $E_\nu>E_\nu^b$ (see Fig. 1). The neutrino break energy E_ν^b is fixed by the threshold energy of protons for photo-production in interaction with the dominant ~1 MeV photons in the GRB,

$$E_\nu^b \approx 5\times10^{14}\Gamma_{300}^2 (E_\gamma^b/1\ \text{MeV})^{-1}\ \text{eV}, \tag{A1}$$

where $\Gamma=300\Gamma_{300}$.

The normalization of the flux is determined by the efficiency of pion production. As shown in [3], the fraction of energy lost to pion production by protons producing the neutrino flux above the break, E^b_ν, is essentially independent of energy and is given by

$$f_\pi = 0.20 \frac{L_{\gamma,51}}{(E^b_\gamma/1\ \text{MeV})\Gamma^4_{300}\Delta t_{\text{ms}}}. \tag{A2}$$

Here $\Delta t = 1\Delta t_{\text{ms}}$ ms and $L_\gamma = 10^{51} L_{\gamma,51}$ erg/s is the observed gamma-ray luminosity. The values of Γ and Δt in Eq. (A2) are determined by the hardness of the γ-ray spectrum and by the flux variability. These parameters are also constrained by the fact that the characteristic observed photon energy is ~ 1 MeV. Internal collisions are expected to be ''mildly'' relativistic in the fireball rest frame [28], i.e. characterized by the Lorentz factor $\gamma_i - 1 \sim 1$, since adjacent shells within the wind are expected to expand with similar Lorentz factors. The internal shocks would therefore heat the protons to random velocities (in the wind frame) $\gamma_p - 1 \sim 1$. The characteristic frequency of synchrotron emission is determined by the characteristic energy of the electrons and by the strength of the magnetic field. These are determined by assuming that the fraction of energy carried by electrons is ξ_e, implying a characteristic rest frame electron Lorentz factor $\gamma_e = \xi_e(m_p/m_e)$, and that a fraction ξ_B of the energy is carried by the magnetic field, implying $4\pi r_d^2 c\Gamma^2 B^2/8\pi = \xi_B L$ where L is the total wind luminosity. Since the electron synchrotron cooling time is short compared to the wind expansion time, electrons lose their energy radiatively and $L \approx L_\gamma/\xi_e$. The characteristic observed energy of synchrotron photons, $E^b_\gamma = \Gamma\hbar\gamma_e^2 eB/m_e c$, is therefore

$$E^b_\gamma \approx 4\xi_B^{1/2}\xi_e^{3/2}\frac{L^{1/2}_{\gamma,51}}{\Gamma^2_{300}\Delta t_{\text{ms}}}\ \text{MeV}. \tag{A3}$$

At present, there is no theory that allows the determination of the values of the equipartition fractions ξ_e and ξ_B. However, for values close to equipartition, the model photon break energy E^b_γ is consistent with the observed E^b_γ for $\Gamma = 300$ and $\Delta t = 1$ ms.

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