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LOCALIZED HELIOSEISMIC CONSTRAINTS ON SOLAR STRUCTURE

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ABSTRACT

Localized differences between the real Sun and standard solar models are shown to be small. The sound speeds of the real and the standard model Suns typically differ by less than 0.3% for regions of radial width $\simeq 0.1\ R_\odot$ in the solar core.

Subject headings: nuclear reactions, nucleosynthesis, abundances — Sun: interior — Sun: oscillations

1. INTRODUCTION

Since the first discrepancy was reported almost 30 years ago between observations of solar neutrinos (Davis, Harmer, & Hoffman 1968) and the predicted fluxes of solar neutrinos (Bahcall, Bahcall, & Shaviv 1968), many authors have proposed ad hoc changes in the input data to solar models or have hypothesized evolutionary scenarios in which the real Sun differs from computed standard solar models. On detailed examination, the suggested modifications of the solar models have failed because of specific conflicts with other astronomical data or with laboratory measurements. Nevertheless, the pressure to examine all allowable modifications of solar models has intensified as new solar neutrino experiments, combined with more precise solar model predictions and the development of attractive particle physics explanations of the solar neutrino discrepancies, have made it clear that solar neutrinos might be revealing new physics beyond what is described by the minimal standard electroweak theory.

In the past few years, helioseismic measurements have become increasingly precise (see, e.g., Hill et al. 1996; Kosovichev et al. 1997) and now include accurate measurements of the low-degree p-mode frequencies that extend well into the core of the Sun ($r \leq 0.3\ R_\odot$), where nuclear fusion occurs and solar neutrinos are produced. Several groups have shown recently that the standard solar model predicts results for the sound speeds that are in remarkably good agreement with those measured by helioseismological techniques: the typical deviations between the models and the measurements are less than or of the order 0.2% in the solar core and 0.1% rms throughout nearly all of the Sun (see Bahcall et al. 1997; see also Guenther & Demarque 1997; Antia & Chitre 1997; Kosovichev et al. 1997; Basu et al. 1996b; Gough et al. 1996).

These previous studies show that the standard solar model represents well the overall structure of the Sun. The sound-speed differences between the Sun and standard solar models inferred from helioseismology are convolved with weighting kernels that average the available information over appreciable volumes of the Sun (see Fig. 1).

In this Letter, we set general constraints on possible localized deviations from standard solar models by making use of an accurate data set of helioseismic frequencies. We rule out localized differences between sound speeds in the Sun and in standard solar models that are large enough to affect significantly predictions of solar neutrino fluxes. For specificity, we consider Gaussian perturbations, but our numerical results are expected to be also valid for oscillatory perturbations for which the wavelength is comparable to or larger than the typical width of the averaging kernels (i.e., perturbation wavelengths $\gtrsim 0.1\ R_\odot$).

Helioseismic inversions yield values for the sound speed in the Sun that are essentially proportional to $(T/\mu)^{1/2}$ (where T is the temperature and μ is the mean molecular weight). In the absence of unexpected fine-tuned cancellations between perturbations to T and μ, even tiny fractional errors in the models values of T or μ would produce measurable discrepancies in the precisely determined helioseismic sound speeds. Localized modifications of the temperature that produce changes exceeding a few tenths of a percent in the sound-speed profiles of standard solar models are ruled out in the regions tested by existing helioseismic measurements. By contrast, the internal temperature in the standard solar model must be changed by typically 5%–10% in order to affect solar neutrino calculations in an important way.[1] We concentrate here on assumed departures from the standard solar model that are confined to shells of characteristic widths of order $0.1\ R_\odot$ since that is the typical width in which the solar neutrinos are produced. However, we also consider perturbations in shells of width somewhat larger and somewhat smaller than $0.1\ R_\odot$ and show that the observational upper limit to the amplitude of a sound-speed perturbation, obtained from helioseismology, is inversely proportional to the thickness of the shell.

[1] As the referee points out, there is a mathematical possibility that sound velocities in a nonstandard solar model could approximately equal the sound velocities in the standard model to an accuracy of better than 0.2%, but nevertheless the temperature differences are substantially larger. We do not investigate such scenarios in this Letter since no physical basis for so precise a fine tuning has ever been suggested.

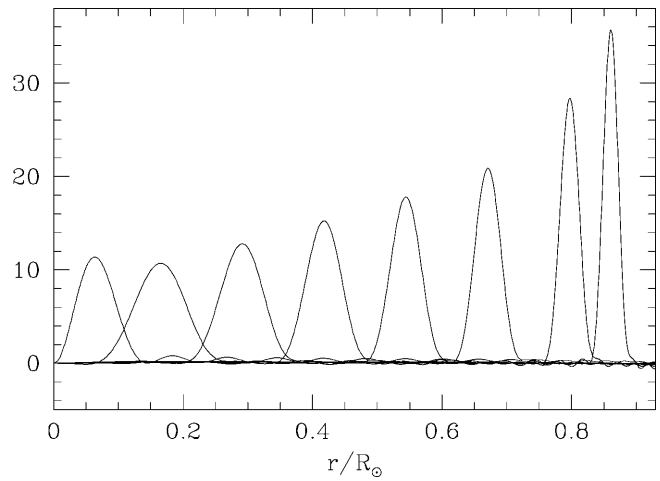

FIG. 1.—Sample of the normalized averaging kernels for the relative sound-speed difference obtained with the first year's data from the LOWL instrument (Tomczyk et al. 1995). These correspond to the inversion results in Basu et al. (1996b).

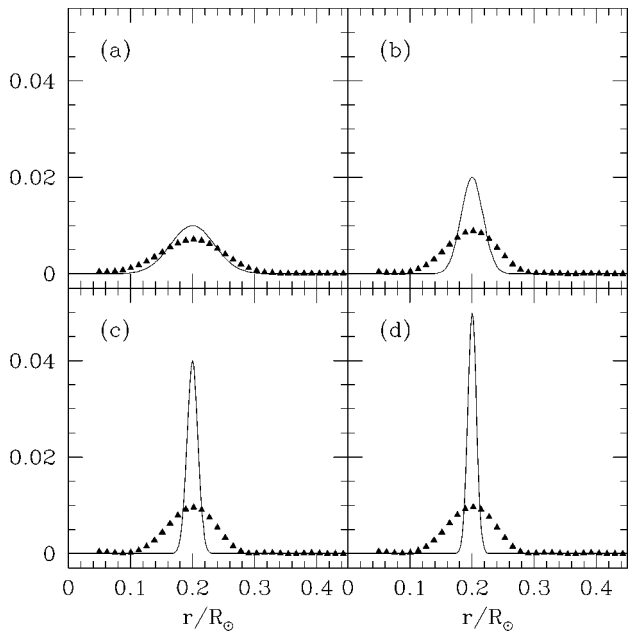

FIG. 3.—Fractional Gaussian perturbations with identical areas applied at $0.2\ R_\odot$ (*curves*) and the result of convolving the perturbation with the averaging kernels (*symbols*). The detectability of a hypothesized perturbation depends primarily on the assumed area of the perturbation.

We present the results of our calculations in § 2 and Figures 2 and 3 and then summarize and discuss the applications of our results in § 3.

2. CALCULATIONS

The observed solar *p*-mode frequencies can be inverted to determine localized averages of the relative sound-speed differences between the Sun and a specified solar model. The averages are taken over the averaging kernels, which are linear combinations of the functions that relate the sound-speed differences between the Sun and a model to the differences of the frequencies. The averaging kernels are constructed so that they are as narrow as allowed by the data, and their finite radial widths define the resolution of the inversion. The narrower the averaging kernel, the closer the inferred sound-speed difference to the true sound-speed difference at that point. Details on how the *p*-modes are inverted and the averaging kernels are formed are given in Basu et al. (1996a). The sidelobes of the kernels are small.

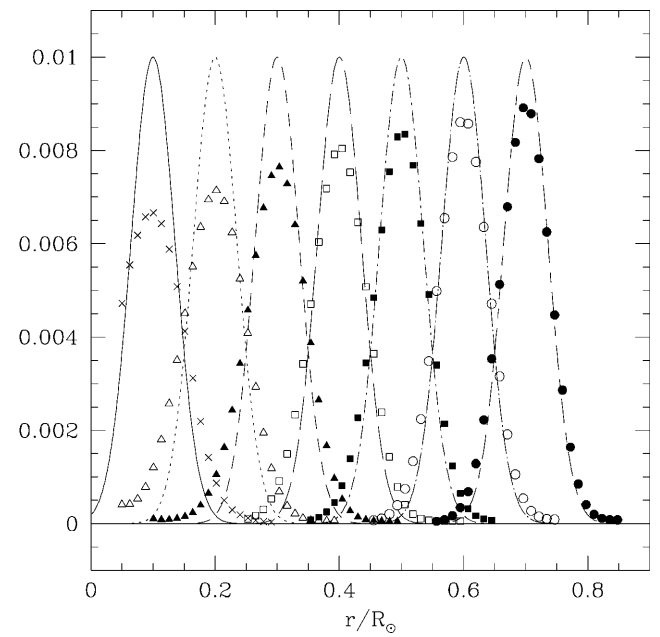

FIG. 2.—Fractional Gaussian perturbation in sound speed with 2 σ widths of $0.1\ R_\odot$ and height 0.01 applied at various radii (*curves*) and the result obtained by convolving the actual perturbations with the averaging kernels (*symbols*). The perturbations and the convolved results for the different radii are shown with different line and point styles for clarity.

The averaging kernels we use for this work are those obtained by inverting solar oscillation frequencies obtained by the LOWL instrument (Tomczyk et al. 1995) in its first year of operations. Figure 1 shows a representative sample of the averaging kernels. All *p*-modes in the LOWL data set with angular degree between $l = 0$ and 99 and frequency in the range 1–3.5 mHz were included in the construction of these kernels; the fractional errors in the frequency measurements for the LOWL data are of order a few times 10^{-5}. Reliable kernels cannot yet be constructed in the subsurface area from $0.95\ R_\odot$ to $1.0\ R_\odot$, but techniques have been found that minimize the effects of the surface layers on the inferred interior properties (see Basu et al. 1996a).

The measurement errors lead to an error of $\sim$0.05% in the inverted sound-speed difference in the solar core and $\sim$0.01% elsewhere. The LOWL data set was used to infer the difference in the sound-speed profile of the Sun and that of a standard solar model (Basu et al. 1996b) and has also been used to determine the sound-speed differences between the Sun and the Bahcall & Pinsonneault solar model used to predict solar neutrino fluxes (see Bahcall et al. 1997).

By direct comparison, we find that the differences in the sound speeds computed from the best standard solar model of Bahcall & Pinsonneault (1995) and the solar model S presented by Christensen-Dalsgaard et al. (1996) decrease from

0.3% at $R = 0.05\ R_\odot$ to 0.1% at $R = 0.15\ R_\odot$. Outside this innermost region of the Sun, the differences in computed sound speeds between the two models are always less than 0.1% and are typically less than a few hundredths of a percent. We conclude that these solar models constructed with independent numerical codes and with different implementations of the input physics nevertheless lead to almost the same sound speeds. Nevertheless, it will be important to determine in future work the specific reasons for the discrepancies between the two standard models, especially for $R \leq 0.15\ R_\odot$, where the model-to-model variations are comparable to the differences between the helioseismological values for the sound speeds and the model values.

We have introduced artificial perturbations in the sound speeds at different solar radii and have evaluated the sensitivity of the helioseismic data to these departures from the standard solar model. Any change to the sound-speed profile will, of course, be accompanied by other changes to the structure that are required to satisfy the constraint of hydrostatic equilibrium. However, the inversion procedure is such that we can invert for the sound-speed perturbations only.

Figure 2 shows the results of a series of seven perturbations introduced at different locations in the standard solar model, with the peaks of the perturbations appearing at 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 $R_\odot$. For the calculations summarized in Figure 2, we introduced a Gaussian perturbation with an amplitude of 1% and a FWHM of 0.1 $R_\odot$.

The effects of the perturbations are more easily detected in the outer region of the Sun than in the core, because relatively few modes penetrate the solar core and acoustic waves spend most of the travel time in the outer layers of the Sun. Nevertheless, for the solar core defined as $0.1\ R_\odot < r < 0.3\ R_\odot$, the 1% peak perturbations introduced in the solar model are detected by helioseismic inversion with an amplitude of ~0.7% after averaging over the inversion kernels. In the intermediate regions, $0.4\ R_\odot < r < 0.7\ R_\odot$, the 1% peak perturbations translate into observed discrepancies of ~0.8%.

Since the standard solar model predicts sound speeds in the solar core that agree with the helioseismically inferred sound speeds to better than 0.2% rms (Bahcall et al. 1997), it is clear that any perturbation with an amplitude bigger than $(0.2/0.7) \times 1\% = 0.3\%$ would be detectable. Thus, the existing data limit localized departures from the standard model to be less than 0.3% in peak amplitude if they extend over ~0.1 $R_\odot$. The agreement between the standard solar model and helioseismic sound speed is even better, 0.1% rms, in the intermediate region. Thus, we can place upper limits of order $(0.1/0.8) \times 1\% = 0.12\%$ for perturbations of width 0.1 $R_\odot$ in the intermediate regions of the Sun. We have verified that this conjecture is in fact correct to a high degree of approximation for perturbations introduced at the seven representative peak positions considered in Figure 2.

A localized 1% perturbation in the density in the solar core is recovered by the helioseismic inversion with an amplitude of ~0.7%. The errors in the inverted density differences due to uncertainties in the frequency measurements are ~0.22% in the solar core and 0.15% elsewhere.

If nothing happens to solar neutrinos after they are created in the core of the Sun, then the existing four solar neutrino experiments suggest that the flux of ^{7}Be neutrinos reaching the Earth may be much less than is predicted by the standard solar model. This inference is very difficult to understand within the constraints of standard electroweak theory since the flux of ^{7}Be neutrinos can be calculated much more accurately than the flux of the rare ^{8}B neutrinos that are observed directly in the Kamiokande neutrino experiment (see, e.g., Bahcall 1994). The ^{7}Be and the ^{8}B neutrinos are both produced by capture reactions on the ambient ^{7}Be ions (via an accurately known electron capture reaction for ^{7}Be neutrinos and by a much less frequent and more poorly known proton capture reaction for ^{8}B neutrinos).

We have therefore tested for the sensitivity to ad hoc perturbations that are fine-tuned to give the maximum effect on the crucial ^{7}Be neutrinos. The production of the ^{7}Be neutrinos is peaked at a radius of 0.06 $R_\odot$ and FWHM of $\approx 0.075\ R_\odot$ (Bahcall 1989). We find that for a perturbation of amplitude x in the sound-speed difference, the perceived change in the sound-speed difference is about $x/2$. Thus, we can rule out differences between the Sun and the standard solar model centered on the ^{7}Be neutrino production radius and of width 0.075 $R_\odot$, which produce sound-speed differences with an amplitude greater than ~0.4%; this corresponds to a change in the ^{7}Be neutrino flux of a few percent.

How do the limits on the allowable perturbations depend on the assumed width of the perturbations? Figure 3 illustrates the effect of varying the width of the perturbations while keeping fixed the area enclosed in the $\delta c/c$ versus r plane. The figure shows that the detectability of a hypothesized perturbation depends primarily on the area. In constructing Figure 3, we have considered four different perturbations all normalized to the same area. The peak amplitudes for the discrepancies introduced were 1%, 2%, 4%, and 5%; the 2 σ widths were, respectively, 0.1, 0.05, 0.025, and 0.02 $R_\odot$. The narrow perturbations are considerably broadened in the process of inversion and have a width similar to the kernel width, and the inferred perturbation amplitude is correspondingly smaller. The narrowest perturbations result in an implied discrepancy of almost 1%, while the broadest perturbation, of the same underlying area, gives rise to a discrepancy of ~0.65% (see Fig. 3).

3. SUMMARY AND DISCUSSION

We have shown that there can be no large, reasonably localized differences between the predictions of the standard solar model and the sound speeds obtained from helioseismology in the region between 0.1 $R_\odot$ and 0.7 $R_\odot$. Numerically, we find that differences in the sound speed from the standard solar model with characteristic radial widths of order 0.1 $R_\odot$ would have been detected in the solar core if they had peak fractional amplitudes greater than ~0.3% and would have been observed in the intermediate solar region if they had peak fractional amplitudes in excess of ~0.1%. The detectability of presumed departures from the standard solar model predictions depends primarily on the assumed area in the $\delta c/c$ versus r plane; the smaller the assumed width, the larger the required peak amplitude (see Fig. 3).

In order to affect significantly the most important solar neutrino fluxes, changes in the nuclear burning temperatures of order 5%–10% are required (see, e.g., Bahcall & Ulmer 1996; Bahcall 1989). The squares of the sound speeds are approximately proportional to the local temperature, so an uncompensated 5% change in the central temperature corresponds to a change of ~2.5% in the sound speed, which is about an order of magnitude larger than the upper limits set here. We conclude, therefore, that the standard solar model

describes the structure of the Sun more accurately than is required to predict well the solar neutrino fluxes. Moreover, the arguments presented here are general, and the limit on hypothesized departures from the standard solar model (e.g., from conjectured mixing, rotational instabilities, or magnetic fields) is small.

Thus, the interior of the Sun is one of the few examples known in astronomy in which the conditions in the actual astronomical system appear to be almost as simple as imagined by most theorists.

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