ULTRA–HIGH-ENERGY COSMIC RAYS MAY COME FROM CLUSTERED SOURCES

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ABSTRACT

Clustering of cosmic-ray sources affects the flux observed beyond the cutoff imposed by the cosmic microwave background and may be important in interpreting the AGASA, Fly's Eye, and HiRes data. The standard deviation, σ, in the predicted number N of events above 10^{20} eV is $\sigma/N = 0.9(r_0/10$ Mpc$)^{0.9}$, where r_0 is the unknown scale length of the correlation function ($r_0 \simeq 10$ Mpc for field galaxies, $H_0 = 50$ km s^{-1} Mpc^{-1}). Future experiments will allow the determination of r_0 through the detection of anisotropies in arrival directions of $\sim 10^{20}$ eV cosmic rays over angular scales of $\Theta \sim r_0/30$ Mpc.

Subject headings: acceleration of particles — cosmic rays — cosmology: theory — large-scale structure of universe

1. INTRODUCTION

The conventional astronomical picture for the origin of ultra–high-energy cosmic rays, namely, proton acceleration to high energy in extragalactic objects, predicts a sharp suppression of the cosmic-ray flux beyond the Greisen-Zatsepin-Kuzmin (GZK) cutoff at $\sim 5 \times 10^{19}$ eV (Greisen 1966; Zatsepin & Kuzmin 1966), due to interaction of protons with photons of the cosmic microwave background. The absence of a GZK cutoff might suggest the presence of a new source of ultra–high-energy cosmic rays, possibly related to the decay of exotic particles (for reviews see Cronin 1996; Hillas 1998).

The suppression of flux beyond the GZK cutoff is most often discussed assuming a uniform source distribution. However, the distribution of other astrophysical systems (e.g., galaxies, clusters of galaxies) is inhomogeneous on scales of tens of megaparsecs, comparable to the propagation distance of protons of energy greater than 5×10^{19} eV. Therefore, significant deviations from the predictions for a uniform distribution may be expected beyond the GZK cutoff (Giler, Wdowczyk, & Wolfendale 1980; Hill & Schramm 1985; Waxman 1995).

There are at least two possible approaches to evaluating the effects of source inhomogeneity on the energy spectrum and spatial direction of high-energy cosmic rays. In the first approach, one assumes that the source density of ultra–high-energy cosmic rays is proportional to (possibly with some bias factor) the galaxy density in some particular survey of the distribution of relatively nearby galaxies. This approach has been used by Waxman, Fisher, & Piran (1997), Giler et al. (1980), and Hill & Schramm (1985) and illustrates some of the principal features of source clustering. In the present paper, we adopt a different and complementary approach. We use an analytic model that summarizes the clustering properties of the unknown source of ultra–high-energy cosmic rays by a single parameter, r_0, the correlation length.

The analytic model that we adopt has the advantage of generality. No specific source population has to be assumed. Also, the results of source clustering on the energy spectrum and on the spatial distribution of the highest energy cosmic rays can be summarized in terms of the single

unknown correlation length of the source population, r_0. Thus the correlation length provides a concise and simple characterization of measurable clustering effects on observational parameters for high-energy cosmic rays. Future experiments that measure the arrival directions of a large number of high-energy cosmic rays will determine r_0 and permit a clear assessment of whether or not the departures from expectations based upon a homogeneous source distribution can plausibly be explained by any nonuniform population.

The source correlation function, $\xi(dr)$, is defined by an average over the observable universe volume, $\langle n(r)n(r+dr)\rangle \equiv \langle n\rangle^2[1 + \xi(dr)]$ where n is the source density and angular brackets denote a volume average. Thus, the variance that we derive in the expected cosmic-ray number due to source clustering is, strictly speaking, the variance of the distribution of cosmic-ray number observed by observers randomly distributed over the universe. This is the best that one can do, but it is not exactly what we want. We really want the variance in conceivable realizations of the universe in our vicinity. However, we show in § 2 that the distance from our Galaxy over which sources contribute to the observed flux above 10^{20} eV is ~ 40 Mpc (see Fig. 1). Since the clustering properties of astronomical objects (e.g., galaxies) within this volume are similar to that obtained by averaging over larger volumes (e.g., Peebles 1999), we expect our result to provide a reasonable approximation for the variance in the number of cosmic rays observed at Earth due to different realizations of cosmic-ray source distributions that satisfy the universal clustering properties of the sources.

One may argue that a more accurate estimate of the variance can be derived by using the known galaxy catalogs to model the effects of source inhomogeneity. However, in order to use such an approach, one must choose a specific model to derive the probability of a given cosmic-ray source distribution under the constraints provided by a given galaxy catalog. The variance calculated in this approach would depend primarily on the resultant cosmic-ray source correlation function. While this correlation function would indeed depend on the clustering properties of the particular galaxies chosen, it would be strongly dependent on the
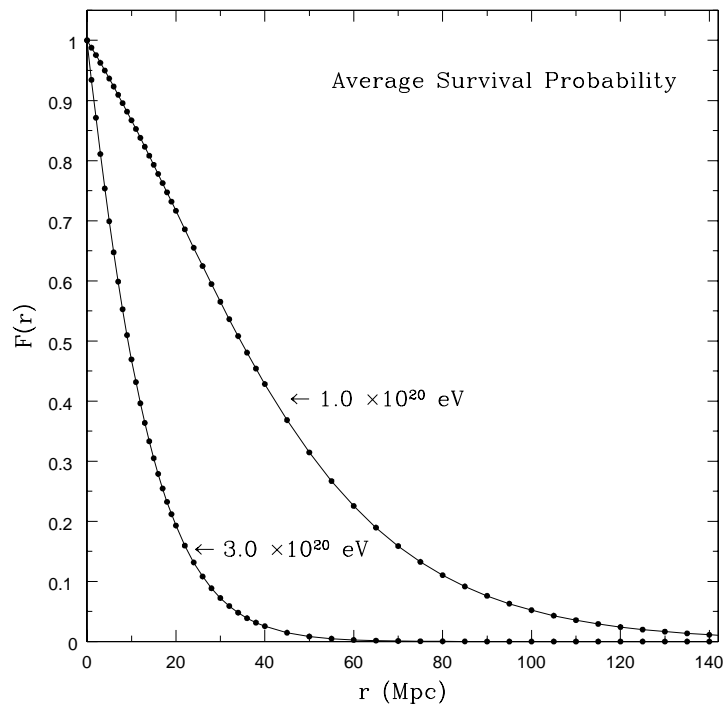
Fig. 1.—Survival probability averaged over energies. The plotted function, $F(r)$, represents the probability that a proton created at a distance r will reach the Earth with an energy in excess of the cutoff energy E_c. Results are shown for $E_c = 1 \times 10^{20}$ eV and $E_c = 3 \times 10^{20}$ eV with an energy spectrum $\partial S/\partial E \propto E^{-2}$.

assumed model. It should be emphasized here that one does not know which particular rare subpopulation of the total observed galaxy population actually produces the highest energy cosmic rays. The local density of field galaxies is of order 0.1 Mpc^{-3}, whereas the local density of sources of ultra–high-energy cosmic rays may be as low as 0.00002 Mpc^{-3}. (The lower limit to the source density is set by the requirement that at least a few sources exist out to a distance of ~ 40 Mpc in order to account for the observed events above 10^{20} eV.) Thus, results derived in this approach would not yield a more accurate estimate of the variance, but rather reflect the assumptions under which the cosmic-ray source distribution is constrained given some chosen galaxy catalog. Moreover, even after one makes a specific assumption about the functional relation between the density of sources of ultra–high-energy cosmic rays and an observed sample of galaxies, one has to perform and analyze many Monte Carlo simulations with galaxy catalogs in order to determine the statistical effects on the observed characteristics of ultra–high-energy cosmic rays.

We believe therefore that the simplicity and conciseness of the analytic model approach justifies its application, as one of the possible approaches, to the evaluation of the effects of source inhomogeneity.

In this paper we consider the implications of clustering of cosmic-ray sources, adopting the conventional picture of protons as the ultra–high-energy (UHE) cosmic rays. We assume that the correlation function between the sources of UHE cosmic rays has the same functional form as for galaxies, clusters of galaxies, and quasars, but we do not require—as was done in previous work (Waxman et al. 1997; Giler et al. 1980; Hill & Schramm 1985)—that the distribution of cosmic-ray sources be the same as for nearby galaxies. As described above, the conditions that must be satisfied in order to produce ultra–high-energy cosmic rays

are so exceptional that the spatial distribution of sources of UHE cosmic rays could be very different from the distribution of average, nearby galaxies like the *IRAS* sources discussed in Waxman et al. (1997). Also, we are primarily concerned in this paper with the energy spectrum of UHE cosmic rays, whereas the *IRAS* galaxy distribution was used in Waxman et al. (1997) to discuss their possible angular distribution on the sky.

Our principal result is that the standard deviation due to clustering, $\sigma_{\text{clustering}}(E_c)$, in the number of cosmic rays detected above energy E_c is proportional to the number $N_{\text{smooth}}(E_c)$ predicted for a uniform source density. This behavior contrasts with the standard deviation due to shot noise, which becomes proportional to $N_{\text{smooth}}^{1/2}$. The constant of proportionality between $\sigma_{\text{clustering}}$ and N_{smooth} depends upon energy and is calculated in § 2. For $E_c \sim 10^{20}$ eV, the constant of proportionality between $\sigma_{\text{clustering}}$ and N_{smooth} is of order unity for plausible values of the correlation length, r_0, of the correlation function of the sources of the UHE cosmic rays. Anisotropies in the source distribution on a scale of $\Theta \sim (r_0/100$ Mpc$)$ should be detectable in the angular distribution of $\sim 10^{19.7}$ eV cosmic rays with the high rates that will be observed in the HiRes (Corbato et al. 1992), the Auger (Cronin 1992; Watson 1993), and the Telescope Array (Teshima et al. 1992) experiments. Unlike the predictions of many particle physics explanations (for recent reviews see Berezinsky 1998; Bhattacharjee & Sigl 1998) of the Fly's Eye (Bird et al. 1993, 1994) and AGASA (Hayashida et al. 1994; Yoshida et al. 1995; Takeda et al. 1998) results, no characteristic dependence on Galactic coordinates is expected on the basis of this conventional extragalactic scenario.

The computed large value of $\sigma_{\text{clustering}}$ reflects the fact that the universe, and hence the UHE source population, is inhomogeneous over distances comparable to the mean free

path of UHE protons that move through the cosmic microwave background radiation. The ratio $\sigma_{clustering}/N_{smooth}$ cannot be reduced by observing longer or doing different experiments. We only have one "nearby universe," and we do not know a priori the local clustering properties of the sources of UHE cosmic rays.

The angular scale of anisotropies due to magnetic scattering of UHE cosmic rays from individual sources is smaller than what is expected from the clustering of the sources themselves unless the intergalactic field is close to the maximum value allowed with the available data on Faraday rotation. The upper limit on the contribution of an intergalactic magnetic field to the Faraday rotation of distant sources, RM < 1 rad m^{-2} for sources at $z = 2.5$ (Vallée 1990), implies an upper limit $B < 10^{-10}$ G on a uniform intergalactic field, and an upper limit $B\lambda^{1/2} < 10^{-8}$ G Mpc$^{1/2}$ on a field with correlation length λ.[1] A simple random walk calculation shows that the upper limit on $B\lambda^{1/2}$ sets an upper limit to the magnetic deflections, $\theta_s < 0.04(d/1\ \text{Mpc})^{1/2}(E/10^{20}\ \text{eV})^{-1}$ for propagation distance d.[2] Galactic magnetic fields contribute only relatively small deflections.

We present our principal results in § 2. We show in § 3 that the large value of $\sigma_{clustering}/N_{smooth}$ is important for the interpretation of the existing AGASA and Fly's Eye experiments. We also list in § 3 the effects of clustering that may be detectable with the HiRes, Auger project, and the Telescope Array experiment.

2. RESULTS

The number, N, of high-energy cosmic rays predicted on average above a threshold energy E_c is

$$N_{\text{smooth}}(E_c) \equiv \langle N(E \geq E_c)\rangle = \int_{E_c}^{\infty} dE \int d^3r\, P(r, E; E_c)$$
$$\times \langle (\partial S/\partial E)_r\, n(r)\rangle/4\pi r^2 , \qquad (1)$$

where $P(r, E; E_c)$ is the probability that a proton created at r with energy E arrives at Earth with an energy above threshold. The energy spectrum generated by the sources is $\partial S/\partial E$, and the luminosity-weighted source density is $n(r)$. We present calculations for $\partial S/\partial E \propto E^{-2}$ and $\partial S/\partial E \propto E^{-3}$, which spans the range usually considered in the

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[1] The value we quote is a few times larger than that quoted by Vallée, since the upper limit is inversely proportional to the electron density n_e and we choose $n_e = 3 \times 10^{-7}$ cm^{-3}, corresponding to $\Omega_b h^2 = 0.03$, while Vallée assumed $n_e = 10^{-6}$ cm^{-3}. Our upper limit is much smaller than that recently claimed by Farrar & Piran (2000), mainly because they use older radio data (Kronberg & Simard-Normandin 1976) for which the RM upper limit for $z = 2.5$ sources is weaker, RM < 5 rad m^{-2}, and assume that the free electron density is only 0.3 of the baryon density.

[2] Following the suggestion of Kulsrud et al. (1997), that magnetic fields could be amplified by turbulence associated with the formation of large-scale filaments and sheets, several authors have recently considered propagation of cosmic rays in a large-scale intergalactic magnetic field (e.g., Sigl, Lemoine, & Biermann 1999; Farrar & Piran 2000). The upper limit on magnetic field coherent on 1 Mpc scale in such structures is smaller by a factor $f^{1/2}$ than the upper limit of 10^{-8} G on a volume filling field, where f is the fraction of the volume occupied by large-scale filaments and sheets. This is due to the fact that the upper limit is inversely proportional to the electron density, $n_e \propto f^{-1}$, and proportional to the square root of the path length $l \propto f$ of light through magnetized plasma. While this reduction of the upper limit may be partly compensated by assuming strong negative evolution of the magnetic field with redshift (e.g., Farrar & Piran 2000), the assumption that the intergalactic magnetic field is confined to large-scale structures does not lead to an increase in the upper limit on the deflection of UHE cosmic rays.

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literature. The AGASA and Fly's Eye results above 10^{19} eV can be shown to require a spectrum less steep than $E^{-2.8}$ (Waxman 1995).

It is convenient to rewrite equation (1) in terms of a function $F(r)$, where

$$\langle N(E \geq E_c)\rangle = \int_0^{\infty} dr\, F(r)\langle n\rangle , \qquad (2)$$

and F is the survival probability averaged over energies,

$$F(r) \equiv \int_{E_c}^{\infty} dE\, P(r, E; E_c)(\partial S/\partial E) . \qquad (3)$$

The constant of proportionality between $\partial S/\partial E$ and E^{-m} can be chosen arbitrarily, since the constant cancels out of the ratio of standard deviation to expected number, which is the primary quantity we calculate (see eq. [11]). For convenience, we fix the constant so that $F(0) = 1.0$; we also suppress the dependence of F on E_c.

For ultra–high-energy cosmic rays ($E > 5 \times 10^{19}$ eV), the probability $P(r, E; E_c)$ is determined by the energy loss of protons due to pair and pion production in interaction with the microwave background. The energy loss in a single pair-production interaction is small, of order m_e/m_p, and for protons at energy greater than 10^{19} eV the characteristic energy loss time due to this interaction is comparable to the Hubble time, $\approx 5 \times 10^9$ yr (Blumenthal 1970). Hence, pair production has only a small effect on $P(r, E; E_c)$ for $E_c \geq 10^{20}$ eV, where the proton lifetime due to pion production is $\leq 3 \times 10^8$ yr. We calculated therefore $P(r, E; E_c)$ taking into account pion-production interactions only. We used the compilation of cross sections given in Hikasa et al. (1992), and assumed that the total cross section corresponds to the isotropic production of a single pion (this assumption has been shown to be adequate in Yoshida & Teshima 1993; Aharonian & Cronin 1994). The resulting probability distribution may be approximated well over the energy range of interest by a function of the form

$$P(r, E; E_c) = \exp\{-a(E_c)r^2 \exp[b(E_c)/E]\} . \qquad (4)$$

We give below results obtained using a precise numerical evaluation, of $P(r, E; E_c)$. The approximation in equation (4) provides a simple description of the functional dependence of P on r and E and can be used above 8×10^{19} eV to evaluate to an accuracy of 10% the average quantities considered in this paper. The appropriate values of a and b for $E_c/(10^{20}$ eV$) = 1$, 3, and 6 are, respectively, $a/(10^{-4}$ Mpc$^{-2}) = 1.4$, 9.2, and 11, $b/(10^{20}$ eV$) = 2.4$, 12, and 28.

The function $F(r)$ is shown in Figure 1 for a spectrum $\partial S/\partial E \propto E^{-2}$. The average distance from which a proton originates if it is observed at Earth to have an energy in excess of E_c,

$$\langle r\rangle \equiv \frac{\int dr\, rF(r)}{\int dr\, F(r)} , \qquad (5)$$

can be calculated analytically using equations (3) and (4). For an energy spectrum with $\partial S/\partial E \propto E^{-2}$,

$$\langle r\rangle = \frac{1 - \exp(-b/E_c)}{\sqrt{4\pi a}[1 - \exp(-b/2E_c)]} . \qquad (6)$$

A similar expression can be obtained for $\partial S/\partial E \propto E^{-3}$. Inserting the appropriate values of a and b in equation (6), $\langle r\rangle = 31.2$ Mpc for $E_c = 1 \times 10^{20}$ eV and $\langle r\rangle = 10.6$ Mpc

for $E_c = 3 \times 10^{20}$ eV [the values obtained using the numerical calculation of $F(r)$ are 31.8 and 10.9 Mpc, respectively]. The observed universe is inhomogeneous on these distance scales.

The variance in the number of cosmic rays observed above an energy E_c can be computed from the expression

$$\sigma^2(E \geq E_c) = \langle N^2(E \geq E_c) \rangle - \langle N(E \geq E_c) \rangle^2 . \quad (7)$$

Since galaxies, clusters of galaxies, and quasars are all clustered with correlation functions that have the same shape, it is natural to suppose that the sources of UHE cosmic rays are also clustered with a similar correlation function. Explicitly, we assume that

$$\langle n(r')n(r'') \rangle \equiv \langle n \rangle^2 [1 + \xi(r' - r'') + \delta(r' - r'')/\langle n \rangle] , \quad (8)$$

where the correlation function ξ is

$$\xi(r) \equiv \left(\frac{r_0}{r} \right)^{1.8} \quad (9)$$

and the δ-function contribution represents shot noise. The correlation length, r_0, is not known for the sources of UHE cosmic rays, but it is about 10 Mpc for galaxies (Groth & Peebles 1977; Shectman et al. 1996; Tucker et al. 1997) and about 40 Mpc for rich clusters of galaxies (Bahcall 1988; Bahcall & Soneira 1983; Peacock & West 1992) (for $H_0 = 50$ km s^{-1} Mpc^{-1}).

When the variance is calculated using equation (7), the only term that survives, in addition to shot noise, is proportional to the correlation function, i.e.,

$$\sigma^2(E \geq (E_c) = \int \int dE'dE''(\partial S/\partial E')(\partial S/\partial E'')$$

$$\times \int \int \frac{d^3 r'}{4\pi r'^2} \frac{d^3 r''}{4\pi r''^2}$$

$$\times P(r', E'; E_c)P(r'', E''; E_c)$$

$$\times \langle n(r') \rangle \langle n(r'') \rangle \xi(r' - r'') + \sigma_{\text{shot noise}}^2 . \quad (10)$$

For simplicity, we will not display $\sigma_{\text{shotnoise}}$ explicitly in what follows. After some algebra, we find

$$\frac{\sigma_{\text{clustering}}}{N_{\text{smooth}}} = \left(\frac{5}{2} \right)^{1/2} r_0^{0.9}$$

$$\times \frac{\{\int_0^\infty \int_0^\infty (dx\,dy/xy)F(x)F(y)[(x+y)^{0.2} - |(x-y)|^{0.2}]\}^{0.5}}{\int_0^\infty dr\,F(r)} .$$

$$(11)$$

The ratio $\sigma_{\text{clustering}}/N_{\text{smooth}}$ can be evaluated numerically using equations (3) and (11). For cosmic rays above 1.0×10^{20} eV, we find

$$\frac{\sigma_{\text{clustering}}}{N_{\text{smooth}}} = 0.9 \left(\frac{r_0}{10 \text{ Mpc}} \right)^{0.9} , \quad (12)$$

and for cosmic rays above 3.0×10^{20} eV,

$$\frac{\sigma_{\text{clustering}}}{N_{\text{smooth}}} = 2.6 \left(\frac{r_0}{10 \text{ Mpc}} \right)^{0.9} . \quad (13)$$

The results given in equations (12) and (13) were calculated assuming a spectrum $\propto E^{-2}$. If the spectrum is instead assumed to be $\propto E^{-3}$, then the coefficient 0.9 in equation (12) is increased to 1.0 and the coefficient in equation (13)

becomes 3.6. For cosmic rays above 3.0×10^{20} eV, the dispersion is so large (see eq. [13]) that it will be difficult to interpret statistically the observed number.

Due to the rapid decrease with r of $F(r)$ at large r, shown in Figure 1, the integral in equation (10) is dominated by the contribution from small separations $|r' - r''|$. Neglecting, for example, the contribution from large separations $|r' - r''| > 20$ Mpc, where the galaxy-galaxy correlation function drops faster than $r^{-1.8}$, reduces $\sigma_{\text{clustering}}/N_{\text{smooth}}$ by 20% for $E_c = 10^{20}$ eV and by 3% for $E_c = 3 \times 10^{20}$ eV. Thus our results for $\sigma_{\text{clustering}}/N_{\text{smooth}}$ are not sensitive to the r dependence of $\xi(r)$ at large r. On the other hand, $\sigma_{\text{clustering}}/N_{\text{smooth}}$ is more sensitive to the r dependence at small r. For $\xi(r) \propto r^{-1.8}$ and $r_0 = 10$ Mpc, approximately half the contribution to the integral in equation (10) comes from separations $|r' - r''| < 3$ Mpc for $E_c = 10^{20}$ eV and $|r' - r''| < 1$ Mpc for $E_c = 3 \times 10^{20}$ eV. The galaxy-galaxy correlation function, for example, is known to follow the form of equation (9) down to $r \sim 0.1$ Mpc, while the cluster-cluster correlation function is not well measured for $r < 10$ Mpc.

In principle, the contribution to the variance from shot noise, $\sigma_{\text{shotnoise}}^2$, could be large because of the possibility of having a nearby source. However, an extremely close source would dominate the all-sky flux, which is not the case for the observed cosmic-ray events. If the average source density is much larger than the minimum value required to explain the observations of UHE cosmic rays, i.e., $\langle n \rangle \gg 10^{-4}$ Mpc^{-3}, then the shot-noise contribution is relatively small. The shot noise could be significant, of order N_{smooth}, if the local source density is of order 10^{-4} Mpc^{-3}.

3. DISCUSSION AND PREDICTIONS

Are new particles required to explain the observed number of ultra–high-energy cosmic rays? The numbers of events observed beyond 10^{20} eV by AGASA (Takeda et al. 1998), by the Fly's Eye (Bird et al. 1993, 1994), and by Yakutsk (Efimov et al. 1991) are, respectively, 6, 1, and 1. The average observed number of events, 2.7, is to be compared with conventional estimates of the number of events from a smooth model of between $N_{\text{smooth}} = 0.7$ (for $\partial S/\partial E \propto E^{-3}$) (cf. Takeda et al. 1998) and $N_{\text{smooth}} = 2.2$ (for $\partial S/\partial E \propto E^{-2}$) (cf. Waxman 1995). The total standard deviation in the expected number of events beyond 10^{20} eV is

$$\sigma_{\text{total}} = [0.8(r_0/10 \text{ Mpc})^{1.8} N_{\text{smooth}}^2 + N_{\text{smooth}}]^{0.5} , \quad (14)$$

where the first term in the brackets is the calculated uncertainty due to clustering and the second term is due to Poisson statistics. For $N_{\text{smooth}} = 2.2$ and $r_0 = 10$ Mpc, $\sigma_{\text{total}} = 2.4$. Even for $N_{\text{smooth}} = 0.7$, $\sigma_{\text{total}} = 1.0$. No matter how one combines the experiments and the smooth models, there is not a 3 σ discrepancy with the conventional picture of protons as the source of ultra–high-energy cosmic rays. If one considers, as many recent authors have done, only the AGASA data, then $N_{\text{observed}} = 6$ and for $\partial S/\partial E \propto E^{-2}$, $N_{\text{smooth}} = 2.2$, and $\sigma_{\text{total}} = 2.4$, which is a 1.6 σ discrepancy. On the basis of the available evidence, we conclude that new particles are not required to explain the observed cosmic-ray energy spectrum.

However, a number of authors have interpreted the same data as suggesting the possibility that new particles are producing the highest energy cosmic rays (for a recent discussion and review of this point of view see Ellis 1999 and the references contained therein). One of the principal

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reasons for the difference in conclusions is the uncertainty over whether the extrapolation of the "conventional high-energy component" should be made using $\partial S/\partial E \propto E^{-2}$, as we have done, or whether the fall-off of the spectrum at energies just below the GZK cutoff is stronger, e.g., $\propto E^{-3}$. This is just one of the important questions that will have to be answered in the future by a much larger data set.

We also note that preliminary results from the HiRes experiment were recently presented in a talk at TAUP99 (Matthews 1999), reporting seven events beyond 10^{20} eV for an exposure similar to that of the Fly's Eye. It is difficult to decide how this result should be interpreted, since the discrepancy between HiRes and the Fly's Eye results is present not only above 10^{20} eV but also at lower energy, where Fly's Eye, AGASA, and Yakutsk experiments are in agreement: 13 events above 6×10^{19} eV are reported in the preliminary HiRes analysis, while only five events at that energy range are reported by the Fly's Eye. We therefore believe that unambiguous conclusions based on the recent HiRes data can be drawn only after a complete analysis of the HiRes data is published (which would include, e.g., corrections due to realistic atmospheric conditions).

One possible interpretation of the suggested excess of events beyond the GZK cutoff is that this may be a hint that ultra–high-energy cosmic rays are produced by sources whose density in the nearby universe is somewhat higher than their average cosmological density. Of course, other interpretations are possible, including the hypothesis— amply represented in the literature—that new physics is involved and that the GZK cutoff is bypassed by this new physics. If the GZK cutoff is ultimately convincingly measured, then the most likely explanation for the small number of already observed UHE cosmic rays will be the effects of source clustering.

Future measurements of high statistical significance with AGASA, HiRes, Auger, and the Telescope Array are required to determine whether source clustering affects significantly the observed number and angular distribution of UHE cosmic rays. The predicted effects of clustering depend upon the unknown correlation length r_0.

We summarize below the principal observational implications of assuming a correlation length between 10 Mpc (observed for galaxies) and 40 Mpc (observed for the rich clusters of galaxies).

1. Clustering gives rise to a standard deviation that is proportional to N_{smooth} (see eqs. [12] and [13]). When more events beyond the GZK cutoff are available, clustering will be more important than shot noise in determining the shape of the energy spectrum of the highest energy cosmic rays. Even for the existing Fly's Eye and AGASA samples, the variance due to clustering may dominate shot noise (see eq. [14]).

2. The fractional calculated variance, $\sigma_{clustering}/N_{smooth}$, increases with the cutoff energy, E_c, which simply reflects the fact that the mean distance from which protons originate decreases with the observed energy (see eq. [6]). The fractional variance is large at all energies beyond the cutoff. With the approximations of the present paper, $\langle r \rangle \simeq 120$ Mpc and $\sigma_{clustering}/N_{smooth} = 0.25(r_0/10 \text{ Mpc})^{0.9}$ for $E_c = 6 \times 10^{19}$ eV (the exact value of $\sigma/N_{smooth} [\langle r \rangle]$ is somewhat larger, since at this energy pair-production energy loss, neglected in our calculations, becomes important). The fractional variance does not increase significantly as E_c is increased above 3×10^{20} eV, since the average distance at which a proton originates is approximately constant at higher energy (e.g., $\langle r \rangle = 9.5$ Mpc for $E_c = 6 \times 10^{20}$ eV, comparable to $\langle r \rangle = 10.9$ Mpc for $E_c = 3 \times 10^{20}$ eV).

3. Anisotropies should be observed on large angular scales $\Theta_0 \sim r_0/r_{origin}$, where r_{origin} decreases from ~ 200 Mpc to ~ 30 Mpc as E_c increases from 0.4×10^{20} eV (just below the GZK cutoff) to 1×10^{20} eV. A comparison of anisotropies observed below and above the GZK cutoff will be a crucial test of whether clustering is important. For $r_0 \sim 10$ Mpc, one can show that one will have to observe 10^2 (10^3 events) above 10^{20} (4×10^{19}) eV to detect a 3 σ effect. Thus, the detection of anisotropies would require the large exposure of the Auger project.

4. No characteristic dependence on Galactic coordinates is expected. This is in contrast to many exotic particle physics scenarios for the production of ultra–high-energy cosmic rays in which a strong dependence on Galactic coordinates is predicted, in particular, a peaking toward the Galactic center. The Fly's Eye experiment reports an angular distribution peaked toward the Galactic disk for lower energy cosmic rays, which we interpret to be due to a different, more local set of sources than the sources of ultra–high-energy cosmic rays.

5. The suggested correlation of the directions of ultra–high-energy cosmic rays with the directions of compact radio-loud quasars (Farrar & Biermann 1998) should disappear as more events become available. The candidates suggested are all at distances greater than 10^3 Mpc, much beyond the allowed range of ~ 40 Mpc for ultra–high-energy protons.

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