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Standard Solar Models

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I review recent developments that affect standard solar model predictions of solar neutrino fluxes.

1. INTRODUCTION

A lot of progress has been made in understanding the robustness of solar model predictions since Neutrino 96 [1]. In this talk, I will first summarize the new ingredients and then give the current best-estimates. Then I will discuss the uncertainties in the predictions.

Many of the results given here are adopted from the recent BP98 paper [2]. As we shall see in Section 3, there is excellent agreement between standard solar models calculated by different groups with different codes.

If you want to obtain the numerical data that are discussed in this talk, you can copy them from my Web site: http://www.sns.ias.edu/∼jnb .

2. NEW INGREDIENTS

In this section, I will first summarize the new and relevant results on nuclear fusion reactions and on the screening of nuclear reactions and then summarize the situation with respect to neutrino cross sections. Finally, I will mention a few miscellaneous improvements that have been made since Neutrino 96.

2.1. Nuclear reaction cross sections

In January, 1997, the Institute for Nuclear Theory (INT) hosted a workshop devoted to determining the best estimates and the uncertainties in the most important solar fusion reactions. Thirty-nine experts in low energy nuclear experiments and theory, representing many different research groups and points of view, participated in the workshop and evaluated the existing experimental data and theoretical calculations. Their conclusions have been summarized in a detailed article authored jointly by the participants and published by the Reviews of Modern Physics [3]. In general outline, the conclusions of the INT workshop paper confirmed and strengthened previous standard analyses of nuclear fusion rates, although in a few important cases (for the $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$, $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$, and $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ reactions) the estimated uncertainties were determined to be larger than previously believed.

The largest change from what was used in the results presented at Neutrino 96 is the lower $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$ cross section adopted by Adelberger et al. [3]. Previously, most authors constructing standard solar models used the Caltech (CIT) value for the $^{8}\mathrm{B}$ production cross section [4]. The difference between the INT and the CIT estimates of the $^{8}\mathrm{B}$ production cross section is due almost entirely to the decision by the INT group to base their estimate on only one (the best documented) of the six experiments analyzed by the CIT collaboration.

As we go alone, I will indicate how the principal predictions of solar models depend upon the assumed $^{8}\mathrm{B}$ production cross section.

2.2. Screening of nuclear reactions

In one respect, the calculation of neutrino fluxes has simplified from Neutrino 96 to Neutrino 98. The rather complicated expressions in the literature for the screening of nuclear fusion reactions by electrons and ions have been replaced by a simple analytic expression that was originally derived by Salpeter [5] for the case of "weak screening" only.

Gruzinov and Bahcall [6] employed a mean field formalism to calculate the electron density of the screening cloud using the appropriate density matrix equation of quantum statistical mechanics. Because of well understood physical effects that are included for the first time in this treatment, the calculated enhancement of reaction rates does not agree with the frequently used interpolation formulae. For the sun, screening effects cause only small uncertainties in the predicted neutrino fluxes if the appropriate Salpeter formula is used.

2.3. Neutrino cross sections and energy spectra

Improved cross sections for neutrino absorption by gallium and by chlorine are now available as well as somewhat more precise standard (undistorted) neutrino energy spectra. I summarize the results here.

A number of authors do not use the best available data for the neutrino cross sections and energy spectra and some authors even give event rates (or SNU values) but do not say which cross sections and energy spectra they use so that one cannot interpret their results precisely.

I have calculated [7] neutrino absorption cross sections for ^{71}Ga for all solar neutrino sources with standard energy spectra, and for laboratory sources of ^{51}Cr and ^{37}Ar; the calculations including, where appropriate, the thermal energy of fusing solar ions and use improved nuclear and atomic data. The ratio, R, of measured (in GALLEX and SAGE) to calculated ^{51}Cr capture rate is $R = 0.95 \pm 0.07$ (exp) $+ \ ^{+0.04}_{-0.03}$ (theory) and was discussed extensively at Neutrino 98 by Gavrin and by Kirsten.

I also calculated cross sections for specific neutrino energies chosen so that a spline fit determines accurately the event rates in a gallium detector even if new physics changes the energy spectrum of solar neutrinos. In order to make possible more precise analyses of event rates for neutrino scenarios which change the shape of the neutrino energy spectra from individual neutrino sources, I evaluated and presented, for the first time, theoretical uncertainties for absorption cross sections at specific energies, as well as for the standard (undistorted) neutrino energy spectra. Also for use by people doing neutrino oscillation calculations, I calculated standard energy spectra for pp and CNO neutrino sources and presented the results in Appendices and on my Web site.

I note in passing that neutrino fluxes predicted by standard solar models, corrected for diffusion, have been in the range 120 SNU to 141 SNU since 1968 [7].

A group of us have recently redetermined the standard shape for the ^{8}B neutrino energy spectrum [8]. The available data all seem to be consistent with each other within rather small uncertainties, so we were able to determine not only a best-fit energy spectrum but also two extreme spectra that are different by what we estimate is effectively $\pm 3\sigma$. The uncertainties include estimates of the radiative and forbidden corrections. This improved spectrum yields a slightly different ^{8}B absorption cross section [8].

I am somewhat nervous about the standard (undistorted) ^{8}Be neutrino energy spectrum since it does depend upon rather old data [8]. It would be very good if the α-particle energy spectrum from ^{8}Be decay could be remeasured accurately in a new laboratory experiment.

The situation for neutrino-electron scattering is good; cross sections are available [9] that include electroweak radiative corrections.

2.4. Miscellaneous improvements

The standard solar model, BP98, that is discussed in this report includes somewhat improved radiative opacities calculated by the Livermore National Laboratory group, the so-called OPAL96 opacities [10], and the improved OPAL equation of state [11].

One improvement that I am rather proud of is new publicly available software that I have made available on my Web site (under Neutrino Software and Data) is a program to calculate solar neutrino rates and uncertainties. The problem of calculating the uncertainties in the predicted fluxes is somewhat complicated, especially since the uncertainties are asymmetric for some of the important input parameters. I decided to make this software publicly available (and therefore polished it significantly) so that people could see explicitly what uncertainties were included for each parameter and how the uncertainties were combined.

I have also polished my nuclear energy generation code. The changes made in this code, al-

Table 1
Standard Model Predictions (BP98): solar neutrino fluxes and neutrino capture rates, with 1σ uncertainties from all sources (combined quadratically).

Source	Flux $\left(10^{10}\ \mathrm{cm^{-2}s^{-1}}\right)$	Cl (SNU)	Ga (SNU)
pp	$5.94\left(1.00^{+0.01}_{-0.01}\right)$	0.0	69.6
pep	$1.39\times10^{-2}\left(1.00^{+0.01}_{-0.01}\right)$	0.2	2.8
hep	2.10×10^{-7}	0.0	0.0
^{7}Be	$4.80\times10^{-1}\left(1.00^{+0.09}_{-0.09}\right)$	1.15	34.4
^{8}B	$5.15\times10^{-4}\left(1.00^{+0.19}_{-0.14}\right)$	5.9	12.4
^{13}N	$6.05\times10^{-2}\left(1.00^{+0.19}_{-0.13}\right)$	0.1	3.7
^{15}O	$5.32\times10^{-2}\left(1.00^{+0.22}_{-0.15}\right)$	0.4	6.0
^{17}F	$6.33\times10^{-4}\left(1.00^{+0.12}_{-0.11}\right)$	0.0	0.1
Total		$7.7^{+1.2}_{-1.0}$	129^{+8}_{-6}

though they probably took me altogether a few weeks of programming and debugging time, ended up not changing neutrino flux predictions by more than a percent.

3. BEST-ESTIMATE FLUXES AND EVENT RATES

Table 1 gives the neutrino fluxes and their uncertainties for our best standard solar model, hereafter BP98. As discussed in the previous section, the solar model makes use of the INT nuclear reaction rates, recent (1996) Livermore OPAL radiative opacities, the OPAL equation of state, and electron and ion screening as determined by the recent density matrix calculation.

Figure 1 displays the calculated ^{7}Be and ^{8}B neutrino fluxes for all 19 standard solar models with which we are familiar which have been published in the last 10 years in refereed science journals. The fluxes are normalized by dividing each published value by the flux from the BP98 solar model [2]; the abscissa is the normalized ^{8}B flux and the ordinate is the normalized ^{7}Be neutrino flux. The rectangular box shows the estimated 3σ uncertainties in the predictions of the BP98 solar model. The abbreviations, which indicate references to individual models, are identified in the caption of Figure 1.

All of the solar model results from different groups fall within the estimated 3σ uncertainties in the BP98 analysis (with the exception of the Dar-Shaviv model whose results have not been reproduced by other groups). This agreement demonstrates the robustness of the predictions since the calculations use different computer codes (which achieve varying degrees of precision) and involve a variety of choices for the nuclear parameters, the equation of state, the stellar radiative opacity, the initial heavy element abundances, and the physical processes that are included.

The largest contributions to the dispersion in values in Figure 1 are due to the choice of the normalization for S_{17} (the production cross-section factor for ^{8}B neutrinos) and the inclusion, or non-inclusion, of element diffusion in the stellar evolution codes. The effect in the plane of Fig. 1 of the normalization of S_{17} is shown by the difference between the point for BP98 (1.0,1.0), which was computed using the INT normalization, and the point at (1.18,1.0) which corresponds to the BP98 result with the CIT normalization.

Helioseismological observations have shown [1, 13] that diffusion is occurring and must be included in solar models, so that the most recent models shown in Fig. 1 now all include helium and heavy element diffusion. By comparing a large number of earlier models, it was shown that all published standard solar models give the same results for solar neutrino fluxes to an accuracy of better than 10% if the same input parameters and physical processes are included [14,15].

The theoretical predictions in Table 1 disagree with the observed neutrino event rates, which are, see Ref. [16] and the results presented at this conference by Lande, Gavrin, Kirsten, and Suzuki: 2.56 ± 0.23 SNU (chlorine), 72.2 ± 5.6 SNU (GALLEX and SAGE gallium experiments), and $(2.44 \pm 0.10) \times 10^{6}\mathrm{cm^{-2}s^{-1}}$ (^{8}B flux from SuperKamiokande).

Bahcall, Krastev, and Smirnov [17] have compared the observed rates with the calculated, standard model values, combining quadratically

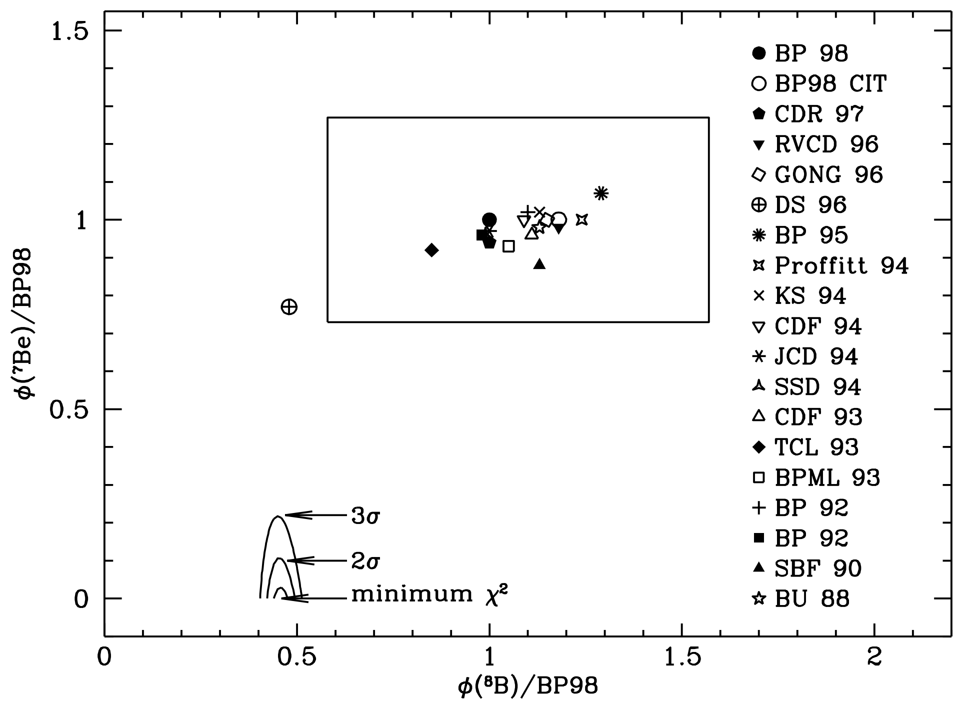

Figure 1. Predictions of standard solar models since 1988. The figure shows the predictions of 19 standard solar models in the plane defined by the ^{7}Be and ^{8}B neutrino fluxes. The abbreviations that are used in the figure to identify different solar models are defined in the bibliographical item, Ref. [12]. We include all standard solar models with which we are familiar that were published in refereed journals in the decade 1988-1998. All of the fluxes are normalized to the predictions of the Bahcall-Pinsonneault 98 solar model, BP98 [2]. The rectangular error box defines the 3σ error range of the BP98 fluxes. The best-fit ^{7}Be neutrino flux is negative. At the 99% C.L., there is no solution with all positive neutrino fluxes if the fluxes of CNO neutrinos are arbitrarily set equal to zero. There is no solution at the 99.9% C.L. if the CNO neutrinos are fixed at their standard solar model values. All of the standard model solutions lie far from the best-fit solution, even far from the 3σ contour.

the theoretical solar model and experimental uncertainties, as well as the uncertainties in the neutrino cross sections. Since the GALLEX and SAGE experiments measure the same quantity, we treat the weighted average rate in gallium as one experimental number. We adopt the SuperKamiokande measurement as the most precise direct determination of the higher-energy ^{8}B neutrino flux.

Using the predicted fluxes from the BP98 model, the χ^2 for the fit to the three experimental rates (chlorine, gallium, and SuperKamiokande) is

$$\chi^2_{\rm SSM}(3\ \text{experimental rates}) = 61\ . \qquad (1)$$

The result given in Eq. (1), which is approximately equivalent to a 20σ discrepancy, is a quantitative expression of the fact that the standard model predictions do not fit the observed solar neutrino measurements.

The principal differences between the results shown in Table 1 and the results presented in our last systematic publication of calculated solar neutrino fluxes [15] is a 1.3σ decrease in the ^{8}B neutrino flux and 1.1σ decreases in the ^{37}Cl and ^{71}Ga capture rates. These decreases are due mainly to the lower ^{7}Be$(p,\gamma)^{8}$B cross section adopted by Adelberger et al. [3]. If we use, as in our recent previous publications, the Caltech (CIT) value for the ^{8}B production cross section [4], then the ^{8}B flux is $\phi\left(^{8}\mathrm{B},\mathrm{CIT}\right) = 6.1^{+1.1}_{-0.9} \times 10^{6}\ \mathrm{cm^{-2}s^{-1}}$, $\Sigma\left(\phi\sigma\right)_i\Big|_{\mathrm{Cl,\ CIT}} = 8.8^{+1.4}_{-1.1}$ SNU, and $\Sigma\left(\phi\sigma\right)_i\Big|_{\mathrm{Gallium}} = 131^{+9}_{-7}$ SNU, all of which are within ten percent of the Bahcall-Pinsonneault 1995 best-estimates.

4. UNCERTAINTIES

In this section, I will first discuss the formal uncertainties in the solar model flux calculations and then review the strong constraints that helioseismology places on perturbations of the standard solar model.

4.1. Uncertainties in the flux calculations

Table 2 summarizes the uncertainties in the most important solar neutrino fluxes and in the Cl and Ga event rates due to different nuclear fusion reactions (the first four entries), the heavy element to hydrogen mass ratio (Z/X), the radiative opacity, the solar luminosity, the assumed solar age, and the helium and heavy element diffusion coefficients. The ^{14}N $+ p$ reaction causes a 0.2% uncertainty in the predicted pp flux and a 0.1 SNU uncertainty in the Cl (Ga) event rates.

The predicted event rates for the chlorine and gallium experiments use recent improved calculations of neutrino absorption cross sections [7,8]. The uncertainty in the prediction for the gallium rate is dominated by uncertainties in the neutrino absorption cross sections, +6.7 SNU (7% of the predicted rate) and −3.8 SNU (3% of the predicted rate). The uncertainties in the chlorine absorption cross sections cause an error, ±0.2 SNU (3% of the predicted rate), that is relatively small compared to other uncertainties in predicting the rate for this experiment. For non-standard neutrino energy spectra that result from new neutrino physics, the uncertainties in the predictions for currently favored solutions (which reduce the contributions from the least well-determined ^{8}B neutrinos) will in general be less than the values quoted here for standard spectra and must be calculated using the appropriate cross section uncertainty for each neutrino energy [7,8].

The nuclear fusion uncertainties in Table 2 were taken from Adelberger et al. [3], the neutrino cross section uncertainties from [7,8], the heavy element uncertainty was taken from helioseismological measurements [19], the luminosity and age uncertainties were adopted from BP95 [15], the 1σ fractional uncertainty in the diffusion rate was taken to be 15% [20], which is supported by helioseismological evidence [13], and the opacity uncertainty was determined by comparing the results of fluxes computed using the older Los Alamos opacities with fluxes computed using the modern Livermore opacities [14]. To include the effects of asymmetric errors, the now public-available code for calculating rates and uncertainties (see discussion in previous section) was run with different input uncertainties and the results averaged. The software contains a description of how each of the uncertainties listed in Table 2 were determined and used.

The low energy cross section of the ^{7}Be $+ p$ reaction is the most important quantity that must be determined more accurately in order to decrease the error in the predicted event rates in solar neutrino experiments. The ^{8}B neutrino flux that is measured by the Kamiokande [16], Super-Kamiokande [21], and SNO [22] experiments is, in all standard solar model calculations, directly proportional to the ^{7}Be$+p$ cross section. If the 1σ uncertainty in this cross section can be reduced by a factor of two to 5%, then it will no longer be the limiting uncertainty in predicting the crucial ^{8}B neutrino flux (cf. Table 2).

4.2. How large an uncertainty does helioseismology suggest?

Could the solar model calculations be wrong by enough to explain the discrepancies between predictions and measurements for solar neutrino experiments? Helioseismology, which confirms predictions of the standard solar model to high precision, suggests that the answer is probably “No.”

Figure 2 shows the fractional differences between the most accurate available sound speeds measured by helioseismology [23] and sound

Table 2
Average uncertainties in neutrino fluxes and event rates due to different input data. The flux uncertainties are expressed in fractions of the total flux and the event rate uncertainties are expressed in SNU. The ^{7}Be electron capture rate causes an uncertainty of $\pm 2\%$ [18] that affects only the ^{7}Be neutrino flux. The average fractional uncertainties for individual parameters are shown.

<Fractional uncertainty>	pp 0.017	^{3}He^{3}He 0.060	^{3}He^{4}He 0.094	^{7}Be $+ p$ 0.106	Z/X 0.033	opac	lum 0.004	age 0.004	diffuse
Flux									
pp	0.002	0.002	0.005	0.000	0.002	0.003	0.003	0.0	0.003
^{7}Be	0.0155	0.023	0.080	0.000	0.019	0.028	0.014	0.003	0.018
^{8}B	0.040	0.021	0.075	0.105	0.042	0.052	0.028	0.006	0.040
SNUs									
Cl	0.3	0.2	0.5	0.6	0.3	0.4	0.2	0.04	0.3
Ga	1.3	0.9	3.3	1.3	1.6	1.8	1.3	0.20	1.5

speeds calculated with our best solar model (with no free parameters). The horizontal line corresponds to the hypothetical case in which the model predictions exactly match the observed values. The rms fractional difference between the calculated and the measured sound speeds is 1.1×10^{-3} for the entire region over which the sound speeds are measured, $0.05 R_\odot < R < 0.95 R_\odot$. In the solar core, $0.05 R_\odot < R < 0.25 R_\odot$ (in which about 95% of the solar energy and neutrino flux is produced in a standard model), the rms fractional difference between measured and calculated sound speeds is 0.7×10^{-3}.

Helioseismological measurements also determine two other parameters that help characterize the outer part of the sun (far from the inner region in which neutrinos are produced): the depth of the solar convective zone (CZ), the region in the outer part of the sun that is fully convective, and the present-day surface abundance by mass of helium ($Y_{\rm surf}$). The measured values, $R_{\rm CZ} = (0.713 \pm 0.001) R_\odot$ [24], and $Y_{\rm surf} = 0.249 \pm 0.003$ [19], are in satisfactory agreement with the values predicted by the solar model BP98, namely, $R_{\rm CZ} = 0.714 R_\odot$, and $Y_{\rm surf} = 0.243$. However, we shall see below that precision measurements of the sound speed near the transition between the radiative interior (in which energy is transported by radiation) and the outer convective zone (in which energy is transported by convection) reveal small discrepancies between the model predictions and the observations in this region.

If solar physics were responsible for the solar neutrino problems, how large would one expect the discrepancies to be between solar model predictions and helioseismological observations? The characteristic size of the discrepancies can be estimated using the results of the neutrino experiments and scaling laws for neutrino fluxes and sound speeds.

All recently published solar models predict essentially the same fluxes from the fundamental pp and pep reactions (amounting to 72.4 SNU in gallium experiments, cf. Table 1), which are closely related to the solar luminosity. Comparing the measured gallium rates (reported at Neutrino 98) and the standard predicted rate for the gallium experiments, the ^{7}Be flux must be reduced by a factor N if the disagreement is not to exceed n standard deviations, where N and n satisfy $72.4 + (34.4)/N = 72.2 + n\sigma$. For a 1σ (3σ) disagreement, $N = 6.1(2.05)$. Sound speeds scale like the square root of the local temperature divided by the mean molecular weight and the ^{7}Be neutrino flux scales approximately as the 10th power of the temperature [25]. Assuming that the temperature changes are dominant, agreement to within 1σ would require fractional changes of or-

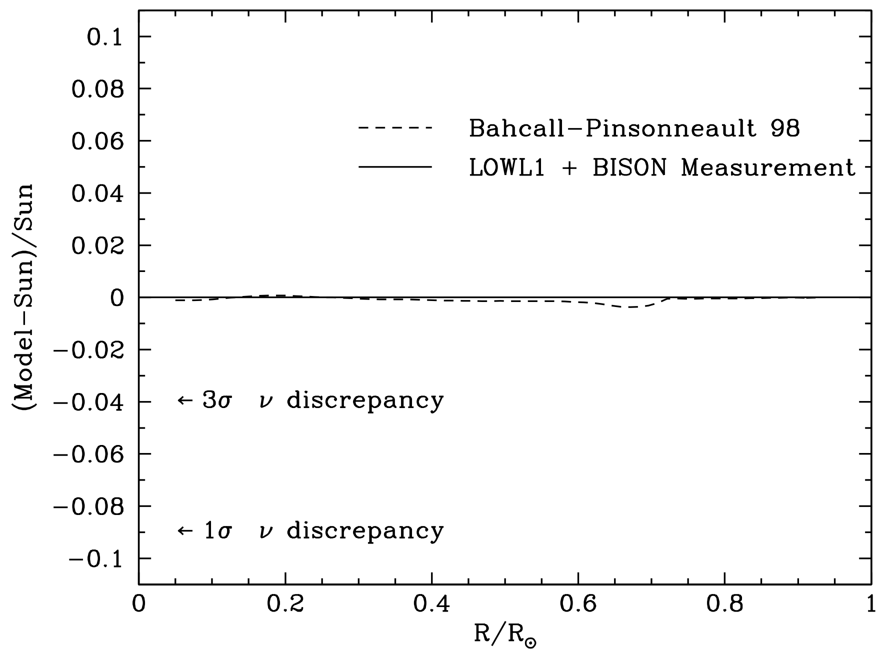

Figure 2. Predicted versus Measured Sound Speeds. This figure shows the excellent agreement between the calculated (solar model BP98, Model) and the measured (Sun) sound speeds, a fractional difference of 0.001 rms for all speeds measured between $0.05R_\odot$ and $0.95R_\odot$. The vertical scale is chosen so as to emphasize that the fractional error is much smaller than generic changes in the model, 0.03 to 0.08, that might significantly affect the solar neutrino predictions.

der 0.09 in sound speeds (3σ could be reached with 0.04 changes), if all model changes were in the temperature[1]. This argument is conservative because it ignores the contributions from the ^{8}B and CNO neutrinos which contribute to the observed counting rate (cf. Table 1) and which, if included, would require an even larger reduction of the ^{7}Be flux.

I have chosen the vertical scale in Fig. 1 to be appropriate for fractional differences between measured and predicted sound speeds that are of order 0.04 to 0.09 and that might therefore affect solar neutrino calculations. Fig. 1 shows that the characteristic agreement between solar model predictions and helioseismological measurements is more than a factor of 30 better than would be expected if there were a solar model explanation of the solar neutrino problems.

5. DISCUSSION AND CONCLUSION

Three decades of refining the input data and the solar model calculations has led to a predicted standard model event rate for the chlorine experiment, 7.7 SNU, which is very close to the best-estimate value obtained in 1968 [26], which was 7.5 SNU. The situation regarding solar neutrinos is, however, completely different now, thirty years later. Four experiments have confirmed the detection of solar neutrinos. Helioseismological mea-

[1] I have used in this calculation the GALLEX and SAGE measured rates reported by Kirsten and Gavrin at Neutrino 98. The experimental rates used in BP98 were not as precise and therefore resulted in slightly less stringent constraints than those imposed here. In BP98, we found that agreement to within 1σ with the then available experimental numbers would require fractional changes of order 0.08 in sound speeds (3σ could be reached with 0.03 changes.)

surements show (cf. Fig. 1) that hypothetical deviations from the standard solar model that seem to be required by simple scaling laws to fit just the gallium solar neutrino results are at least a factor of 40 larger than the rms disagreement between the standard solar model predictions and the helioseismological observations. This conclusion does not make use of the strong evidence which points in the same direction from the chlorine, Kamiokande, and SuperKamiokande experiments.

The improvement in helioseismological measurements over the past two years, from Neutrino 96 to Neutrino 98 (cf. Figure 2 of the Neutrino 96 talk [1] with Figure 2 of this talk), has resulted in a five-fold improvement in the agreement with the calculated standard solar model sound speeds and the measured solar velocities! I believe that this improved agreement is yet another reason to believe that standard solar models reliably predict solar neutrino fluxes.

I am grateful to Y. Suzuki for special efforts that made possible my attendance at Neutrino 98 and which enriched the scientific experience of this extraordinarily exciting conference. I acknowledge support from NSF grant #PHY95-13835.

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