The luminosity constraint on solar neutrino fluxes

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A specific linear combination of the total solar neutrino fluxes must equal the measured solar photon luminosity if nuclear fusion reactions among light elements are responsible for solar energy generation. This luminosity constraint, previously used in a limited form in testing the no neutrino oscillation hypothesis, is derived in a generality that includes all of the relevant solar neutrino fluxes and which is suitable for analyzing the results of many different solar neutrino experiments. With or without allowing for neutrino oscillations, the generalized luminosity constraint can be used in future analyses of solar neutrino data. Accurate numerical values for the linear coefficients are provided.

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I. INTRODUCTION

If nuclear fusion reactions among light elements are responsible for the solar luminosity, then a specific linear combination of solar neutrino fluxes must equal the solar constant. More explicitly, we shall see that

$$\frac{L_\odot}{4\pi(\mathrm{AU})^2} = \sum_i \alpha_i \Phi_i, \qquad (1)$$

where $Ł_\odot$ is the solar luminosity measured at the earth's surface and 1 AU is the average earth-sun distance. The coefficient α_i is the amount of energy provided to the star by nuclear fusion reactions associated with each of the important solar neutrino fluxes, Φ_i. Equation (1) is known as the luminosity constraint.

The sum in Eq. (1) should be taken over all the neutrino fluxes whose associated nuclear fusion reactions could in principle contribute significantly to the energy budget of the sun. If no detailed knowledge derived from solar models is used to limit the appearance of terms in Eq. (1), then the luminosity constraint can be applied to tests of the hypothesis of no neutrino oscillations. The luminosity constraint provides an additional condition that must be satisfied by neutrino fluxes that are otherwise allowed to have arbitrary amplitudes when fit to the available solar neutrino data [1–10].

A. Previous work

Spiro and Vignaud [1] first proposed, in a lucid and insightful paper, the use of the luminosity constraint as a test of the null hypothesis for solar neutrino propagation, i.e., as a test independent of solar models of the assumption of no neutrino oscillations. Following these authors, most of the pioneering applications of the luminosity constraint (see, e.g., [2–6,8]) have approximated the solar neutrino spectrum by grouping the neutrinos into three sets, the low-energy (principally pp) neutrinos, intermediate-energy neutrinos (usually taken to be either ^{7}Be neutrinos only or the ^{7}Be neutrinos plus the CNO and pep neutrinos), and finally the

high-energy (^{8}B) neutrinos. This approximation was necessary and appropriate when the number of solar neutrino experiments was small (two or four), but is no longer required or optimal now that the number of solar neutrino experiments is six (chlorine [11], Kamiokande [12], SAGE [13], GALLEX and GNO [14,15], Super-Kamiokande [16], and SNO [17]) and growing (BOREXINO [18] and ICARUS [19]), with additional experiments in the planning stages. The derivation given here explains (and, in some cases, corrects) the results that were simply stated in Ref. [9].

All previous discussions with which I am familiar, including Ref. [9], have implemented the luminosity constraint in the context of showing that solar fluxes with arbitrary amplitudes, but without the distortion of the energy spectrum implied by neutrino oscillations, do not fit the available data on the measured rates of solar neutrino experiments. The papers by Hata, Bludman, and Langacker [3], Parke [6], Heeger and Robertson [8], and Bahcall, Krastev, and Smirnov [10] were important in persuading many physicists who are not familiar with solar models that a particle physics solution was required for the solar neutrino problem.

B. What is this paper about?

My goal in this paper is to provide a general formulation for the luminosity constraint that can be used in future analyses, with or without allowing for neutrino oscillations, that may include six or more experiments. I also want to provide a specific derivation, lacking in the literature, for the coefficients in Eq. (1). The lack of a general derivation in the literature has led to a confusion about the basis for the luminosity constraint and to significant errors in the published values of some coefficients.

In this paper, I derive the coefficients for the luminosity constraint for all seven of the important neutrino fluxes shown in Table I. After deriving the coefficients for the general form of the luminosity relation, I discuss the most appropriate approximations to make in analyzing data sets in which the number of measured neutrino event rates is not sufficient to allow a statistically meaningful application of the full luminosity constraint. The discussion in this paper is a natural generalization of the treatment given in the very important paper by Hata, Bludman, and Langacker [3],

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TABLE I. Luminosity constraint, neutrino characteristics. The average neutrino energies are taken from Ref. [22] for ^{7}Be and from Ref. [23] for all other sources. The neutrino energies include thermal effects from electron and ion motion and from the solar temperature profile, as well as atomic ionization effects. The nuclear data are taken from [24]. The quantities α and a are defined in Eqs. (2)–(4), the dimensionless form of the luminosity constraint.

Flux	Reaction	$\langle E_\nu \rangle_\odot$ (MeV)	α (10 MeV)	a
$\phi(pp)$	$p+p\rightarrow\,^2\mathrm{H}+e^++\nu_e$	0.2668	1.30987	0.6978
$\phi(pep)$	$p+e^-+p\rightarrow\,^2\mathrm{H}+\nu_e$	1.445	1.19193	0.001 642
$\phi(hep)$	$^3\mathrm{He}+p\rightarrow\,^4\mathrm{He}+e^++\nu_e$	9.628	0.37370	1.09E-07
$\phi(^7\mathrm{Be})$	$^7\mathrm{Be}+e^-\rightarrow\,^7\mathrm{Li}+\nu_e$	0.814 [a]	1.26008	0.055 94
$\phi(^8\mathrm{B})$	$^8\mathrm{B}\rightarrow\,^8\mathrm{Be}+e^++\nu_e$	6.735	0.66305	0.000 059 2
$\phi(^{13}\mathrm{N})$	$^{13}\mathrm{N}\rightarrow\,^{13}\mathrm{C}+e^++\nu_e$	0.706	0.34577	0.006 426
$\phi(^{15}\mathrm{O})$	$^{15}\mathrm{O}\rightarrow\,^{15}\mathrm{N}+e^++\nu_e$	0.996	2.15706	0.005 629

[a]89.7%, 0.8631 MeV and 10.3%, 0.3855 MeV.

which presented cogently the argument that the measured solar neutrino event rates (chlorine [11], Kamiokande [12], SAGE [13], and GALLEX [14]) required new physics, even before the epochal Super-Kamiokande [16] and SNO [17] measurements.

The formulations presented in previous discussions can be recovered from the present analysis by assuming that one or all of the following assumptions is valid: (i) certain neutrino fluxes are zero; (ii) the CNO neutrino fluxes (from ^{13}N and ^{15}O β decay) are equal; or (iii) the standard solar model ratio of pep neutrino flux to pp neutrino flux is correct.

In the future, the generalized luminosity constraint can and should be implemented in analyses that determine solar neutrino parameters. The additional constraint provided by the measured solar luminosity will be especially important when pp and ^{7}Be neutrino fluxes are measured as well as the ^{8}B neutrino flux. As more experimental data become available, the analyses of neutrino oscillations will become more independent of the standard solar model and it will be natural and convenient to incorporate the luminosity constraint, Eq. (1).

The luminosity constraint can be written conveniently in a dimensionless form by considering the ratios of all the neutrino fluxes to the values predicted by the standard solar model, and dividing both sides of Eq. (1) by $L_\odot/[4\pi(\mathrm{AU})^2]$. We obtain

$$1=\sum_i \left(\frac{\alpha_i}{10\ \mathrm{MeV}}\right)a_i\phi_i, \qquad (2)$$

where the dimensionless neutrino fluxes, ϕ_i, are the ratios of the true neutrino fluxes to the neutrino fluxes predicted by the BP2000 standard solar model [20], i.e.,

$$\phi_i\equiv\Phi_i/\Phi_i(\mathrm{BP2000}). \qquad (3)$$

The quantities a_i are the ratios of $\Phi_i(\mathrm{BP2000})$ to the characteristic solar photon flux defined by $L_\odot/[4\pi(\mathrm{AU})^2(10\ \mathrm{Mev})]$. Thus

$$a_i\equiv\Phi_i(\mathrm{BP2000})/(8.5272\times10^{10}\mathrm{cm}^{-2}\,\mathrm{s}^{-1}). \qquad (4)$$

In calculating the characteristic solar flux, I have used the recent best-estimate solar luminosity (see Ref. [21]), 3.842×10^{33} erg s^{-1}, that is derived from all the available satellite data.

Depending on the context, I shall use ϕ_i to refer to either the neutrino flux produced locally or integrated over the entire sun. This dual usage will not cause any confusion since the specific meaning of ϕ_i will be clear in all cases. Moreover, because of the linearity of the averaging process, equations that are valid locally have the same form when the results are integrated over the entire sun.

Table I provides the numerical values for the dimensionless form of the luminosity constraint.

In Sec. II, I derive explicit expressions for the α coefficients that appear in Eq. (2) in terms of measured atomic mass differences and computed neutrino energy losses. I summarize in Sec. III the principal assumptions that are used in the derivation and discuss the results and application strategies in Sec. IV.

II. DERIVATION OF THE LUMINOSITY CONSTRAINT

In this section, I shall derive expressions for the energy coefficients, α_i, of the luminosity constraint. Section II A treats the simpler case of the CNO neutrinos and Sec. II B derives the coefficients for neutrino fluxes produced by reactions in the pp chain. The numerical values given in Table I were calculated using Eqs. (7) and (8) of Sec. II A and Eqs. (13)–(21) of Sec. II B.

In carrying out the calculations, we will use R_{ij} to represent the reaction rate per unit of time per unit of volume between two fusing ions, i and j, where

$$R_{ij}=\frac{\langle i,j\rangle n(i)n(j)}{(1+\delta_{ij})}. \qquad (5)$$

Here $\langle i,j\rangle$ is the local thermal average of σv, the product of the relative velocity of particles i,j and their interaction cross section, and the Kronecher δ prevents double counting of identical particles. For example, $R_{34}=\langle^3\mathrm{He},^4\mathrm{He}\rangle n(^3\mathrm{He})n(^4\mathrm{He})$ and $R_{pp}=\langle p,p\rangle n(p)^2/2$.

In the following, I shall denote the average neutrino energy from a particular nuclear reaction, X, by $\langle E_\nu\rangle(X)$. Table I lists accurate values for these neutrino energies. The values given in Table I are averaged over the energy spectrum from each source and include corrections for solar effects such as contributions from the thermal motion of the fusing ions, averages over ionization states, and the temperature profile of the sun. The average solar neutrino energy losses are taken from Refs. [22,23]. The masses, for example $M(^{13}\mathrm{C})$ or $M(^1\mathrm{H})$, that appear in the equations of Sec. II A and Sec. II B are atomic masses that I have taken from Ref. [24].

For convenience in the calculations, we will introduce a fictitious neutrino flux density,

$$\Phi_{33}\equiv\langle^3\mathrm{He},^3\mathrm{He}\rangle n\langle^3\mathrm{He},^3\mathrm{He}\rangle/2, \qquad (6)$$

which is the rate of the $^3\text{He}(^3\text{He},2p)^4\text{He}$ reaction (and would be the flux density of neutrinos produced by this reaction if the ^{3}He-^{3}He reaction gave rise to neutrinos). This fictitious flux will not appear in any of the final formulas.

A. CNO neutrinos

The ^{13}N β decay corresponds to the thermal energy derived from the reactions $^{12}\text{C}(p,\gamma)^{13}\text{N}$ and $^{13}\text{N}\rightarrow^{13}\text{C}+e^+ +\nu_e$. Hence,

$$\alpha(^{13}\text{N})=M(^{12}\text{C})+M(^1\text{H})-M(^{13}\text{C})-\langle E_\nu\rangle(^{13}\text{N}). \quad (7)$$

Similarly, the energy associated with the ^{15}O β decay derives from the reactions $^{13}\text{C}(p,\gamma)^{14}\text{N}$, $^{14}\text{N}(p,\gamma)^{15}\text{O}$, $^{15}\text{O}\rightarrow^{15}\text{N}+e^++\nu_e$, and $^{15}\text{N}(p,\alpha)^{12}\text{C}$. Therefore,

$$\alpha(^{15}\text{O})=3M(^1\text{H})+M(^{13}\text{C})$$
$$-M(^4\text{He})-M(^{12}\text{C})-\langle E_\nu\rangle(^{15}\text{O}). \quad (8)$$

The neutrino flux from ^{17}F β decay is a potential measure of the primordial ^{16}O abundance in the sun (see Ref. [25]), but does not play a significant role in the generation of the solar luminosity. For completeness, I include here the coefficients α and a that describe the reactions $^{16}\text{O}(p,\gamma)^{17}\text{F}$ and $^{17}\text{F}\rightarrow^{17}\text{O}+e^++\nu_e$. The appropriate linear coefficients for use in Eq. (2) are

$$\alpha(^{17}\text{F})=2.363 \ \text{MeV}, \quad a(^{17}\text{F})=1.09E-07. \quad (9)$$

The value of $a(^{17}\text{F})$ is so small (because of the high Coulomb barrier for this reaction) that the flux $\phi(^{17}\text{F})$ is not relevant for practical applications of the luminosity constraint.

B. pp neutrinos

The analysis of reactions in the pp chain is simplified by the fact that ^{2}H and ^{3}He are burned quickly at solar temperatures [26]. The lifetime for nuclear burning of ^{2}H is $\sim 10^{-8}$ yr and the lifetime of ^{3}He is $\sim 10^5$ yr. These values are both very small compared to the 10^{10} yr lifetime of a proton (which is destroyed primarily by the pp reaction). Therefore, it is an excellent approximation to assume that both ^{2}H and ^{3}He are in local kinetic equilibrium (rate of destruction equals rate of production).

From the local equilibrium of ^{2}H, $dn(^2\text{H})/dt=0$, one has

$$\langle ^1\text{H},^1\text{H}\rangle n(^1\text{H})^2/2+\langle ^1\text{H},e^-+\text{H}\rangle n(e)n(\text{H})$$
$$=\langle ^1\text{H},^2\text{H}\rangle n(^1\text{H})n(^2\text{H}). \quad (10)$$

Equation (10) states that the production of deuterium via the pp and pep reactions is balanced by the destruction of deuterium via the $^2\text{H}(p,\gamma)^3\text{He}$ reaction. The equilibrium of ^{3}He, $dn(^3\text{He})/dt=0$, implies

$$\langle ^1\text{H},^2\text{H}\rangle n(^1\text{H})n(^2\text{H})$$
$$=\langle ^3\text{He},^3\text{He}\rangle n(3)^2+\langle ^3\text{He},^4\text{He}\rangle n(^3\text{He})n(^4\text{He})$$
$$+\langle ^3\text{He},^1\text{H}\rangle n(^3\text{He})n(^1\text{H}). \quad (11)$$

Equation (11) describes the fact that the rate of production of ^{3}He via the $^2\text{H}+^1\text{H}$ reaction is balanced by the destruction of ^{3}He via the ^{3}He-^{3}He, ^{3}He-^{4}He, and hep reactions. For the term describing the ^{3}He-^{3}He reaction in Eq. (11), the factor of one-half from the identity of the fusing particles is canceled by the factor of 2 representing the destruction of two ^{3}He ions.

Combining Eqs. (6), (10), and (11), we find

$$\phi(pp)+\phi(pep)=2\times\phi(3,3)+\phi(hep)+\phi(^7\text{Be})+\phi(^8\text{B}). \quad (12)$$

Let ϵ_i represent the thermal energy released to the star as a result of the nuclear fusion reactions associated directly with each neutrino producing reaction. Then we can write

$$L_\odot/[4\pi(\text{AU})^2]=\text{CNO terms}+\epsilon_{pp}\phi(pp)+\epsilon_{pep}\phi(pep)$$
$$+(\epsilon_{33}/2)[\phi(pp)+\phi(pep)-\phi(hep)$$
$$-\phi(^7\text{Be})-\phi(^8\text{B})]+\epsilon_{hep}\phi(hep)$$
$$+(\epsilon_{34}+\epsilon_{e7})\phi(^7\text{Be})+(\epsilon_{34}+\epsilon_{p,7})\phi(^8\text{B}). \quad (13)$$

I have used Eq. (12) to eliminate the fictitious flux $\phi(3,3)$ from Eq. (13). This substitution associates with each of the real neutrino fluxes in the pp chain an additional energy contribution proportional to ϵ_{33}, the energy released to the star via the ^{3}He-^{3}He reaction. Physically, these terms proportional to ϵ_{33} represent a way of keeping track of how much energy from the the ^{3}He-^{3}He reaction should be associated with the other neutrino fluxes.

The values of ϵ_i can be calculated by writing explicitly the reaction equations that are associated with the production of each neutrino flux. One finds

$$\epsilon_{pp}=3M(^1\text{H})-M(^3\text{He})-\langle E_\nu\rangle(pp), \quad (14)$$

$$\epsilon_{pep}=3M(^1\text{H})-M(^3\text{He})-\langle E_\nu\rangle(pep), \quad (15)$$

$$\epsilon_{33}=2M(^3\text{He})-M(^4\text{He})-2M(^1\text{H}), \quad (16)$$

$$\epsilon_{34}=M(^3\text{He})+M(^4\text{He})-M(^7\text{Be}), \quad (17)$$

$$\epsilon_{e7}=M(^7\text{Be})+M(^1\text{H})-2M(^4\text{He})-\langle E_\nu\rangle(^7\text{Be}), \quad (18)$$

$$\epsilon_{p,7}=M(^7\text{Be})+M(^1\text{H})-2M(^4\text{He})-\langle E_\nu\rangle(^8\text{B}), \quad (19)$$

and

$$\epsilon_{hep}=M(^3\text{He})+M(^1\text{H})-M(^4\text{He})-\langle E_\nu\rangle(hep). \quad (20)$$

In calculating ϵ_{e7}, one must average over the two ^{7}Be neutrino lines with the appropriate weighting and include the γ-ray energy from the 10.3% of the decays that go to the first excited state of ^{7}Li.

The values of the α's can be determined using the following relations between the α coefficients and the ϵ coefficients that follow from Eq. (13). We have

$$\alpha(pp) = \epsilon_{pp} + 0.5\epsilon_{33}, \quad \alpha(pep) = \epsilon_{pep} + 0.5\epsilon_{33}, \quad \alpha(^7\text{Be})$$
$$= \epsilon_{34} + \epsilon_{e7} - 0.5\epsilon_{33},$$

$$\alpha(^8\text{B}) = \epsilon_{34} + \epsilon_{p,7} - 0.5\epsilon_{33}, \quad \alpha(hep) = \epsilon_{hep} - 0.5\epsilon_{33}.$$
$$(21)$$

III. WHAT ASSUMPTIONS ARE MADE IN DERIVING THE LUMINOSITY CONSTRAINT?

The basic assumption made in deriving the luminosity constraint is that nuclear fusion reactions among light elements are responsible for the observed solar luminosity. More specifically, I assume in Sec. II that the specific nuclear reactions that have been recognized [26–29] over the six decades since Hans Bethe's epochal work on the subject as being most important at temperatures of order of a keV are indeed the fusion reactions that power the sun. The characteristic temperature of the sun can be estimated relatively well without making use of a detailed model [26,27].

In order to derive Eqs. (10) and (11), it is necessary to assume that both ^{2}H and ^{3}He are in local kinetic equilibrium (rate of creation equal rate of destruction). As discussed in Sec. II B, this is an excellent approximation because the lifetimes for nuclear burning of these isotopes are short compared to the evolutionary time scale for the sun.

IV. DISCUSSION

I have given in Sec. II an explicit derivation of the luminosity constraint and have presented in Table I coefficients that can be used in the dimensionless form of the constraint, Eq. (2). The coefficients that are given in Table I are calculated accurately and include small corrections to the neutrino energy release that result from the high temperatures ($\sim$keV) in the region in which fusion reactions occur in the sun.

The form of the luminosity constraint given here includes all the important solar neutrino fluxes. One can recover approximately the coefficients used by previous authors who have combined neutrino fluxes, or who have considered only a reduced set of fluxes, by making the relevant choices among the fluxes listed in Table I. However, the reader is warned not to expect precise agreement; there are many inaccurate numerical values in the published papers.[1]

The generalized form of the luminosity constraint presented here can be used, as the more restricted constraint has been used in the past, to help test the validity of the null hypothesis for solar neutrino oscillations. In this test (cf. Refs. [1–9]), the neutrino fluxes are allowed to have arbitrary amplitudes subject to Eq. (2) and the condition that the energy spectra are undistorted by neutrino oscillations.

In the past, applications of the luminosity constraint have been limited to tests of the no oscillation hypothesis. Nothing in the derivation of the luminosity constraint requires this limitation in the range of applications. In the future, when more experimental data are available, the luminosity constraint can be used together with the measured solar neutrino interaction rates, energy spectra, and time dependences, to help determine neutrino oscillation parameters.

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[1]In fact, I am responsible for an egregious but unimportant error. In Ref. [9], the listed value for $\alpha(hep)$ is wrong because I neglected to subtract the value of $\epsilon_{33}/2 = 6.4298$ MeV in going from ϵ_{hep} to $\alpha(hep)$ [see Eq. (21) of the present paper]. The error is unimportant, although embarrassing, because the Super-Kamiokande experiment has placed [16] a strong upper limit on the hep flux; this upper limit is also in agreement with the predicted hep flux for the standard solar model [20].

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