STATUS OF SOLAR MODELS [a]

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The neutrino fluxes calculated from 14 standard solar models published recently in refereed journals are inconsistent with the results of the 4 pioneering solar neutrino experiments if nothing happens to the neutrinos after they are created in the solar interior. The sound speeds calculated from standard solar models are in excellent agreement with helioseismological measurements of sound speeds. Some statements made by Dar at Neutrino 96 are answered here.

1 Introduction

I was asked by Matts Roos to review in this talk the status of solar models as they relate to the solar neutrino problems. I will therefore not discuss any of the solutions that suggest new physics; that subject has just been covered beautifully by Alexei Smirnov and there will be a further careful discussion by Serguey Petcov this afternoon. I do, however, want to make a few introductory remarks in order to put my talk in the appropriate context.

Solar neutrino research has achieved its primary goal, the detection of solar neutrinos, and is now entering a new phase in which large electronic detectors will yield vast amounts of diagnostic data. The new experiments, [1,2,3] which will be described after lunch today in talks by Suzuki, McDonald, Bellotti, Vogelaar, and Bowles, will test the prediction of standard electroweak theory [4,5,6] that essentially nothing happens to electron type neutrinos after they are created by nuclear fusion reactions in the interior of the sun.

The four pioneering experiments—chlorine [7,8] (reviewed by Ken Lande at this conference), Kamiokande [9] (reviewed by Y. Suzuki), GALLEX [10] (reviewed by T. Kirsten), and SAGE [11] (reviewed by V. Gavrin)—have all observed neutrino fluxes with intensities that are within a factors of a few of those predicted by standard solar models. Three of the experiments (chlorine, GALLEX, and SAGE) are radiochemical and each radiochemical experiment

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[a] This talk is based upon continuing collaborative research of John Bahcall and M. H. Pinsonneault. The first stages of this work were described at the symposium on The Inconstant Sun, Naples, Italy, March 18, 1996, to be published in Memorie della Societa, eds. G. Cauzzi and C. Marmolino. This talk will appear in Neutrino 96, Proceedings of the 17 International Conference on Neutrino Physics and Astrophysics, Helsinki, Finland, ed. K. Huitu, K. Enqvist, and J. Maalampi (World Scientific, Singapore, 1996). For both conferences, the talks were given by Bahcall. Further information about solar neutrinos is available at http://www.sns.ias.edu/~jnb .

measures one number, the total rate at which neutrinos above a fixed energy threshold (which depends upon the detector) are captured. The sole electronic (non-radiochemical) detector among the initial experiments, Kamiokande, has shown that the neutrinos come from the sun, by measuring the recoil directions of the electrons scattered by solar neutrinos. Kamiokande has also demonstrated that the observed neutrino energies are consistent with the range of energies expected on the basis of the standard solar model.

Despite continual refinement of solar model calculations of neutrino fluxes over the past 35 years (see, e.g., the collection of articles reprinted in the book edited by Bahcall, Davis, Parker, Smirnov, and Ulrich [12]), the discrepancies between observations and calculations have gotten worse with time. All four of the pioneering solar neutrino experiments yield event rates that are significantly less than predicted by standard solar models. Moreover, there are well known inconsistencies between the different experiments if the observations are interpreted assuming that nothing happens to the neutrinos after they are created.

This talk is organized as follows. I will first summarize the results of all the recently published standard solar model calculations and compare them with the results of the four solar neutrino experiments. This survey of the literature is, to the best of my knowledge, complete until June 1, 1996, just prior to the beginning of the Neutrino 96 conference. Next I shall discuss the excellent agreement between the sound speeds predicted by standard solar models and the sound speeds measured by helioseismological techniques. Finally, I shall discuss briefly some of the remarks about solar models that were made at the conference by A. Dar.

2 Observation versus Calculation: Neutrino Fluxes

Figure 1 displays the calculated ^{7}Be and ^{8}B solar neutrino fluxes for all 14 of the standard solar models with which I am familiar that have been published in refereed science journals since 1988 and until June 1, 1996. I choose to start in 1988 since, as we shall see below, helioseismology plays an important role in validating and constraining solar models and the first systematic discussion of the relation between helioseismology and solar neutrino research was published in 1988. [13] I normalize the fluxes by dividing each published value by the flux from the most recent Bahcall and Pinsonneault [14] standard solar model which makes use of improved input parameters and includes heavy element and helium diffusion. The abscissa is the normalized ^{8}B flux and the numerator is the normalized ^{7}Be neutrino flux. The sides of the rectangular box represent the separate 3σ uncertainties in the predicted ^{7}Be and ^{8}B neutrino fluxes

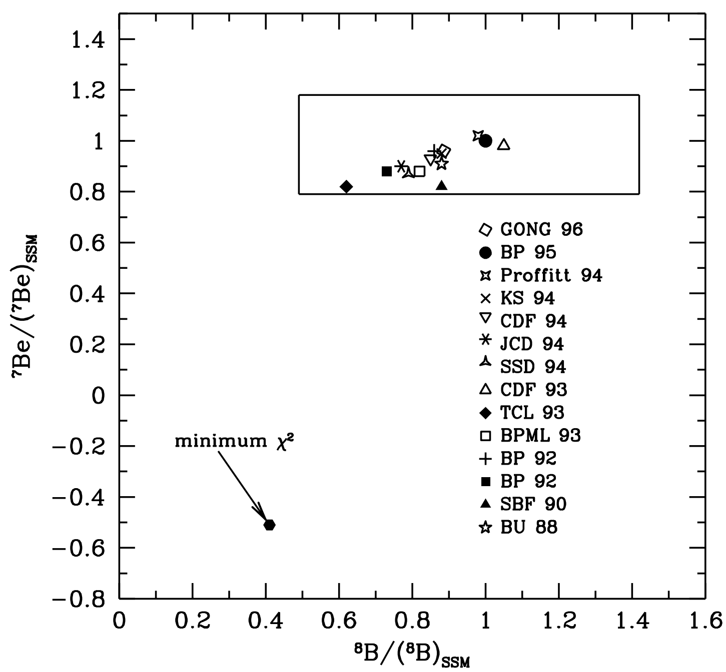

Figure 1: The calculated ^{7}Be and ^{8}B solar neutrino fluxes for all 14 of the standard solar models. The sides of the rectangular box represent the estimated 3σ uncertainties in the predicted ^{7}Be and ^{8}B neutrino fluxes of the standard solar model. [14] All of the fluxes have been normalized by dividing by the Bahcall and Pinsonneault [14] standard solar model (SSM) values. The abbreviations of the various solar models are GONG (Christensen-Dalsgaard et al. [15]), BP 95 (Bahcall and Pinsonneault [14]), KS 94 (Kovetz and Shaviv [16]), CDF 94 (Castellani et al. [17]), JCD 94 (Christensen-Dalsgaard [18]), SSD 94 (Shi, Schramm, and Dearborn [19]), CDF 93 (Castellani, Degl'Innocenti, and Fiorentini [20]), TCL 93 (Turck-Chièze and Lopes [21]), BPML 93 (Berthomieu, Provost, Morel, and Lebreton [22]), BP 92 (Bahcall and Pinsonneault [23]), SBF 90 (Sackman, Boothroyd, and Fowler [24]), and BU 88 (Bahcall and Ulrich [13]).

of the standard solar model. [14] The abbreviations that indicate references to individual models are identified in the caption of Figure 1.

All of the solar model results from different groups fall within the rectangular error box, i.e., within the estimated 3σ uncertainties in the standard model predictions. This agreement between the results of 14 groups demonstrates the robustness of the predictions since the calculations use different computer codes and involve a variety of choices for the nuclear parameters, the equation of state, the stellar radiative opacity, the initial heavy element abundances, and the physical processes that are included. In fact, all published standard solar models give the same results for solar neutrino fluxes to an accuracy of better than 10% if the same input parameters and physical processes are included.[23,14]

The largest contribution to the dispersion in values in Figure 1 is caused by the inclusion, or non-inclusion, of element diffusion in the stellar evolution codes. The Proffitt,[25] the Bahcall and Pinsonneault,[14] and the Christensen-Dalsgaard et al.[15] models all include helium and heavy element diffusion. The predicted fluxes in these three models agree to within $\pm 10\%$, although the models are calculated using different mathematical descriptions of diffusion (and somewhat different input parameters), The calculated value that is furtherest from the center of the box is by Turck-Chièze and Lopes,[21] which does not include either helium or heavy element diffusion. However, the Turck-Chièze and Lopes best estimate is still well within the 3σ box.

We shall now see that helioseismology shows that diffusion must be included in the solar model in order to obtain agreement with observations.

3 Comparison with Helioseismological Measurements

Helioseismology has recently sharpened the disagreement between observations and the predictions of solar models with standard (non-oscillating) neutrinos. The solar models that include diffusion predict[14] somewhat higher event rates in the chlorine and Kamiokande solar neutrino experiments and thereby exacerbate the well known solar neutrino problems that arise when standard neutrino physics (no neutrino oscillations) is assumed.

By including element diffusion, the four solar models near the center of the box in Figure 1 (models of Bahcall and Pinsonneault,[23] Proffitt,[25] Bahcall and Pinsonneault,[14] and Christensen-Dalsgaard et al.[15]) yield values for the depth of the convective zone and the primordial helium abundance that are in agreement with helioseismological measurements. (The model of Richard et al.[26] yields results in good agreement with the four solar models just mentioned that include element diffusion, but was not yet published in Astron. and Astrophys. by the cutoff date, June 1, 1996.)

Figure 2 compares the values of P/ρ (pressure divided by density) obtained from helioseismology and the values calculated for three different solar models. The helioseismological values were kindly supplied to us by W. A. Dziembowski; they are based upon the Dziembowski et al. (1994) method.[27] The specific calculations leading to these improved values of P/ρ are described

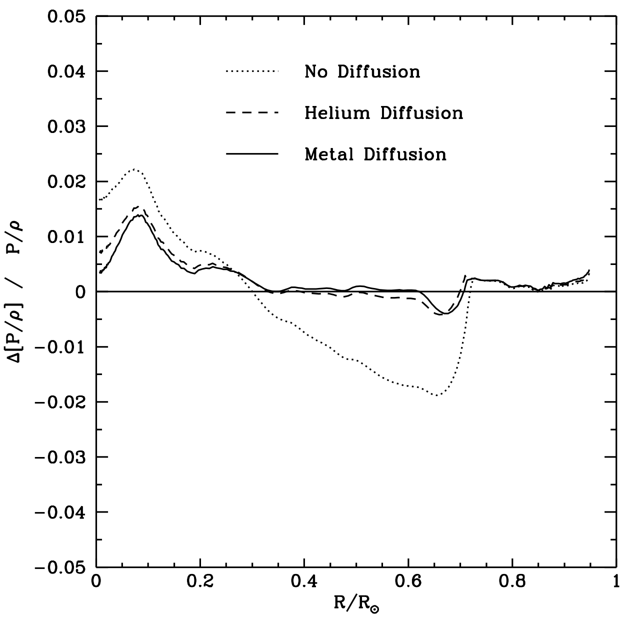

Figure 2: Comparison of the profile of (pressure/density) predicted by different standard solar models with the values inferred from helioseismology. There are no free parameters in the models; the microphysics is successively improved by first including helium diffusion and then by using helium and heavy element diffusion. The figure shows the fractional difference, $[x - x\odot]/x_{\odot}$, between the predicted Model values of $x = P/\rho$ and the measured Solar values of P/ρ, as a function of radial position in the sun ($R_{\odot}$ is the solar radius). The dotted line refers to a model [14] in which diffusion is not included and the dashed line was computed from a model [14] in which helium diffusion was included. The dark line represents our best 1995 solar model which includes both helium and heavy element diffusion.

in Richard et al. (1996). [26] The calculations make use of new data for the low degree modes, $l \leq 3$, from the BISON network. [28]

For the models that include helium diffusion or helium plus heavy element diffusion, the agreement is excellent between model predictions and the solar values of P/ρ. Over the entire region of the sun for which the helioseismological values are well determined, from $0.3 \leq (r/R_{\odot}) \leq 0.95$, the model values of P/ρ agree with the helioseismological values to much better than 1%. To a good approximation, $P/\rho \propto T/\mu$, where T is the local value of the temperature and μ is the local mean molecular weight.The temperature in the standard solar model changes by a factor of 24 from $R = 0.3R_{\odot}$ to $R = 0.95R_{\odot}$, while the molecular weight only changes by a few percent.

The excellent agreement shown in Figure 2 between solar models that include diffusion and the helioseismological observations demonstrates that solar models correctly predict the temperature profile of the sun to a few tenths of a percent over most of the sun. The agreement is less precise, of the order of 1%, in the deep interior, but in this region the observations are not yet very reliable.

Helioseismology, as summarized in Figure 2, has effectively shown that the solar neutrino problems cannot be ascribed to errors in the temperature profile of the sun. It is well known [29,30] that in order to change the predicted neutrino fluxes by amounts sufficient to affect significantly the discrepancies with neutrino observations the temperatures must differ from the values in the standard solar model by at least 5%. Figure 2 shows that helioseismology constrains the differences from standard models to be everywhere less than or of order 1%, and much less than 1% over most of the sun.

Solar models that do not include diffusion are not consistent with the helioseismological evidence (for previous evidence supporting this conclusion see the discussions in Christensen-Dalsgaard, Proffitt, and Thompson,[31] Guzik and Cox,[32] Bahcall and Pinsonneault,[14] and Christensen-Dalsgaard et al.[15]). Figure 2 shows that solar models in which diffusion is not included are grossly inconsistent with the helioseismological observations in the region in which the observations are most reliable and precise.

In my view, only solar models that include element diffusion should, in the future, be called "standard solar models". These "standard models" all lie close to the center of the rectangular error box in Figure 1. The physics of diffusion is simple and there is an exportable subroutine available for calculating diffusion in stars (see http://www.sns.ias.edu/~jnb). Observation requires, and computing technology easily permits, the inclusion of diffusion in any standard stellar evolution code.

4 Recent Improvements in the Equation of State and Opacity

In preparation for this meeting, we have calculated new solar models that include recent improvements in opacity [33] and equation of state [34] on the predicted solar neutrino fluxes. Table 1 gives the neutrino fluxes computed for three different standard solar models, all of which include helium and heavy element diffusion. The model labeled BP95 is from Bahcall and Pinsonneault;[14] the models labeled New Opac and OPAL EOS include, respectively, the improved opacities discussed in Iglesias and Rogers [33] and the improved opacities plus the new OPAL equation of state discussed in Rogers, Swenson, and Iglesias.[34]

Table 1: Neutrino Fluxes for Solar Models with Diffusion. All fluxes, except for ^{8}B and ^{17}F, are given in units of 10^{10} per $cm^{-2}s^{-1}$ at the earth's surface. The ^{8}B and ^{17}F fluxes are in units of 10^6 per $cm^{-2}s^{-1}$.

Model	*pp*	*pep*	^{7}Be	^{8}B	^{13}N	^{15}O	^{17}F
BP95	5.91	0.014	0.515	0.662	0.062	0.055	0.065
New Opac	5.91	0.014	0.516	0.662	0.062	0.055	0.065
OPAL EOS	5.91	0.014	0.513	0.660	0.062	0.054	0.065

The neutrino fluxes computed with the improved opacity and equation of state differ from the previously published values[14] by amounts that are negligible in solar neutrino calculations. The predicted event rate, for all three models, is

$$\text{Cl Rate} = 9.5^{+1.2}_{-1.4} \text{ SNU} \quad (1)$$

for the chlorine experiment and

$$\text{Ga Rate} = 137^{+8}_{-7} \text{ SNU} \quad (2)$$

for the gallium experiments. The only noticeable change in the predicted event rates for the chlorine and the gallium experiment is a slightly increased (by 2%) event rate for chlorine, which is due to a small improvement[35] in the calculation of the neutrino absorption cross sections for ^{8}B.

It is obviously important to compare the improved solar models with helioseismological measurements to see if the better equation of state and opacity used in these most recent models affect significantly the calculated sound velocities. Unfortunately, we were not able to complete those calculations in time for the meeting.

5 Quantitative Comparison with Neutrino Experiments

How do the observations from the four pioneering solar neutrino experiments agree with the solar model calculation? Plamen Krastev and I (see Bahcall and Krastev[36] for a description of the techniques) have recently compared the predicted standard model fluxes, with their estimated uncertainties, and the observed rates in the chlorine, Kamiokande, GALLEX, and SAGE experiments. The theoretical solar modeland experimental uncertainties, as well as the uncertainties in the neutrino cross sections, have been combined quadratically. Using the predicted fluxes from the Bahcall and Pinsonneault [14] model, the χ^2 for the fit to the four experiments is

$$\chi^2_{\rm SSM}(\text{all 4 experiments}) = 56\ . \qquad (3)$$

The theoretical uncertainties (from the solar model and the neutrino cross section calculations) and the experimental errors (statistical and systematic, combined quadratically) have been taken into account in obtaining Eq. 3.

Suppose we now ignore what we have learned from solar models and allow the important ^{7}Be and ^{8}B fluxes to take on any non-negative values. What is the minimum value of χ^2 for the 4 experiments, when the only constraint on the fluxes is the requirement that the luminosity of the sun be supplied by nuclear fusion reactions among light elements? We include the nuclear physics inequalities between neutrino fluxes (see section 4 of Bahcall and Krastev [36]) that are associated with the luminosity constraint and maintain the standard value for the almost model-independent ratio of *pep* to *pp* neutrinos.

The best fit for arbitrary ^{7}Be and ^{8}B neutrino fluxes is obtained for $^7{\rm Be}/(^7{\rm Be})_{\rm SSM} = 0$ and $^8{\rm B}/(^8{\rm B})_{\rm SSM} = 0.40$, where

$$\chi^2_{\rm minimum}(\text{all 4 experiments;\ \ arbitrary }^7\text{Be, }^8\text{B}) = 14.4\ . \qquad (4)$$

The CNO neutrinos were assumed equal to their standard model values in the calculations that led to Eq. 4. The fit can be further improved if we set the CNO neutrino fluxes equal to zero. Then, the same search for arbitrary ^{7}Be and ^{8}B neutrino fluxes leads to

$$\chi^2_{\rm minimum}(\text{all 4 experiments;\ \ arbitrary }^7\text{Be, }^8\text{B; CNO} = 0) = 5.9\ . \qquad (5)$$

If we drop the physical requirement that the ^{7}Be flux be positive definite, the minimum χ^2 occurs (cf. Figure 1) for a negative value of the ^{7}Be flux; this unphysical result is a reflection of what has become known in the physics literature as " the missing ^{7}Be solar neutrinos.". The reason that the ^{7}Be neutrinos appear to be missing (or have a negative flux) is that the two gallium experiments, GALLEX and SAGE, have an average event rate of 74 ± 8 SNU, which is fully accounted for in the standard model by the fundamental $p - p$ and *pep* neutrinos (best estimate 73 ± 1 SNU). In addition, the ^{8}B neutrinos that are observed in the Kamiokande experiment will produce about 7 SNU in the gallium experiments, unless new particle physics affects the neutrinos.

To me, these results suggest strongly that the assumption on which they are based—nothing happens to theneutrinos after they are created in the interior of the sun—is incorrect. A less plausible alternative (in my view) is that some of the experiments are wrong; this must be checked by further experiments.

6 Comments on Some Remarks by Dar

In the closing session on solar neutrinos at Neutrino 96, Arnon Dar made a number of surprising statements about solar models and the input data used in their construction.[37] I state below in italics some of Dar's most remarkable claims. The resolution of each of the issues he raised is given in a paragraph following the relevant italicized statement.

• *Final state interactions in ^{37}Cl and ^{71}Ga may invalidate the neutrino cross sections of Bahcall for low energy pp and ^{7}Be neutrinos.*

Dar cites electron screening, overlap and exchange effects, nuclear recoil, and radiative corrections as final state interactions that might be important.

Electron screening is included explicitly in Bahcall's calculations with the aid of Hartree-Fock wave functions and amounts to an effect of order 1% for ^{37}Cl and 4% for ^{71}Ga. Overlap and exchange effects, as well as bound-state beta-decay, were evaluated in Section III of Bahcall (1978) and found to be less than 1%. These results are summarized in Section 8.1A of the book *Neutrino Astrophysics.*[38] Radiative corrections have been calculated explicitly for some cases and are about 1% (i.e, of order the fine structure constant, α). Nuclear recoil effects are $\sim$ [nuclear recoil energy/(electron kinetic energy)] and are less than 0.1% for ^{37}Cl and ^{71}Ga.

• *A strong magnetic field may polarize the electrons in the solar interior and affect the branching ratios of electron capture by ^{7}Be.*

In order to polarize electrons in the solar interior with typical kinetic energies of order a keV, a magnetic field of order 10^{12}G is required. A field of 10^{12}G would produce a total pressure in the solar interior 10^{5} times larger than the pressure in standard solar models and is therefore ruled out by the excellent agreement (to within 1%) between the standard models and the helioseismological measurements (see Figure 2 and Section 5.6 of *Neutrino Astrophysics*).

• *Something must be wrong because it is known that the OPAL equation of state causes significant changes in the calculated neutrino fluxes.*

In his talk, Dar cited calculations in which the use of the OPAL equation of state significantly affected the calculated neutrino fluxes. He suggested that something must be wrong with our calculations because we did not find large changes when we usedthe new equation of state.

The previous equation of state and opacity values that we have been using are quite close in the solar interior to the newer OPAL equation of state and opacity tables. This explains why we find only small changes in the neutrino fluxes (see Table 1). Presumably, for the codes Dar cited, the new OPAL data caused larger changes in the input physics and hence larger changes in the calculated neutrino fluxes.

• *The differences between the Bahcall-Pinsonneault and the Dar-Shaviv nuclear reaction cross sections represent personal judgment.*

We use the cross section factors published by the experimentalists who did the measurements. When multiple measurements are made of a given reaction, we use the weighted average of the measurements that is published by nuclear physicists.

Dar described in his talk his proposed method of extrapolation, which is apparently different from what nuclear experimentalists have traditionally used. The analysis by Dar has been criticized by Langanke, [39] who argues that a proper treatment with Dar's method must lead to the same results as obtained by the more traditional extrapolation. Dar and Shaviv use six cross section factors that are significantly different from the conventional values that we have taken from the literature. [37] All of the choices that Dar and Shaviv have made are in the direction of reducing the calculated event rates in the solar neutrino experiments.

• *The pp reaction cross section can be calculated accurately from measured reactions involving deuterium.*

As justification for his choice of the cross section factor for the $^1H(p, e^+ + \nu)^2$H reaction, Dar cites [37] the experimental cross sections for anti-neutrinos and gamma-rays on deuterium. He states that these measurements were used to obtain a cross section for $p + p \rightarrow {}^2H + e^+ + \nu$. No equations or other details are given (see page 938 of ref. 37). The most relevant measurement to which Dar refers is the reaction $\bar{\nu}_e + {}^2H \rightarrow n + n + e^+$, for which the quoted 1σ experimental uncertainty is 26%. [40] The matrix element for the γ-disintegration reaction he cites is not the same as the matrix element for the neutrino reaction. Dar states that his procedure yields a value consistent with the Caughlan and Fowler (1988) rate, [41] which was based upon the recalculation of the *pp* cross section factor by Bahcall and Ulrich (1988). [13] Since the publication of the 1988 work, Kamionkowski and Bahcall(1994) [42] included vacuum polarization in the calculation of this reaction and reevaluated the nuclear matrix elements using improved data for the *pp* scattering and for the deuteron wave function. In their published paper, Kamionkowski and Bahcall tabulated the numerical results they obtained by solving the Schroedingerequation with seven different nuclear potentials that have been used by different nuclear physics groups. Combining the theoretical and experimental uncertainties, Kamionkowski and Bahcall find $S_{pp}(0) = 3.89(1 \pm 0.011)$ MeV barns. Dar gives a value of ≈ 4.07, with no quoted uncertainty, instead of $3.89(1 \pm 0.011)$. The neutrino cross sections to which Dar refers as his justification are uncertain by much more than the 4% difference between his estimated value and the detailed Kamionkowski and Bahcall calculation. From the published literature, one cannot determine how Dar obtained the value he quotes.

• *The pep and ^{7}Be electron capture rates of Bahcall and his collaborators are not as accurate as the 1988 tabulation by Fowler and Caughlan .*

This statement is based upon a misunderstanding of the purpose of the Fowler-Caughlan tabulations.

The Fowler and Caughlan expressions are simple analytic approximations to the complicated expressions derived by Bahcall and his collaborators. The Fowler and Caughlan expressions are designed to be approximately valid, as they state, over an enormous range of temperatures, 10^6 K to 10^9 K. They are not designed to reproduce precisely, for solar temperatures, the expressions of Bahcall et al. from which they are derived.

For both the *pep* and the ^{7}Be electron capture reactions, all of the references by Fowler, Caughlan, and their collaborators in their 5 review articles [41] are to results by Bahcall and his collaborators (see ref. [41]).

• *The normalization of the heavy element abundances is not handled properly by Bahcall and Pinsonneault.*

Dar states that Bahcall and Pinsonneault assume that the "present photospheric abundances equal the meteoritic abundances."

This is not only an incorrect statement of what we do, it is impossible to implement. Meteorites are rocks; they do not contain hydrogen. Therefore, one cannot fix the normalization of the heavy elements from the meteoritic abundances.

As described at the bottom of page 87 of *Neutrino Astrophysics*, we take the *relative* abundances of the heavy elements (except for He,C,N,O, and Ne) from the meteorites. We assume that this set of relative abundances applies to the *initial* sun. The connection between the meteoritic abundances and the measured solar photospheric abundances, which do include hydrogen, is made by Anders and Grevesse[43] using a series of elements for which abundances are measured accurately in both the photosphere and the meteorites. This fixes Z/X on the surface of the sun today, where X and Z are the mass fractions of hydrogen and heavy elements. Of course, $X + Y + Z = 1$, where Y is the helium mass fraction. We fix theabsolute values of the abundances by requiring that the current solar model have a luminosity at the present solar epoch equal to the observed solar luminosity. With the normalization of the three fractions, the observed ratio of Z/X, and the luminosity constraint, we have three equations for three unknowns. In models in which diffusion is included, the current surface abundance of heavy elements is different from the initial surface abundance of heavy elements.

It is not clear how Dar and Shaviv normalize their heavy element abundances since Dar states that they assume that the "initial solar abundance equals the meteoritic abundance." As explained above, the meteorites only determine relative heavy element abundances.

7 Discussion

The combined predictions of the standard solar model and the standard electroweak theory disagree with the results of the four pioneering solar neutrino experiments. The same solar model calculations are in good agreement with the helioseismological measurements.

Comparing the solar model predictions to the existing solar neutrino data, we obtain values for $\chi^2_{standard}$ of ~ 56. The fits are much improved if neutrino oscillations, which are described by two free parameters, are included in the calculations. With neutrino oscillations, the characteristic value for $\chi^2_{\mathrm{min},\ osc.} \sim 1$. New experiments [1,2,3] involving large electronic detectors of individual neutrino events will decide in the next few years if neutrino oscillations are indeed important in interpreting solar neutrino experiments.

We may ask: What have solar neutrino experiments taught us about astronomy? Most importantly, the experiments have detected solar neutrinos with approximately the fluxes and in the energy range predicted by solar models. The operating experiments have achieved the initial goal of solar neutrino astronomy by showing empirically that the sun shines via nuclear fusion reactions. This achievement by a large community of physicists, chemists, engineers, and astronomers puts the theory of stellar evolution on a firm empirical basis.

Moreover, the observed and the standard predicted neutrino interaction rates agree within factors of a few, providing— even if we ignore the effects of possible neutrino oscillations— semi-quantitative confirmation of the calculations of temperature-sensitive nuclearfusion rates in the solar interior.

Acknowledgments

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