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NEUTRINO AFTERGLOW FROM GAMMA-RAY BURSTS: $\sim 10^{18}$ eV

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ABSTRACT

We show that a significant fraction of the energy of a γ-ray burst (GRB) is probably converted to a burst of 10^{17}–10^{19} eV neutrinos and multiple GeV γ-rays that follow the main GRB by ~ 10 s. If GRBs accelerate protons to $\sim 10^{20}$ eV, a suggestion that recently gained support from observations of GRB afterglows, then both the neutrinos and the γ-rays may be detectable.

Subject headings: acceleration of particles — elementary particles — gamma rays: bursts

1. INTRODUCTION

The widely accepted interpretation of the phenomenology of γ-ray bursts (GRBs) is that the observable effects are due to the dissipation of the kinetic energy of a relativistically expanding fireball whose primal cause is not yet known (see Mészáros 1995 and Piran 1996 for reviews). The physical conditions in the dissipation region imply that protons can be Fermi accelerated to energies greater than 10^{20} eV (Waxman 1995a; Vietri 1995; see Waxman 1999 for a recent review).

Adopting the conventional fireball picture, we showed previously that the prediction of an accompanying burst of $\sim 10^{14}$ eV neutrinos is a natural consequence (Waxman & Bahcall 1997). The neutrinos are produced by π^+ created in interactions between fireball γ-rays and accelerated protons. The key relation is between the observed photon energy, ϵ_γ, and the accelerated proton's energy, ϵ_p, at the photo-meson threshold of the Δ-resonance. In the observer's frame,

$$\epsilon_\gamma \epsilon_p = 0.2 \text{ GeV}^2 \, \Gamma^2 \, , \tag{1}$$

where phenomenologically the Lorentz factors of the expanding fireball are $\Gamma > 10^2$. Inserting a typical observed γ-ray energy of 1 MeV, we see that characteristic proton energies $\sim 2 \times 10^6$ GeV are required to produce neutrinos from pion decay. Typically, the neutrinos receive $\sim 5\%$ of the proton energy, leading to neutrinos of $\sim 10^{14}$ eV as stated.[3] In the standard picture, these neutrinos result from internal shocks within the fireball.

In the last two years, afterglows of GRBs have been discovered in X-ray, optical, and radio (Costa et al. 1997; van Paradijs et al. 1997; Frail et al. 1997). These observations confirm (Waxman 1997a; Wijers, Rees, & Mészáros 1997) standard model predictions (Paczyński & Rhoads 1993; Katz 1994; Mészáros & Rees 1997; Vietri 1997a) of afterglows that result from the collision of the expanding fireball with the surrounding medium. Inserting in equation (1) a typical afterglow photon energy $\sim 10^2$ eV, we see that characteristic neutrino energies of order 10^9 GeV may be expected.[4] Gamma rays of similar energies are produced by π^0 decay, but because the fireball is optically thick at these energies the γ-rays probably leak out only at much lower energies, ~ 10 GeV.

The ultra–high-energy neutrinos, $\sim 10^{18}$ eV, are produced in the initial stage of the interaction of the fireball with its surrounding gas, which occurs over a time, T, comparable to the duration of the GRB itself. Optical–UV photons are radiated by electrons accelerated in shocks propagating backward into the ejecta. Protons are accelerated to high energy in these "reverse" shocks. The combination of low-energy photons and high-energy protons produces ultra–high-energy neutrinos via photo-meson interactions, as indicated by equation (1).

Afterglows have been detected in several cases; reverse shock emission has only been identified for GRB 990123 (Akerlof et al. 1999). Both the detections and the nondetections are consistent with shocks occurring with typical model parameters (Sari & Piran 1999; Mészáros & Rees 1999), suggesting that reverse shock emission may be common. The predicted neutrino and γ-ray emission depends upon parameters of the surrounding medium that can only be estimated once more observations of the prompt optical afterglow emission are available.

We discuss in § 2 likely plasma conditions in the collisions between the fireball and the surrounding medium, and in § 3 the physics of how the ultra–high-energy neutrinos are produced. We first discuss in § 3.1 ultra–high-energy cosmic-ray (UHECR) production in GRBs in the light of recent afterglow observations. We show that afterglow observations provide further support for the model of UHECR production in GRBs, and address some criticism of the model recently made in the literature (Gallant & Achterberg 1999). Neutrino production is then discussed in § 3.2. The expected neutrino flux and spectrum are derived in § 4, and the implications of our results for future experiments are discussed in § 5.

2. PLASMA CONDITIONS AT THE REVERSE SHOCKS

We concentrate in this section on the epoch between the time the expanding fireball first strikes the surrounding

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[3] The claim that the spectrum of neutrinos produced in interaction with burst (rather than afterglow) photons extends to 10^{19} eV (Vietri 1998) is due to calculational errors.

[4] Afterglow photons are produced over a wide range of energies, from radio to X-rays, leading to a broad neutrino spectrum. As we show in § 4, however, the flux is dominated by neutrinos in the energy range 10^{17}–10^{19} eV (cf. eq. [14]), which are produced in proton interactions with 10 eV–1 keV photons.

medium and the time when the reverse shocks have erased the memory of the initial conditions. After this period, the expansion approaches the Blandford & McKee (1976) self-similar solutions. The purpose of this section is to derive the plasma parameters and the UV luminosity and spectrum expected for typical GRB parameters. We use the results derived here to estimate in the following sections the energy to which protons can be accelerated and to calculate the production of ultra–high-energy neutrinos and multiple GeV photons.

During self-similar expansion, the Lorentz factor of plasma at the shock front is $\Gamma_{\rm BM} = (17E/16\pi n m_p c^2)^{1/2} r^{-3/2}$, where E is the fireball energy and n is the surrounding gas number density. The characteristic time at which radiation emitted by shocked plasma at radius r is observed by a distant observer is $t \approx r/4\Gamma_{\rm BM}^2 c$ (Waxman 1997b).

The transition to self-similar expansion occurs on a timescale T (measured in the observer frame) comparable to the longer of the two timescales set by the initial conditions: the (observer) GRB duration $t_{\rm GRB}$ and the (observer) time T_Γ at which the self-similar Lorentz factor equals the original ejecta Lorentz factor Γ_i, $\Gamma_{\rm BM}(t = T_\Gamma) = \Gamma_i$. Since $t = r/4\Gamma_{\rm BM}^2 c$,

$$T = \max\left[t_{\rm GRB},\ 6\left(\frac{E_{53}}{n_0}\right)^{1/3}\left(\frac{\Gamma_i}{300}\right)^{-8/3}\ {\rm s}\right]. \quad (2)$$

During the transition, plasma shocked by the reverse shocks expands with Lorentz factor close to that given by the self-similar solution, $\Gamma \simeq \Gamma_{\rm BM}(t = T)$, i.e.,

$$\Gamma \simeq 245\left(\frac{E_{53}}{n_0}\right)^{1/8} T_1^{-3/8}\ , \quad (3)$$

while the unshocked fireball ejecta propagate with the original expansion Lorentz factor, $\Gamma_i > \Gamma$. We write equation (3) in terms of dimensionless parameters that are characteristically of order unity in models that successfully describe observed GRB phenomena. Thus $E = 10^{53} E_{53}$ ergs, $T = 10 T_1$ s, $n = 1 n_0$ cm^{-3}, and typically $\Gamma_i \approx 300$. Lorentz factors of the reverse shocks in the frames of the unshocked plasma are mildly relativistic, $\Gamma_R - 1 \sim \Gamma_i/\Gamma_{\rm BM}$. If the initial Lorentz factor is extremely large, $\Gamma_i \gg 300$, the transition Lorentz factor computed from equations (2) and (3) remains unchanged, $\Gamma \approx 250$, while the reverse shocks become highly relativistic, $\Gamma_R \sim \Gamma_i/\Gamma \gg 1$.

The observed photon radiation is produced in the fireball model by synchrotron emission of shock-accelerated electrons. We now summarize the characteristics of the synchrotron spectrum that leads to ultra–high-energy neutrinos and GeV photons.

Let ξ_e and ξ_B be the fractions of the thermal energy density U (in the plasma rest frame) that are carried, respectively, by electrons and magnetic fields. The characteristic electron Lorentz factor (in the plasma rest frame) is $\gamma_m \simeq \xi_e(\Gamma_R - 1) m_p/m_e \simeq \xi_e(\Gamma_i/\Gamma) m_p/m_e$, where the thermal energy per proton in the shocked ejecta is $\simeq (\Gamma_R - 1) m_p c^2$. The energy density U is $E \approx 4\pi r^2 c T \Gamma^2 U$, and the number of radiating electrons is $N_e \approx E/\Gamma_i m_p c^2$. The characteristic (or peak) energy of synchrotron photons (in the observer frame) is

$$\epsilon_{\gamma m}^{\rm ob} \approx \hbar \Gamma \gamma_m^2 \frac{eB}{m_e c} = 0.6 \xi_{e,-1}^2\, \xi_{B,-2}^{1/2}\, n_0^{1/2}\left(\frac{\Gamma_i}{300}\right)^2 {\rm eV}\ , \quad (4)$$

and the specific luminosity, $L_\epsilon = dL/d\epsilon_\gamma^{\rm ob}$, at $\epsilon_{\gamma m}^{\rm ob}$ is

$$L_m \approx (2\pi\hbar)^{-1}\Gamma \frac{e^3 B}{m_e c^2} N_e \approx 6 \times 10^{60} \xi_{B,-2}^{1/2} E_{53}^{5/4} T_1^{-3/4}\left(\frac{\Gamma_i}{300}\right)^{-1} n_0^{1/4}\ {\rm s}^{-1}\ , \quad (5)$$

where $\xi_e = 0.1\xi_{e,-1}$, and $\xi_B = 0.01\xi_{B,-2}$. Hereafter, we denote particle energy in the observer frame with the superscript "ob," and particle energy measured at the plasma frame with no superscript (e.g., $\epsilon_{\gamma m}^{\rm ob} = \Gamma\epsilon_{\gamma m}$).

Since the reverse shocks are typically mildly relativistic, electrons are expected to be accelerated in these shocks to a power-law energy distribution, $dN_e/d\gamma_e \propto \gamma_e^{-p}$ for $\gamma_e > \gamma_m$, with $p \simeq 2$ (Axford, Leer, & Skadron 1977; Bell 1978; Blandford & Ostriker 1978). The specific luminosity extends in this case to energy $\epsilon_\gamma > \epsilon_{\gamma m}$ as $L_\epsilon = L_m(\epsilon_\gamma/\epsilon_{\gamma m})^{-1/2}$, up to photon energy $\epsilon_{\gamma c}$. Here $\epsilon_{\gamma c}$ is the characteristic synchrotron frequency of electrons for which the synchrotron cooling time, $6\pi m_e c/\sigma_{\rm T}\gamma_e B^2$, is comparable to the ejecta (rest-frame) expansion time, $\sim r/\Gamma c$. At energy $\epsilon_\gamma > \epsilon_{\gamma c}$,

$$\epsilon_{\gamma c}^{\rm ob} \approx 0.3 \xi_{B,-2}^{-3/2} n_0^{-1} E_{53}^{-1/2} T_1^{-1/2}\ {\rm keV}\ , \quad (6)$$

the spectrum steepens to $L_\epsilon \propto \epsilon_\gamma^{-1}$.

3. UHECR AND NEUTRINO PRODUCTION

3.1. *UHECR Production*

Protons are expected to be accelerated to high energies in mildly relativistic shocks within an expanding ultrarelativistic GRB wind (Waxman 1995a; Vietri 1995). Energies as high as $\epsilon_p^{\rm ob} = 10^{20}\epsilon_{p,20}^{\rm ob}$ eV may be achieved provided the fraction of thermal energy density carried by magnetic fields, ξ_B, is large enough, and provided shocks occur at large enough radii, so that proton energy loss by synchrotron emission does not affect acceleration (Waxman 1995a, 1999). The condition that needs to be satisfied by ξ_B,

$$\frac{\xi_B}{\xi_e} > 10^{-2}(\epsilon_{p,20}^{\rm ob}\,\Gamma/250)^2 L_{\gamma,52}^{-1}\ , \quad (7)$$

where Γ is the wind expansion Lorentz factor and $L_\gamma = 10^{52} L_{\gamma,52}$ ergs s^{-1} its γ-ray luminosity, is consistent with constraints imposed by afterglow observations. Afterglow observations imply $\xi_e \sim 0.1$ and $\xi_B \sim 0.01$ (e.g., eq. [4]). The observed distribution of GRB redshifts, which suggests that most detected GRBs occur at the redshift range of 1–3 (Krumholtz, Thorsett, & Harrison 1998; Mao & Mo 1998; Hogg & Fruchter 1999), imply that the characteristic GRB γ-ray luminosity is $L_\gamma \approx 10^{52}$ ergs s^{-1}: For characteristic GRB γ-ray flux, $F_\gamma \approx 10^{-6}$ ergs cm^{-2} s^{-1} in the BATSE 20 keV–2 MeV range, and adopting the cosmological parameters $\Omega = 0.2$, $\Lambda = 0$, and $H_0 = 75$ km s^{-1} Mpc^{-1}, the luminosity for a $z = 1.5$ burst is $L_\gamma \approx 10^{52}$ ergs s^{-1}. This result is consistent with the more detailed analysis of Mao & Mo (1998), who obtain a median GRB luminosity in the 50–300 keV range (which accounts for $\sim 1/3$ of the BATSE range luminosity) of $\approx 10^{51}$ ergs s^{-1} for $\Omega = 1$, $\Lambda = 0$, and $H_0 = 100$ km s^{-1} Mpc^{-1}.

The condition that needs to be satisfied to avoid proton synchrotron energy loss (Waxman 1995a),

$$r > r_{\rm syn} = 10^{12}(\Gamma/250)^{-2}(\epsilon_{p,20}^{\rm ob})^3\ {\rm cm}\ , \quad (8)$$

is clearly satisfied in the present context, as reverse shocks arise at $r \approx 4\Gamma^2 cT \approx 10^{17}$ cm $\gg 10^{12}$ cm. Thus, synchro-

tron losses of protons accelerated to high energy at the radii where reverse shocks are expected to arise are negligible.

We note that it has recently been claimed (Gallant & Achterberg 1999) that acceleration of protons to $\sim 10^{20}$ eV in the *highly relativistic external* shock driven by the fireball into its surrounding medium is impossible. Regardless of whether this claim is correct or not, it is not relevant to the model proposed in Waxman (1995a) and discussed here, in which protons are accelerated in the *mildly relativistic internal* (*reverse*) shocks.

Finally, improved constraints from afterglow observations on the energy generation rate of GRBs provide further support to the GRB model of UHECR production. For an open universe, $\Omega = 0.2$, $\Lambda = 0$, and $H_0 = 75$ km s^{-1} Mpc^{-1}, the GRB rate per unit volume required to account for the observed BATSE rate is $R_{\rm GRB} \approx 10^{-8}$ Mpc^{-3} yr^{-1}, assuming a constant comoving GRB rate. Present data do not allow us to distinguish between models in which the GRB rate is evolving with redshift, e.g., following star formation rate, and models in which it is not evolving, since in both cases most detected GRBs occur at the redshift range of 1–3 (Hogg & Fruchter 1999). Thus, $R_{\rm GRB}$ provides a robust estimate of the rate at $z \sim 1$, while the present, $z = 0$, rate may be lower by a factor of ~ 8 if strong redshift evolution is assumed. This implies that the present rate of γ-ray energy generation by GRBs is in the range of 10^{44}–10^{45} ergs Mpc^{-3} yr^{-1}, remarkably similar to the energy generation rate required to account for the observed UHECR flux above 10^{19} eV, $\sim 10^{44}$ ergs Mpc^{-3} yr^{-1} (Waxman 1995b; Waxman & Bahcall 1999).

3.2. *Neutrino Production*

The photon distribution in the wind rest frame is isotropic. Denoting by $n_\gamma(\epsilon_\gamma)d\epsilon_\gamma$ the number density (in the wind rest frame) of photons in the energy range ϵ_γ to $\epsilon_\gamma + d\epsilon_\gamma$, the fractional energy loss rate of a proton with energy ϵ_p due to pion production is

$$t_\pi^{-1}(\epsilon_p) \equiv -\frac{1}{\epsilon_p}\frac{d\epsilon_p}{dt} = \frac{1}{2\gamma_p^2}\, c \int_{\epsilon_0}^{\infty} d\epsilon\, \sigma_\pi(\epsilon)\xi(\epsilon)\epsilon \int_{\epsilon/2\gamma_p}^{\infty} dx\, x^{-2} n(x)\,, \quad (9)$$

where $\gamma_p = \epsilon_p/m_p c^2$, $\sigma_\pi(\epsilon)$ is the cross section for pion production for a photon with energy ϵ in the proton rest frame, $\xi(\epsilon)$ is the average fraction of energy lost to the pion, and $\epsilon_0 = 0.15$ GeV is the threshold energy.

The photon density is related to the observed luminosity by $n(x) = L_\epsilon(\Gamma x)/(4\pi r^2 c\Gamma x)$. For proton Lorentz factor $\epsilon_0/2\epsilon_{\gamma c} \ll \gamma_p < \epsilon_0/2\epsilon_{\gamma m}$, photo-meson production is dominated by interaction with photons in the energy range $\epsilon_{\gamma m} < \epsilon_\gamma \ll \epsilon_{\gamma c}$, where $L_\epsilon \propto \epsilon_\gamma^{-1/2}$. For this photon spectrum, the contribution to the first integral of equation (9) from photons at the Δ-resonance is comparable to that of photons of higher energy, and we obtain

$$t_\pi^{-1}(\epsilon_p) \simeq \frac{2^{5/2}}{2.5}\frac{L_m}{4\pi r^2\Gamma}\left(\frac{\epsilon_{\rm peak}}{\gamma_p\epsilon_{\gamma m}}\right)^{-1/2}\frac{\sigma_{\rm peak}\,\xi_{\rm peak}\,\Delta\epsilon}{\epsilon_{\rm peak}}\,. \quad (10)$$

Here $\sigma_{\rm peak} \simeq 5 \times 10^{-28}$ cm^2 and $\xi_{\rm peak} \simeq 0.2$ at the resonance $\epsilon = \epsilon_{\rm peak} = 0.3$ GeV, and $\Delta\epsilon \simeq 0.2$ GeV is the peak width. The time available for proton energy loss by pion production is comparable to the expansion time as measured in the wind rest frame, $\sim r/\Gamma c$. Thus, the fraction of energy lost by protons to pions is

$$f_\pi(\epsilon_p^{\rm ob}) \approx 0.05\left(\frac{L_m}{6\times 10^{60}\ {\rm s}^{-1}}\right)\left(\frac{\Gamma}{250}\right)^{-5} T_1^{-1} \times (\epsilon_{\gamma m,{\rm eV}}^{\rm ob}\,\epsilon_{p,20}^{\rm ob})^{1/2}\,. \quad (11)$$

equation (11) is valid for protons in the energy range

$$4\times 10^{18}\left(\frac{\Gamma}{250}\right)^2 (\epsilon_{\gamma c,{\rm keV}}^{\rm ob})^{-1}\ {\rm eV} < \epsilon_p^{\rm ob} < 4\times 10^{21}\left(\frac{\Gamma}{250}\right)^2 (\epsilon_{\gamma m,{\rm eV}}^{\rm ob})^{-1}\ {\rm eV}\,. \quad (12)$$

Such protons interact with photons in the energy range $\epsilon_{\gamma m}$ to $\epsilon_{\gamma c}$, where the photon spectrum $L_\epsilon \propto \epsilon_{\gamma c}^{1/2}$ and the number of photons above interaction threshold is $\propto \epsilon_p^{1/2}$. At lower energy, protons interact with photons of energy $\epsilon_\gamma > \epsilon_{\gamma c}$, where $L_\epsilon \propto \epsilon^{-1}$ rather then $L_\epsilon \propto \epsilon^{-1/2}$. At these energies, therefore, $f_\pi \propto \epsilon_p^{\rm ob}$.

Since the flow is ultrarelativistic, the results given above are independent of whether the wind is spherically symmetric or jetlike, provided the jet opening angle is greater than $1/\Gamma$. For a jetlike wind, L_m is the luminosity that would have been produced by the wind if it were spherically symmetric.

4. NEUTRINO SPECTRUM AND FLUX

Approximately half of the energy lost by protons goes into π^0's, and the other half to π^+'s. Neutrinos are produced by the decay of π^+'s, $\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$. The mean pion energy is 20% of the proton energy. This energy is roughly evenly distributed between the π^+ decay products. Thus, approximately half the energy lost by protons of energy ϵ_p is converted to neutrinos with energy $\sim 0.05\epsilon_p$. Equation (12) implies that the spectrum of neutrinos below $\epsilon_{\nu b}^{\rm ob} \approx 10^{17}(\Gamma/250)^2(\epsilon_{\gamma c,{\rm keV}}^{\rm ob})^{-1}$ eV is harder by one power of the energy than the proton spectrum, and by half a power of the energy at higher energy. For a power-law differential spectrum of accelerated protons $n(\epsilon_p) \propto \epsilon_p^{-2}$, which is expected for Fermi acceleration and which could produce the observed spectrum of ultra–high-energy cosmic rays (Waxman 1995b), the differential neutrino spectrum is $n(\epsilon_\nu) \propto \epsilon_\nu^{-\alpha}$ with $\alpha = 1$ below the break and $\alpha = 3/2$ above the break.

The energy production rate required to produce the observed flux of ultra–high-energy cosmic rays, assuming that the sources are cosmologically distributed, is (Waxman 1995b)

$$E_{\rm CR}^2\, d\dot{N}_{\rm CR}/dE_{\rm CR} \approx 10^{44}\ {\rm ergs\ Mpc^{-3}\ yr^{-1}}\,. \quad (13)$$

If GRBs are indeed the sources of ultra–high-energy cosmic rays, then equations (11) and (12) imply that the expected neutrino intensity is

$$\epsilon_\nu^2\,\Phi_\nu \approx 10^{-10}\,\frac{f_\pi^{[19]}}{0.1}\left(\frac{\epsilon_\nu^{\rm ob}}{10^{17}\ {\rm eV}}\right)^{\beta} {\rm GeV\ cm^{-2}\ s^{-1}\ sr^{-1}}\,, \quad (14)$$

where $f_\pi^{[19]} \equiv f_\pi(\epsilon_{p,20}^{\rm ob} = 2)$ and $\beta = 1/2$ for $\epsilon_\nu^{\rm ob} > 10^{17}$ eV and $\beta = 1$ for $\epsilon_\nu^{\rm ob} < 10^{17}$ eV. The fluxes of all neutrinos are similar and given by equation (14), $\Phi_{\nu_\mu} \approx \Phi_{\bar{\nu}_\mu} \approx \Phi_{\nu_e} \approx \Phi_\nu$. Equation (14) is obtained by integrating the neutrino generation rate implied by equations (13) and (11) over cosmological time, under the assumption that the generation rate is independent of cosmic time (Waxman & Bahcall 1999). If

the GRB energy generation rate increases with redshift in a manner similar to the evolution of the QSO luminosity density, which exhibits the fastest known redshift evolution, the expected neutrino flux would be ~ 3 times that given in equation (14) (Waxman & Bahcall 1999).

The neutrino flux is expected to be strongly suppressed at energy $\epsilon_\nu^{ob} > 10^{19}$ eV, since protons are not expected to be accelerated to energy $\epsilon_p^{ob} \gg 10^{20}$ eV. If protons are accelerated to much higher energy, the ν_μ flux may extend to $\sim 10^{21}$ eV. At higher energy the ejecta expansion time ΓT is shorter than the pion decay time, leading to strong suppression of ν_μ flux due to adiabatic energy loss at $\epsilon_\nu^{ob} > 10^{21} T_1 (\Gamma/250)^2$ eV. Adiabatic energy loss of muons will suppress the $\bar{\nu}_\mu$ and ν_e flux at $\epsilon_\nu^{ob} > 10^{19} T_1 (\Gamma/250)^2$ eV.

5. DISCUSSION

When the expanding fireball of a GRB collides with the surrounding medium, reverse shocks are created that give rise to observed afterglow by synchrotron radiation. The specific luminosity and energy spectrum describing these processes are given in equations (4)–(6). For typical values of the plasma parameters and fireball Lorentz factor, these equations are consistent with the afterglow observations. The burst GRB 990123 was especially luminous, and also for this burst the relations equations (4)–(6) are consistent with the observation of a reverse shock and the other afterglow phenomena.

If protons are accelerated in GRBs up to energies $\sim 10^{20}$ eV, then the expected flux of high-energy neutrinos is given by equations (14) and (11). Muon energy loss suppresses the $\bar{\nu}_\mu$ and ν_e flux above $\sim 10^{19}$ eV, while pion energy loss suppresses the ν_μ flux only above $\sim 10^{21}$ eV. Since protons are not expected to be accelerated to $\gg 10^{20}$ eV (Waxman 1995a), the energy beyond which the ν_μ flux is suppressed will likely be determined by the maximum energy of accelerated protons. Measuring the maximum neutrino energy will set a lower limit to the maximum proton energy. The predicted flux is sensitive to the value of the Lorentz factor of the reverse shock (see eq. [11]), but this value is given robustly by equation (3) as $\Gamma \simeq 250$.

Will the ultra–high-energy neutrinos predicted in this paper be detectable? The sensitivities of high-energy neutrino detectors have not been determined for ultra–high-energy neutrinos whose time of occurrence is known to within ~ 10 s and whose direction on the sky is known accurately. Special techniques may enhance the detection of GRB neutrinos (see below).

Planned 1 km^3 detectors of high-energy neutrinos include ICECUBE, ANTARES, NESTOR (Halzen 1999), and NuBE (Roy, Crawford, & Trattner 1999). Neutrinos are detected by observing optical Cerenkov light emitted by neutrino-induced muons. The probability $P_{\nu\mu}$ that a neutrino would produce a high-energy muon with the currently required long path within the detector is $P_{\nu\mu} \approx 3 \times 10^{-3} (\epsilon_\nu/10^{17}\ \mathrm{eV})^{1/2}$ (Gaisser, Halzen, & Stanev 1995; Gandhi et al. 1998). Using equation (14), the expected detection rate of muon neutrinos is ~ 0.06 km^{-2} yr^{-1} (over 2π sr), or ~ 3 times larger if GRBs evolve like quasars. GRB neutrinos may be detectable in these experiments because the knowledge of neutrino direction and arrival time may relax the requirement for a long muon path within the detector.

Air showers could be used to detect ultra–high-energy neutrinos. The neutrino acceptance of the planned Auger detector, $\sim 10^4$ km^3 sr (Capelle et al. 1998), seems too low. The effective area of proposed space detectors (Linsley 1985; Takahashi 1995) may exceed $\sim 10^6$ km^2 at $\epsilon_\nu > 2 \times 10^{19}$ eV, detecting several tens of GRB-correlated events per year, provided that the neutrino flux extends to $\epsilon_\nu > 2 \times 10^{19}$ eV. Since, however, the GRB neutrino flux is not expected to extend well above $\epsilon_\nu \sim 10^{19}$ eV, and since the acceptance of space detectors decreases rapidly below $\sim 10^{19}$ eV, the detection rate of space detectors would depend sensitively on their low-energy threshold.

We note that, as mentioned in the Introduction, the predicted neutrino emission depends on parameters of the surrounding medium. We have assumed throughout the paper that the fireball expands into a uniform medium of density typical in the ISM, $n \sim 1$ cm^{-3}, consistent with GRB 990123 observations. However, some GRBs may result from the collapse of a massive star (e.g., Woosley & MacFadyen 1999), in which case the fireball is expected to expand into a preexisting wind. For typical wind parameters (e.g., Chevalier & Li 2000), the transition to self-similar behavior takes place at a radius where the wind density is $n \approx 10^4$ cm$^{-3} \gg 1$ cm^{-3}. The higher density implies a lower Lorentz factor of the expanding plasma during the transition stage, and hence a larger fraction of proton energy lost to pion production. The analysis of §§ 2 and 3, for the density appropriate for the wind case, implies that protons of energy $\varepsilon_p \geq 10^{18}$ eV lose all their energy to pion production. In this case, a typical GRB at $z \sim 1$ is expected to produce a fluence $\sim 10^{-10}$ cm^{-2} of $\sim 10^{17}$ eV neutrinos, implying a detection probability $\sim 10^{-2.5}$ in a 1 km^3 detector and corresponding to several muon-induced neutrinos per year in a 1 km^3 detector.

As explained in Waxman & Bahcall (1997), ν_τ's are not expected to be produced in the GRB. However, the strong mixing between ν_μ and ν_τ favored by super-Kamiokande observations of atmospheric neutrinos indicates that the flux of ν_μ and ν_τ should be equal at Earth. This conclusion would not hold if the less favored alternative of ν_μ to ν_{sterile} occurs.

The decay of π^0's produced in photo-meson interactions would lead to the production of $\sim 10^{19}$ eV photons. For the photon luminosity and spectrum given in equations (4)–(6), the fireball optical depth for pair production is higher than unity for $\epsilon_\gamma > 10$ GeV. Thus, the ultra–high-energy photons would be degraded and will escape the fireball as multi-GeV photons. Since in order for GRBs to be the sources of ultra–high-energy protons similar energy should be produced in ~ 1 MeV photons and $\sim 10^{20}$ eV protons (Waxman 1995b), the expected multi-GeV integrated luminosity is $\approx 10\%$ of the ~ 1 MeV integrated luminosity, i.e., $\sim 10^{-6}$ ergs cm^{-2}. Such multi-GeV emission has been detected in several GRBs on a timescale of greater than 10 s following the GRB, and may be common (Dingus 1995). This is not, however, conclusive evidence for proton acceleration to ultra–high energy. For the parameters adopted in this paper, inverse Compton scattering of synchrotron photons may also produce the observed multi-GeV photons. More wide-spectrum observations, of optical to greater than 10 GeV photons, are required to determine whether the observed multi-GeV emission on ~ 10 s is due to inverse Compton or π^0 decay.

We note here that multi-GeV photon production by synchrotron emission of $\sim 10^{20}$ eV protons accelerated at the *highly relativistic external* shock driven by the fireball into

its surrounding medium has been discussed in Vietri (1997b) and Bottcehr & Dermer (1998). However, protons are not likely to be accelerated to such energy at the external shock (Gallant & Achterberg 1999), and, moreover, even if acceleration is possible, the fraction of proton energy lost by synchrotron emission at the radii where external shocks occur is $\ll 1$ (see the discussion following eq. [8]), and hence the expected flux is much smaller than the inverse Compton or π^0 decay flux.

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