THE ASTROPHYSICAL JOURNAL, 458:L83–L85, 1996 February 20

OBSERVATIONAL SEARCHES FOR SOLAR *g*-MODES: SOME THEORETICAL CONSIDERATIONS

PAWAN KUMAR[1] AND ELIOT J. QUATAERT
Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139
AND
JOHN N. BAHCALL
Institute for Advanced Study, Princeton, NJ 08540
Received 1995 September 25; accepted 1995 December 12

ABSTRACT

We argue that the solar *g*-modes are unlikely to have caused the discrete peaks in the power spectrum of the solar wind flux observed by Thomson et al (1995). The lower limit to the energy of individual *g*-modes, using the amplitudes given by Thomson et al., is estimated to be at least 10^{36} ergs for low-order *g*-modes; the resulting surface velocity amplitude is at least 50 cm s^{-1}, larger than the observational upper limit (5 cm s^{-1}).

We suggest that the most likely source for the excitation of solar *g*-modes is turbulent stresses in the convection zone. The surface velocity amplitude of low-degree and low-order *g*-modes resulting from this process is estimated to be of order 10^{-2} cm s^{-1}. This amplitude is interestingly close to the detection threshold of the *SOHO* satellite. The long lifetime of *g*-modes ($\sim 10^6$ yr for low-order modes) should be helpful in detecting these small-amplitude pulsations.

Subject headings: Sun: interior — Sun: oscillations

1. INTRODUCTION

Gravity mode oscillations of the Sun are primarily confined to its radiative interior, and their observation would thus provide a wealth of information about the energy-generating region, which is poorly probed by the *p*-modes. In the past 20 years a number of different groups have claimed to detect *g*-modes in the Sun (e.g., Brookes, Isaak, & van der Raay 1976; Brown, Stebbins, & Hill 1978; Delache & Scherrer 1983; Scherrer et al. 1979; Severny, Kotou, & Tsap 1976; for a detailed review of the observations, please see the article by Pallé 1991 and references therein), but thus far there is no consensus that *g*-modes have in fact been observed. Recently, Thomson, MacLennan, & Lanzerotti (1995, hereafter TML) have reported detection of *g*-modes in the flux of high-energy particles in the solar wind. In the absence of a detailed model of how pulsation affects the solar wind flux, it is difficult to determine the velocity amplitude of the pulsation in the photosphere from the amplitude of the solar wind flux variation. We, however, use general physical arguments to make an estimate of the photospheric velocity amplitude of *g*-modes from the wind data (§ 2) that can be compared with the results of optical searches. In § 3 we provide estimates of the surface velocity amplitude, arising from excitation by turbulent convection, of *g*-modes of several different degree (l) and order (n) for comparison with this and future observations. The main results are summarized in § 4.

2. OBSERVATION OF *g*-MODES IN THE SOLAR WIND?

The dispersion relation for solar *g*-modes in the limit of $n \gg l$ is (e.g., Christensen-Dalsgaard & Berthomieu 1991)

$$P_{nl} = \frac{P_0}{2\sqrt{l(l+1)}}(2n + l - \delta),$$

where

$$P_0 = \frac{2\pi^2}{\int_0^{r_1} dr\, N_{\rm B}/r} \approx 2160 \text{ s},$$

$N_{\rm B}$ is the Brunt-Väisälä frequency, r_1 is the upper turning point of the mode, and $\delta \approx 5/6$. Thus the order n of a *g*-mode of frequency 5 μHz (the low-frequency end of the observed peaks attributed to *g*-modes by TML) and degree $l = 1$, 2, and 3 is $\sim$130, $\sim$230, and $\sim$320, respectively. Thomson et al. (1995) find several groups of very closely spaced frequencies in their data, for instance, 106.768, 111.938, and 114.042 μHz (see Table 2 of TML). These can not be modes of the same l, if indeed they are *g*-modes, since the observed frequency spacing is considerably smaller than the frequency spacing expected for modes of fixed l but larger than that due to rotational splitting (high-degree *g*-modes have smaller frequency spacings, but they are unlikely to be observed because their surface velocity amplitudes are very small because of large attenuation in the convection zone). Thus, if *g*-mode amplitudes in the solar wind have a frequency dependence similar to that of Table 2 of TML, then one expects there to be $\sim$2000, $l = 1$ and 2, *g*-modes with frequencies in the observed range, which should have amplitudes of order those detected by TML. (This is a conservative estimate since TML attribute some of their peaks to modes of $l > 2$.) TML, however, detect only $\sim$40 such modes, which is a surprisingly small fraction of the total number of observable modes. Although there may be unstated reasons that other frequencies were not observed, it seems unlikely from an a priori point of view that such a small subset of frequencies should be detected.

The mean mass flux in the solar wind is $\sim 1.3 \times 10^{12}$ g s^{-1} (or 2×10^{-14} $M_\odot$ yr^{-1}), and the mean wind speed is 400 km s^{-1}. Thus, the total kinetic energy flux in the wind is $\sim 10^{27}$ ergs s^{-1}; most of this energy is carried by the thermal particles, while

[1] Alfred P. Sloan Fellow and NSF Young Investigator.

the high-energy particles observed by TML carry several orders of magnitude less energy. TML find the fractional variation in the high-energy particle flux in the solar wind associated with g-mode pulsation to be $\sim$2% for low frequencies ($\sim$10 μHz) and $\sim$0.3% for high frequencies ($\sim$100 μHz). Since the fractional variation in the magnetic field is also $\sim$2% (D. J. Thomson, private communication), the flux of thermal particles in the wind is expected to be modulated by an amplitude similar to that of the high-energy particles. Thus, provided that a significant fraction of the flux variation of the thermal particles is due to a change in the wind speed, the observations of TML imply that the g-modes lose energy to the solar wind at a rate of $\sim 10^{24}$ ergs s^{-1} for low-l modes with $n \gtrsim 10$ and $\sim 10^{22}$ ergs s^{-1} for modes with n of order a few (the mean rate of energy loss is proportional to the square of the variation in the particle flux). An independent estimate of the rate of g-mode energy loss can be made using the observed variation of the magnetic field strength associated with a mode in the solar wind. These measurements are made using near-Earth satellites and are also reported by TML. Magnetic field variations are expected to propagate as Alfvén waves, which carry energy away from the Sun at a rate of $\sim d^2(\delta B)^2 V_A/2$, where d is the distance of the spacecraft from the Sun, $\delta B \sim 3 \times 10^{-6}$ G is the magnetic field variation associated with a mode (D. J. Thomson, private communication), and $V_A \sim 50$ km s^{-1} is the Alfvén speed in the solar wind. Thus, we infer that a g-mode loses $\sim 5 \times 10^{21}$ ergs s^{-1} to Alfvén waves in the wind, which is consistent with the above estimate for low-n modes.

If we assume that the damping of the g-modes due to their modulation of the solar wind is not the dominant damping mechanism for these modes, then the lower limit to the g-mode energy is estimated to be $\sim 10^{36}$ ergs for low-l modes with n of order a few and $\sim 10^{35}$ ergs for low-l modes with $n \gtrsim 10$. These estimates were obtained by making use of the dissipation time (turbulent plus radiative) of g-modes, which is of order 10^6 yr for low-order modes and decreases with n as n^2 (n is the mode order). The resultant surface velocity amplitude is at least 2 cm s^{-1} for low-l modes with $n \gtrsim 10$ and 50 cm s^{-1} for modes with n of order a few. The latter amplitudes are larger than the observational upper limit of 5 cm s^{-1} (Kuhn, Libbrecht, & Dicke 1986 find a limit for $l \geq 2$ of $\sim$15 cm s^{-1} at 100 μHz and $\sim$2 cm s^{-1} at 10 μHz; Garcia, Pallé, & Cortés 1988 find a limit of $\sim$4 cm s^{-1} for low l, including $l = 1$; Fröhlich 1990 also finds a limit of $\sim$4 cm s^{-1}; for a review of the results of various optical searches for g-modes, see Pallé 1991 and references therein).

TML also find p-modes in the solar wind electron flux data. The fractional variation of the electron flux associated with a 3 mHz p-mode is $\sim 10^{-4}$ (D. J. Thomson, private communication). Using the argument of the preceding paragraphs, with a mode lifetime of 10 days, we estimate the lower limit to the p-mode surface velocity amplitude to be $\sim$1 cm s^{-1}; the observed amplitude is $\sim$10 cm s^{-1}.

Particle flux perturbations imposed at the solar surface are dispersed out over a distance of a few tens of solar radii because of the distribution of particle speeds (this is damping due to free-streaming of particles). Thus, any periodic variation in the particle flux detected at a distance of $\sim$1 AU must arise because of in situ acceleration of particles. It is possible that g-modes excite Alfvén waves at the solar surface and that these waves in turn accelerate particles locally where they are observed. However, it is unclear if the observed magnetic field variation can lead to the variation of the high-energy particle flux reported by TML. This is a complex problem that should be looked into carefully.

If we assume that perturbations associated with g-modes are imprinted on the solar wind magnetic field near the Sun, which are carried away at the wind speed (away from the Sun, the Alfvén speed is much less than the wind speed), then the phase shift observed in the variation of the magnetic field (and thus in the particle flux variation), at a distance d, over the course of a mode period, P, is $\sim 2\pi(d/PV_w)(\delta V_w/V_w)$, where V_w is the mean thermal wind speed and δV_w is the random variation of the wind speed over a time period P. Thus, the phase coherence time of a mode, observed in the solar wind, is estimated to be $\sim P(PV_w/d)^2(V_w/\delta V_w)^2$. The phase coherence time of a mode of period $\sim$2.6 hr (frequency 105 μHz), due to random variations in the wind speed of 1% in 2.6 hr (J. Belcher, private communication), should therefore be of order 1 day. The coherence time observed by TML (Fig. 2 of their paper) appears, however, to be $\sim$1 yr.

3. VELOCITY AMPLITUDE OF g-MODES DUE TO TURBULENT EXCITATION

A number of people have investigated the linear stability of solar g-modes (e.g., Dilke & Gough 1972; Rosenbluth & Bahcall 1973; Christensen-Dalsgaard, Dilke, & Gough 1974; Shibahashi, Osaki, & Unno 1975; Boury et al. 1975; Saio 1980). All of these investigations find that g-modes of radial-order (n) greater than 3 are stable. However, there is no general agreement about the stability of low-order modes ($n \leq 3$). The reason for this is that there is a delicate balance between driving and damping for very low order modes, whereas higher order modes are stabilized by a rapid increase in damping with n. Thus, it is almost certain that the high-order g-modes reported by TML cannot be excited because of overstability.

Overstability, however, is not ruled out for very low order g-modes ($n \leq 3$ and $\nu \gtrsim 150$ μHz). If overstable, the g-mode amplitude will increase exponentially with time until nonlinear effects become important and saturate their growth. Kumar & Goodman (1996) have recently investigated three-mode parametric interaction, a very efficient nonlinear process. Using their results, we find that the low-order overstable g-modes in the Sun will attain an energy of at least 10^{37} ergs before they are limited by nonlinearities. The velocity at the solar surface corresponding to this energy is $\sim 10^2$ cm s^{-1}, which is an order of magnitude larger than the observational limit of Pallé (1991). Thus even low-order g-modes of frequency greater than about 150 μHz are unlikely to be overstable.

Stable g-modes can be excited by the same process that is responsible for exciting the p-modes, i.e., interaction with the turbulent convection. We follow the work of Goldreich, Murray, & Kumar (1994) in order to estimate the energy and surface velocity amplitude of low-degree g-modes. The excitation is due to the fluctuating Reynolds stress, and the theory has been calibrated so that it fits the energy input rate in the p-modes over the entire observed frequency range. We also estimate the turbulent and radiative damping of g-modes. These damping timescales are of the same order for modes of low n ($\sim 10^6$ yr), but the radiative damping rate increases with n as n^2, whereas the turbulent damping rate is a weak function of mode order. Thus, radiative damping dominates for n greater than a few. Using the energy input rate and the

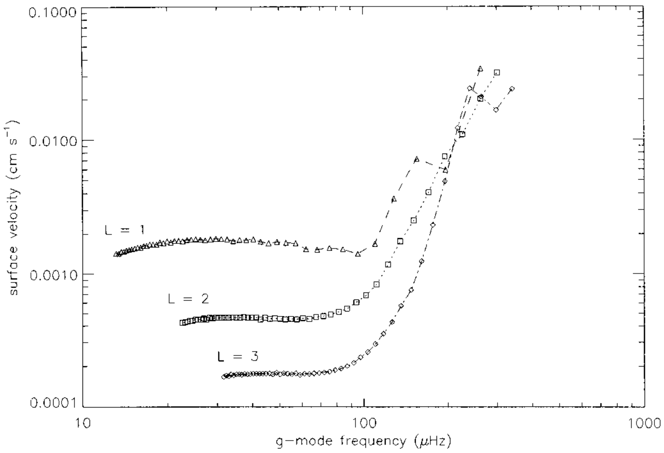

FIG. 1.—Magnitude of the surface velocity amplitude as a function of frequency for low-degree solar *g*-modes excited by coupling with turbulent convection. The surface velocity amplitude falls off strongly with increasing *l* making detection of high-*l* modes unlikely.

damping rates, we calculate the mode energy and the surface velocity amplitude for low-degree *g*-modes.

Figure 1 shows the surface velocity amplitude as a function of frequency for low-degree solar *g*-modes. Note that the surface velocity amplitude is $\sim 10^{-2}$ cm s^{-1} for low-order *g*-modes and $\sim 10^{-3}$ cm s^{-1} for moderate- to high-order modes of $l = 1$, which is a factor of $\sim 10^4$ smaller than the amplitude inferred from the observation of TML. We also find that the surface velocity amplitude falls off rapidly with *l*, which makes detection of high-degree modes ($l \gtrsim 3$) extremely unlikely.

The ratio of the horizontal and radial velocity amplitudes, at the solar surface, can easily be shown to be equal to $[l(l+1)]^{1/2} GM_\odot/(R_\odot^3 \omega^2) \approx 0.13[l(l+1)]^{1/2} P_1^2$, where P_1 is the mode period in units of 1 hr. Thus, it is best to make Doppler measurements near the solar limb to search for low-frequency *g*-modes.

4. CONCLUSION

TML suggest that there are periodic variations in the high-energy particle flux and the magnetic field of the solar wind that they attribute to solar *g*-modes. Using the observed particle flux and magnetic field variations associated with *g*-modes, as reported by TML, we have estimated individual *g*-mode energies and find them to be at least 10^{36} ergs (for comparison, the total energy in all of the solar *p*-modes is $\sim 10^{33}$ ergs). The corresponding surface velocity amplitude for a low-order *g*-mode is at least 50 cm s^{-1}, which is larger than the observational upper limit (Pallé 1991 and references therein) by about a factor of 10.

High-order solar *g*-modes ($n > 3$, or $\nu \lesssim 150$ μHz) are found to be stable by all of the published linear stability calculations. These *g*-modes can, however, be excited by turbulent stresses in the convection zone. The resulting velocity amplitude at the solar surface is estimated to be $\sim 10^{-2}$ cm s^{-1} for low-order, low-degree modes (our calculations of *p*-mode surface velocity amplitudes due to turbulent excitation fit the observed velocity spectrum well, and thus the error in the predicted *g*-mode velocity amplitudes is unlikely to be greater than an order of magnitude). This is about four orders of magnitude smaller than the amplitude implied by the observations of TML. The ground-based network of telescopes designed to observe the solar oscillations continuously (GONG) may be able to detect *g*-mode pulsation down to ~ 1 cm s^{-1}, and *SOHO* is expected to have a detection threshold of $\sim 10^{-1}$ cm s^{-1} (Gabriel et al. 1995). Thus, *g*-modes, if stochastically excited, may be at the threshold of detectability of the forthcoming oscillation instruments. The expected long lifetime of *g*-modes ($\sim 10^6$ yr for low-order *g*-modes and $\sim 10^3$ yr for high-order modes) should help in detecting these small-amplitude pulsations.

We thank John Belcher, Tom Duvall, Peter Goldreich, Peter Sturrock, David Thomson, Louis Lanzerotti, and the referee for useful comments.

REFERENCES

Boury, A., Gabriel, M., Noels, A., Scuflaire, R., & Ledoux, P. 1975, A&A, 41, 279
Brookes, J. R., Isaak, G. R., & van der Raay, H. B. 1976, Nature, 259, 92
Brown, T. M., Stebbins, R. T., & Hill, H. A. 1978, ApJ, 223, 324
Christensen-Dalsgaard, J., & Berthomieu G. 1991, in Solar Interior and Atmosphere, ed. A. N. Cox, W. C. Livingston, & M. S. Matthews (Tucson: Univ. of Arizona Press), 401
Christensen-Dalsgaard, J., Dilke, J. F. W. W., & Gough, D. O. 1974, MNRAS, 169, 429
Delache, P., & Scherrer, P. H. 1983, Nature, 306, 651
Dilke, J. F. W. W., & Gough, D. O. 1972, Nature, 240, 262
Fröhlich, C. 1990, in Lecture Notes in Physics, 367, Oji Seminar on Progress of Seismology of the Sun and Stars, ed. H. Shibahashi & Y. Osaki (New York: Springer), 221
Gabriel, A., et al. 1995, Sol. Phys., in press
Garcia, C., Pallé, P. L., & Cortés, T. Toca. 1988, in Seismology of the Sun and Sun-like Stars (ESA SP-286), ed. E. J. Rolfe (Paris: ESA), 353
Goldreich, P., Murray, N., & Kumar, P. 1994, ApJ, 424, 466
Kuhn, J. R., Libbrecht, K. G., & Dicke, R. H. 1986, Nature, 319, 128
Kumar, P., & Goodman, J. 1996, ApJ, in press
Pallé, P. L. 1991, Adv. Space Res., 11(4), 29
Rosenbluth, M., & Bahcall, J. N. 1973, ApJ, 184, 9
Saio, H. 1980, ApJ, 240, 685
Scherrer, P. H., Wilcox, J. M., Kotov, V. A., Severny, A. B., & Tsap, T. T. 1979, Nature, 277, 635
Severny, A. B., Kotov, V. A., & Tsap, T. T. 1976, Nature, 259, 87
Shibahashi, H., Osaki, Y., & Unno, W. 1975, PASJ, 27, 401
Thomson, D. J., Maclennan, C. G., & Lanzerotti, L. J. 1995, Nature, 376, 139 (TML)