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ON THE POSSIBILITY OF DETECTING REDSHIFTED 21-CM ABSORPTION LINES IN THE SPECTRA OF QUASI-STELLAR SOURCES*

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ABSTRACT

The expected strengths of redshifted 21-cm absorption lines are estimated by using the properties of the observed optical absorption lines in quasi-stellar sources. It is shown that the absence of O I and N I absorption lines in the observed optical absorption spectra of quasi-stellar radio sources indicates that 21-cm absorption lines wider than 100 kHz are likely to be weak unless the heavy-element abundance in the absorbing material is low. Lines narrower than 100 kHz may be strong; their optical counterparts would have escaped detection. A general expression is given for the spin temperature of neutral hydrogen when (following Field) Ly α excitation and de-excitation, 21-cm absorption and emission, and particle collisions are all included. The results are expressed simply in terms of the strength and distance of the radio source and the separation between absorber and emitter. It is shown that the spin temperature of neutral hydrogen is large near a quasi-stellar radio source and is determined either by the ambient 21-cm flux or the ambient Ly α flux for separations between absorber and emitter of less than about 10^5 lt-yr. The expected absorption (and emission) strengths of redshifted 21-cm lines are compared with observational capabilities at radio observatories. Some interesting candidates for study are listed.

I. INTRODUCTION

The detection of redshifted 21-cm absorption lines in the radio spectra of quasi-stellar sources would make possible a number of interesting and important studies. Among the more exciting possibilities are: (1) measurements of the velocity profiles and strengths of the absorption lines of neutral hydrogen which could furnish valuable clues to the physical location of the absorbing material; (2) a direct check on the correctness of the identified (Bahcall 1968; Bahcall, Greenstein, and Sargent 1968) absorption redshifts in PKS 0237 − 23 ($z_{\text{emission}} = 2.22$ [Arp, Bolton, and Kinman 1967]) that are too small ($z_{\text{absorption}} = 1.364, 1.513$) for Ly α to be detected optically; (3) a possible indication of the ratio of heavy elements to hydrogen in the absorbing material (cf. especially eq. [4]); and (4) in combination with the optical data (cf. Savedoff 1956; Bahcall and Schmidt 1967), a measurement of the dimensionless ratio (electron mass/proton mass) in distant objects. We stress that the detection of redshifted 21-cm absorption lines would provide strong evidence that they and their associated optical absorption lines originate far from the quasi-stellar source, since near the quasi-stellar source the spin temperature of neutral hydrogen is very large (cf. § III).

We assume in what follows that matter with the same average properties as the material which produces the optical absorption lines also lies along the line of sight to at least part of the region in which the radio continuum originates. Otherwise, there is no reason

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to expect 21-cm absorption lines at the same redshifts as the observed optical absorption lines. It would not be practical, of course, to search for 21-cm absorption without at least an approximate a priori value for the redshift.

We show in § II that the absence (until the present time) of O I and N I absorption lines in the observed optical absorption spectra of quasi-stellar sources indicates that 21-cm absorption lines wider than 100 kHz are likely to be weak unless the heavy-element abundance in the absorbing material is low. It is also pointed out that 21-cm lines narrower than 100 kHz, possibly due to H I regions, might exist; their optical counterparts could have escaped detection because of their narrowness. In § III we derive a general expression for the spin temperature when (following Field [1958]) Ly α excitation and de-excitation, 21-cm absorption and emission, and particle collisions are all included. Our results are expressed simply in terms of the strength and distance of the radio source and the separation between absorber and emitter. Some typical examples are given. We conclude that for separations less than about 10^5 lt-yr from a quasi-stellar radio source the spin temperature of the neutral hydrogen is large and is determined either by the ambient 21-cm flux or the ambient Ly α flux. We compare, in § IV, the expected strengths of 21-cm absorption lines with the observational capabilities currently available at radio observatories. We discuss in § V the possibility that the optically observed absorbing regions might be detectable in emission at the redshifted wavelengths of 21 cm. In § VI and Table 1 we list some interesting candidates for study and give their relevant characteristics.

Some of our considerations are especially relevant if one is planning experiments to attempt to detect redshifted 21-cm absorption lines. We note that lines wider than $\sim$100 kHz are not expected to be strong. The likelihood of observing 21-cm absorption lines is highest for a strong radio source in which lines from not very highly ionized atoms (e.g., Mg II or Si II) have been observed and in which the absorption redshift is significantly different from the emission redshift. This last criterion follows from the fact that, if the absorption occurs far from the quasi-stellar source, the optical and radio waves reaching us traverse essentially the same path. Moreover, near a quasi-stellar source the spin temperature is very high because of the large ambient Ly α and 21-cm fluxes. We assume that optical absorption redshifts that are very different from their corresponding emission line redshifts have the highest a priori probability of originating far from a quasi-stellar radio source.

II. ESTIMATES OF 21-CM OPTICAL DEPTHS

a) Upper Limits from Observed Optical Spectra

The optical depth, τ_{21}, of redshifted 21-cm radiation can be simply related to the optical depth, $\tau_{\rm ion}$, of a photographically observed absorption line at the same redshift. The relation is

$$\frac{\tau_{21}}{\tau_{\rm ion}} = \epsilon \left(\frac{h\nu_{21}}{4kT_S}\right)\left(\frac{f_{21}}{f_{\rm ion}}\right)\left(\frac{\lambda_{21}}{\lambda_{\rm ion}}\right)\left(\frac{\Delta_{\rm ion}}{\Delta_{\rm H\,I}}\right)\left(\frac{N_{\rm H\,I}}{N_{\rm ion}}\right). \tag{1}$$

In writing equation (1) we have assumed that a fraction ϵ of the radio emitting region is obscured by material with the same average properties as the material which produces the optical absorption lines. If the absorption originates far from the quasi-stellar source, then $\epsilon = 1$. The quantities f_{21} and λ_{21} are the absorption f-value and rest wavelength, respectively, of the 21-cm line; $f_{\rm ion}$ and $\lambda_{\rm ion}$ are the corresponding quantities for an ion with an optically observable absorption line. The Δ's represent the optical line widths. Also, T_S is the spin temperature (cf. Purcell and Field 1956); $N_{\rm H\,I}$ and $N_{\rm ion}$, the neutral hydrogen and ionic number densities, respectively; and ν_{21} is 1420 MHz. Equation (1) can be rewritten in a convenient and suggestive form as follows:

$$\tau_{21} = 2 \times 10^{-8} T_4^{-1} \left[\epsilon \left(\frac{10^3 \text{ Å}}{\lambda_{\text{ion}}} \right) \left(\frac{0.1}{f_{\text{ion}}} \right) \left(\frac{\Delta_{\text{ion}}}{\Delta_{\text{H I}}} \right) \left(\frac{10^{-3} N_{\text{H I}}}{N_{\text{ion}}} \right) \left(\frac{\tau_{\text{ion}}}{0.1} \right) \right] , \tag{2}$$

where T_4 is the spin temperature in units of 10^4 ° K.

It is not possible to use the Ly α line itself to estimate τ_{21}, since the observed Ly α absorption lines are photographically dark in, for example, PKS 1116 + 12 (Bahcall, Peterson, and Schmidt 1966), 3C 191 (Bahcall, Sargent, and Schmidt 1967), and PKS 0237 − 23 (Bahcall *et al.* 1968). Thus, all that can be concluded from such observations is that $\tau_{\text{Ly }\alpha} > 1$; equation (2) shows that $\tau_{\text{Ly }\alpha}$ must be huge in order for τ_{21} to be appreciable.

One can obtain a stringent upper limit on τ_{21} for hydrogen lines of sufficient width, $\Delta\nu/\nu_{21} \gtrsim 10^{-3}$, that the associated ultraviolet absorption lines of other neutral atoms would have been detected in the redshifted optical spectra. A careful check (Bahcall 1968) of the acceptable redshifts in PKS 0237 − 23 revealed no evidence for either O I λ1303.5 or N I λλ1134.6, 1199.9. No evidence was found for either O I or N I absorption in the previously mentioned investigations of PKS 1116 + 12 or 3C 191. (They also were not identified in any of the other investigations referenced in Table 1).[1] The ionization potentials of O I and N I are, respectively, 13.61 and 14.53 eV, compared with 13.60 eV for H I. Hence, $(N_{\text{H I}}/N_{\text{O I}})$ is approximately equal to the abundance ratio of hydrogen to oxygen in the absorbing material, and $(N_{\text{H I}}/N_{\text{N I}})$ is less than or of the order of the abundance ratio of hydrogen to nitrogen. Assuming $\tau_{\text{ion}} \lesssim 0.2$ and inserting the relevant f-values for the oxygen and nitrogen (Wiese, Smith, and Glennon 1966) in equation (2), we find

$$\tau_{21} < \frac{10^{-7}}{T_4} \epsilon \left(\frac{\text{galactic heavy-element abundance}}{\text{heavy-element abundance in absorbing material}} \right) . \tag{3}$$

In deriving the numerical coefficient that appears in equation (3) we assumed (cf. Osterbrock 1963) a galactic abundance ratio of oxygen to hydrogen of 8×10^{-4} and of nitrogen to hydrogen of 2×10^{-4}. We also assumed that $\Delta_{\text{ion}} \sim \Delta_{\text{H I}}$ (for justification, cf. Bahcall *et al.* 1966; Bahcall *et al.* 1968).

The upper limit in equation (3) refers to lines whose total widths are greater than $\Delta\nu = 1.4\ (1 + z_{\text{abs}})^{-1}$ MHz, a value corresponding to a width for the optical lines greater than or of the order of the photographic resolution, ~3 Å. Here z_{abs} is the absorption redshift. The limit can be generalized somewhat by considering equivalent widths instead of optical depths. An equivalent width greater than 1 Å for the absorption lines O I λ1303.5 and N I λλ1134.6, 1199.9 would have been detected optically. Using this fact in equation (2), we conclude that

$$\Delta\nu\tau_{21} \lesssim \frac{6 \times 10^{-7} \text{ MHz}}{(1 + z_{\text{abs}})^2 T_4} \epsilon \left(\frac{\text{galactic heavy-element abundance}}{\text{heavy-element abundance in absorbing material}} \right) , \tag{4}$$

where $\Delta\nu$ is in MHz (the same unit will be used throughout the subsequent discussion). The limit given in equation (4) can be applied only for lines whose optical counterparts could have been detected. An optical absorption line with an infinite τ would have escaped detection if its total width was less than or of the order of 10 per cent the optical resolution. Thus equation (4) is valid only for lines satisfying

$$\Delta\nu > 0.2(1 + z_{\text{abs}})^{-1} \text{ MHz} . \tag{5}$$

[1] Burbidge, Lynds, and Stockton (1968) tentatively identified a strong line observed in PKS 0237 − 23 at λ = 3354.0 with N I λ1134.6 at a redshift of $z_{\text{abs}} = 1.956$. They also identified absorption lines from various metastable states of N I, again at $z = 1.956$. However, this absorption redshift did not pass the tests for statistical and astrophysical acceptability devised by Bahcall (1968) and Bahcall *et al.* (1968). Moreover, N I λ1199.9 has an absorption f-value more than $2\frac{1}{2}$ times as large as N I λ1134.6, and no line is present, in the data of Bahcall *et al.* (1968), at the appropriate redshifted wavelength of λ = 1199.9.

Grewing and Schmidt-Kaler (1968) have made a curve-of-growth analysis of the data of Bahcall *et al.* (1967) for 3C 191. Their results, which are based on a number of assumptions, yield a number of neutral hydrogen atoms $\sim 5 \times 10^{20}$ cm^{-2}, or

$$(\Delta\nu\tau_{21})_{3C\ 191} \sim 4 \times 10^{-5} \epsilon T_4^{-1} \text{ MHz} . \tag{6}$$

It should be emphasized that the number given in equation (6) is uncertain because of ambiguities in the curve-of-growth analysis and difficulties in determining accurately the relevant equivalent widths and f-values.

b) Possible H I *Regions*

No information is available for lines whose radio widths are much less than the limit given in equation (5), since for such lines the optical line widths are much less than the usually available resolutions ($\sim$3 Å for a plate taken at a dispersion of 200 Å mm^{-1}). A completely dark line with a width of less than 0.2 Å would not have been detected optically. It is possible that such narrow lines exist, since in PKS 0237 − 23 many of the absorption lines are not resolved (Bahcall *et al.* 1968). Such a narrow line could be contributed by an H I region on, for example, the outer perimeter of the absorbing material. The thermal width of the absorption lines associated with a hypothetical H I region would be only 0.01 $(1 + z_{\text{abs}})\, T_2^{1/2}$ Å, and the radio widths would be of the order of

$$\Delta\nu_{\text{thermal}} = 0.014(1 + z_{\text{abs}})^{-1} T_2^{1/2} \text{ MHz} . \tag{7}$$

Of course, there may be other mechanisms, such as gross mass motion, which broaden the line. Nevertheless the unit T_2, the temperature in 10^2 ° K, is convenient for characterizing the properties of H I regions. The associated equivalent width of the radio line is

$$\Delta\nu\tau_{21} = \frac{8 \times 10^{-5}}{(1 + z_{\text{abs}})T_2} \epsilon \left(\frac{N_{\text{H I}}}{1 \text{ cm}^3} \frac{d}{1 \text{ pc}} \right) \text{ MHz} , \tag{8}$$

where d is the thickness of the H I region.

III. SPIN TEMPERATURE

a) Introductory Remarks

The spin temperature of neutral hydrogen is defined by the relation (cf. Purcell and Field 1956)

$$\frac{N_2}{N_1} = 3 \exp\left(- h\nu_{21}/kT_S\right) , \tag{9}$$

where N_2 and N_1 are, respectively, number densities in the excited and ground hyperfine states. The principal mechanisms that determine the spin temperature are (the quantity that primarily influences the spin temperature is given in parenthesis); (i) particle collisions (kinetic energies of hydrogen atoms and electrons); (ii) direct excitation and de-excitation by absorption and emission of 21-cm photons (ambient 21-cm flux); and (iii) indirect excitation and de-excitation via absorption of ultraviolet photons, leading to states that ultimately decay to the ground-state hyperfine doublet (ambient Ly α flux). The presence of large photon fluxes near quasi-stellar sources suggests that mechanisms (ii) and (iii) may be unusually important for these objects.

Purcell and Field (1956) included mechanisms (i) and (ii) in their discussion; Wouthuysen (1952) suggested the importance of mechanism (iii) but apparently never published a detailed calculation of its relative importance. All three mechanisms were discussed in a lucid fashion by Field (1958), who estimated for the first time the relative importance of Ly α excitation when particle collisions and direct photon excitation can also occur.

b) Basic Results

In what follows, we present the results of an analysis similar to that of Field's (1958); however, we do not make ab initio assumptions about the form of the photon spectrum or the distribution of Ly α photons in the absorbing region. We can then make clear where and what are the necessary approximations and, more importantly, can relate directly the spin temperature to the physical characteristics of the absorber and the quasi-stellar source.

It will be convenient to have a simplified notation to label the four states of hydrogen that are of principal importance. These states are the ground hyperfine doublet, ${}_0S_{1/2}$ and ${}_1S_{1/2}$ (here we use the conventional representation of ${}_FL_J$), which are separated by 1420 MHz, and the excited levels, ${}_1P_{1/2}$ and ${}_1P_{3/2}$, which lie about 10.2 eV above the ground states. We denote these states by 1, 2, 3, and 4 in order of increasing energy (the order listed above; cf. Fig. 3 of Field 1958). It is also convenient to introduce the average number, n_ν, of photons per available state in photon phase space. In terms of the more familiar specific intensity averaged over all directions, $\langle I_\nu \rangle$, we have

$$n_{ji} = \frac{\lambda_{ij}^3 \langle I_{\nu(i,j)} \rangle}{2hc} , \tag{10}$$

where λ_{ij}, ν_{ij} are the wavelength and frequency of photons connecting states i and j (note $n_{ij} \equiv n_{ji}$). One can show that, in the presence of the three mechanisms listed earlier, the spin temperature T_S can be written in the form

$$T_S = \frac{T_R + y_c T_K + y_{\mathrm{Ly}\,\alpha} T_{\mathrm{Ly}\,\alpha}}{1 + y_c + y_{\mathrm{Ly}\,\alpha}} . \tag{11}$$

Here T_R is the spin temperature due to direct excitation and de-excitation by 21-cm photons,

$$T_R \equiv \frac{h\nu_{21}}{k} (1 + n_{21}) . \tag{12a}$$

For all cases where T_R is important, one can write

$$T_R \cong 6.8 \times 10^{-2} n_{21} \,^\circ \mathrm{K} . \tag{12b}$$

The quantity T_K is the kinetic temperature of the hydrogen atoms and electrons. The appropriate equation for $T_{\mathrm{Ly}\,\alpha}$ is

$$T_{\mathrm{Ly}\,\alpha} \equiv \frac{h\nu_{21}}{k} \frac{(n_{32} + n_{42})}{[n_{32} - n_{31} + n_{42} - n_{41}]} . \tag{13}$$

The quantity y_c has been calculated by Field and is

$$y_c = \frac{h\nu_{21} q_{21}}{k T_K A_{21}} \tag{14a}$$

$$\sim [4 \times 10^1 n_{\mathrm{H\,I}} T_{K,2}^{-0.8} + 10^3 n_e T_{K,2}^{-1/2} (1 + T_{K,2})^{-1/2}] , \tag{14b}$$

where q_{21} is the rate of collisional de-excitation, $T_{K,2}$ is the kinetic temperature in units of $10^2 \,^\circ$ K, and $n_{\mathrm{H\,I}}$ and n_e are, respectively, the number densities of the hydrogen atoms and electrons. We find for $y_{\mathrm{Ly}\,\alpha}$ the simple relation,

$$y_{\mathrm{Ly}\,\alpha} = [n_{32} - n_{31} + n_{42} - n_{41}] \frac{A_{32}}{3A_{21}} . \tag{15}$$

Wouthuysen (1952) and Field (1958) assumed that the shape of the spectrum in the vicinity of Ly α is determined by the kinetic temperature of the absorbing atoms. If this is true, then $n_{ij} \propto [\exp(h\nu_{ij}/kT) - 1]^{-1}$, and

$$[(n_{32} - n_{31} + n_{42} - n_{41})/(n_{32} + n_{42})] \simeq h\nu_{21}/kT \tag{16a}$$

provided that

$$\exp(-h\nu_{31}/kT) \ll 1 . \tag{16b}$$

Inserting equation (16a) in equation (13), one finds (Wouthuysen 1952; Field 1958)

$$T_{\mathrm{Ly}\,\alpha} = T_K . \tag{17}$$

It is not clear that the shape of the spectrum near Ly α will, in all physical situations, be determined by the kinetic temperature of the absorbing atoms (for a detailed discussion of some aspects of this question, see Field 1959; he does not consider the effect of turbulence in the gas or the dependence of profile on position). If we assume that the actual spectrum has a shape $\langle I_\nu \rangle \propto \nu^{-\beta}$, we find from the defining relations (10) and (13) that

$$T_{\mathrm{Ly}\,\alpha} = \frac{h\nu_{\mathrm{Ly}\,\alpha}}{(3+\beta)k} , \tag{18a}$$

or

$$T_{\mathrm{Ly}\,\alpha} \simeq 4(1+\beta/3)^{-1} \times 10^4\ ^\circ\ \mathrm{K} . \tag{18b}$$

For typical quasi-stellar spectra ($0.2 \lesssim \beta \lesssim 2$), $T_{\mathrm{Ly}\,\alpha}$ as calculated from equations (18) is about $3 \times 10^4\ ^\circ$ K. In general, $T_{\mathrm{Ly}\,\alpha}$ can be expected to lie somewhere between or at the values given in equations (17) and (18), with equation (17) giving the single most likely value.

c) Relation of Spin Temperature to Luminosities and Distances

We shall now present some simple relationships from which the radiation temperature, T_R, and the parameter $y_{\mathrm{Ly}\,\alpha}$ can be conveniently estimated. Let D be the luminosity distance (cf. Bondi 1961) of a radio source that has an *observed* flux density S_ν (expressed in flux units $= 10^{-26}$ W m^{-2} Hz^{-1}) and redshift z_{em}. Also, let the absorber with redshift z_{abs} be located at a distance r from the radio emitter. Then (S_ν evaluated at 1420 $(1+z_{\mathrm{abs}})^{-1}$ MHz):

$$n_{21\ \mathrm{cm}} = 1.8 \times 10^{-5}(1+z_{\mathrm{abs}})(D/r)^2 \left(\frac{S_\nu}{1\ \text{flux unit}}\right) . \tag{19}$$

In deriving the above relation, we have assumed that $r \gg$ typical dimension of the source. Combining equations (12) and (19), we obtain a general expression for the radiation temperature at the absorber:

$$T_R = 1.2 \times 10^{-6}\ ^\circ\ \mathrm{K}\ (1+z_{\mathrm{abs}})(D/r)^2 \left(\frac{S_\nu}{1\ \text{flux unit}}\right) . \tag{20}$$

If the intrinsic luminosity, L_ν, of the source is known, then

$$T_R = 1.0 \times 10^{15}\ ^\circ\ \mathrm{K} \left(\frac{1+z_{\mathrm{abs}}}{1+z_{\mathrm{em}}}\right) (L_\nu/10^{35}\ \text{ergs sec}^{-1}\ \text{c/s}^{-1}) \left(\frac{1\ \text{lt-yr}}{r}\right)^2 . \tag{21}$$

In equation (21), L_ν should be evaluated at $\lambda = 21$ cm $(1+z_{\mathrm{abs}})/(1+z_{\mathrm{em}})$.

We consider a typical example for a quasi-stellar radio source. Suppose that $z_{\mathrm{em}} \simeq z_{\mathrm{abs}} \cong 2$ and that $S_\nu \sim 1$ flux unit. Then for $q_0 = +1$, $\Lambda = 0$, $D \equiv cz/(1+z)H_0$,

$(D/r)^2 \simeq 4 \times 10^{19}\, r_{\text{lt-yr}}^{-2}$, and $n_{21\ \text{cm}} \simeq 2 \times 10^{15}\, r_{\text{lt-yr}}^{-2}$. Inserting the required parameters in equation (20), we find

$$T_R \simeq 1.5 \times 10^{14} r^{-2}{}_{\text{lt-yr}}\ {}^\circ\,\text{K}\ . \tag{22}$$

Thus the ambient 21-cm flux produces a high spin temperature for separations r, between emitter and absorber of less than or of the order of 10^5 lt-yrs. The above example could also be considered appropriate for some of the stronger radio galaxies.

To complete our discussion of spin temperatures we must evaluate $y_{\text{Ly}\ \alpha}$. We first indicate how our results for $y_{\text{Ly}\ \alpha}$ compare with those of Field (1958), and then we give a formulation that is expressed more directly in terms of the properties of the radio source and absorber. Combining equations (15) and (16a), we obtain

$$y_{\text{Ly}\ \alpha} = \left(\frac{h\nu_{21}}{kT_K}\right)(n_{32} + n_{42})(A_{32}/3A_{21})\ . \tag{23}$$

Inserting the relation $n_{ij} = (\lambda_{ij}{}^3/4\pi h)\,(h\nu_{ij}/2\nu_{\text{D}})\,N_\alpha$ in equation (23), one obtains Field's (1958) equation (26). The quantity ν_{D} is the Doppler width, N_α is the (unspecified) number of photons per unit volume within $\pm\nu_{\text{D}}$ of the center of Ly α, and $A_{\text{Ly}\ \alpha} = 4.70 \times 10^8\ \text{sec}^{-1}$.

In order to obtain a more useful expression for $y_{\text{Ly}\ \alpha}$, we must estimate n_{ij} for Ly α photons. We find

$$n_\nu = 1.4 \times 10^{-7}{}_\delta \left(\frac{1 + z_{\text{abs}}}{1 + z_{\text{em}}}\right)\left(\frac{F_\nu}{5 \times 10^{30}\ \text{ergs sec}^{-1}\ \text{c/s}^{-1}}\right)\left(\frac{1}{r^2{}_{\text{lt-yrs}}}\right)\ . \tag{24}$$

The quantity $F_\nu d\nu$ is the absolute luminosity at the emitting quasi-stellar source and must be evaluated at $\nu = (1 + z_{\text{em}})\,\nu_{\text{Ly}\ \alpha}/(1 + z_{\text{abs}})$. The value $F_\nu \sim 5 \times 10^{30}$ ergs sec^{-1} c/s^{-1} seems to be typical (Oke 1966; Wampler 1968) for quasi-stellar sources near $\nu = \nu_{\text{Ly}\ \alpha}$. The quantity δ is a geometric factor that occurs because not all of the surface area of the absorber is illuminated directly by the quasi-stellar source. One can show, by using the diffusion approximation for resonant photons (Bahcall 1966), that δ is essentially the ratio of directly illuminated area to total area of the absorber. The proof is simple once one recognizes that the *volume average* of n_ν should be used in equation (24). For a spherical absorber, $\delta = 0.25$. Values of this order are typical for most other geometries.

Combining the defining equations (13) and (15) with equation (24), we find

$$\nu_{\text{Ly}\ \alpha} T_{\text{Ly}\ \alpha} = \frac{3 \times 10^{14}\ {}^\circ\,\text{K}}{r^2{}_{\text{lt-yrs}}}\left[\left(\frac{\delta}{0.25}\right)\left(\frac{F_\nu}{5 \times 10^{30}\ \text{ergs sec}^{-1}\ \text{c/s}^{-1}}\right)\left(\frac{1 + z_{\text{abs}}}{1 + z_{\text{em}}}\right)\right]\ . \tag{25}$$

It should be emphasized that *equation (25) is independent of the uncertainties regarding the actual profile of the photon spectrum within the absorber.* Hence $y_{\text{Ly}\ \alpha}\, T_{\text{Ly}\ \alpha}$ (as given by eq. [25]) may be compared directly with $y_c T_K$ (eqs. [14]) and T_R (eq. [21] or [22]) to see which mechanism actually determines the spin temperature in a given physical situation; the general expression is given in equation (11). If Ly α emission and absorption determines the spin temperature, then one must still decide if equation (17) or equation (18) more accurately represents $T_{\text{Ly}\ \alpha}$.

Note that near a quasi-stellar source ($r \lesssim 10^5$ lt-yrs) the spin temperature is determined by either the ambient 21-cm flux (eq. [21]) or the ambient Ly α flux (eq. [25]).

IV. COMPARISON WITH OBSERVATIONAL CAPABILITIES

The minimum equivalent width that can be detected in a time t with a radiometer that has a system temperature T, an effective area A, and a receiver band width $\Delta\nu_{\text{rec}}$ is

$$(\tau\Delta\nu)_{\min} \approx \frac{2kT}{SA}\left(\frac{\Delta\nu_{\rm rec}}{t}\right)^{1/2} . \tag{26}$$

The optimum sensitivity is achieved when $\Delta\nu_{\rm rec} \approx \Delta\nu$, in which case

$$(\tau\Delta\nu^{1/2})_{\rm optimum} \approx \frac{2kT}{SAt^{1/2}} . \tag{27a}$$

Taking $A = \pi(60)^2$ m^2 and $T = 10^2$ ° K as representing the best currently available radiometers, we have

$$(\tau\Delta\nu^{1/2})_{\rm optimum} \approx 2 \times 10^{-3}\left(\frac{10 \text{ flux units}}{S}\right)\left(\frac{1 \text{ day}}{t}\right)^{1/2} \text{MHz}^{1/2} . \tag{27b}$$

The detection of relatively wide absorption lines, $\Delta\nu > 0.4\ (1 + z_{\rm abs})^{-1}$ MHz, would appear to be very difficult. On the basis of the theoretical expectations (eqs. [3], [22], and [25]) and the present observational capabilities (eq. [27b]) the detection of broad 21-cm absorption lines is unlikely unless the lines originate far ($r > 10^{+5}$ lt-yrs) from the quasi-stellar source *and* oxygen and nitrogen are underabundant in the absorbing material.

An H I region producing narrow lines might be detectable with a narrow-band receiver. Comparing equations (7) and (8) with equation (27b), we find

$$\frac{\tau_{\rm H\,I\ region}}{\tau_{\rm optimum}} \approx \frac{0.3\epsilon}{(1 + z_{\rm abs})^{1/2}T_2^{5/4}}\left(\frac{N_{\rm H\,I}\,d}{3 \times 10^{18}\ \text{cm}^{-2}}\right)\left(\frac{S_\nu}{10 \text{ flux units}}\right)\left(\frac{t}{1 \text{ day}}\right)^{1/2} . \tag{28}$$

We recall that Grewing and Schmidt-Kaler (1968) inferred an integrated neutral-hydrogen number of 5×10^{20} cm^{-2} associated with the relatively wide ($\Delta\lambda/\lambda \sim 3 \times 10^{-3}$) absorption lines in 3C 191. The strong lines in this spectrum probably arise in an H II region (cf. Bahcall *et al.* 1967), and it is therefore unlikely that the spin temperature for this region is much less than 10^4 ° K (even if $r > 10^5$ lt-yrs). However, an equal amount of neutral hydrogen in an H I region with $T \sim 10^2$ ° K and a line width given by equation (7) would produce a 21-cm optical depth much larger than the minimum detectable depth. In fact, for $\epsilon dN_{\rm H\,I} \sim 5 \times 10^{20}$ cm^{-2}, $\tau_{\rm H\,I} \sim 30\ \tau_{\rm optimum}$. The O I and N I absorption lines due to such an H I region would be too narrow to have been observed optically.

V. EMISSION

The possibility that the absorbing region will actually be detectable in emission at the redshifted wavelength of 21 cm, $(1 + z_{\rm abs})\ \lambda_{21}$, cannot be excluded. However, a mass of neutral hydrogen equal to that contained in our Galaxy would produce a small flux for the typical redshifts ($\sim$1) under consideration here. We assume, in order to estimate a maximum effect, that collisional de-excitation is faster than spontaneous emission. We find, in this way, an emitted flux of only

$$S_\nu = 1 \times 10^{-3} \text{ flux units}\left[z_{\rm abs}^{-2}\left(\frac{M}{5 \times 10^9\ M_\odot}\right)\left(\frac{10 \text{ kHz}}{\Delta\nu}\right)\right] , \tag{29}$$

where $\Delta\nu$ is the larger of the two widths, receiver band width and line width, and M is the total mass associated with the absorbing region. We assumed $q_0 = +1$ in deriving equation (29). The interstellar neutral hydrogen in our Galaxy has been estimated to be $\sim 5 \times 10^9\ M_\odot$ (Kerr and Westerhout 1965). It is possible (Bahcall and Salpeter 1966) that some of the absorption lines are produced by gas in clusters of galaxies, in which case S_ν could be comparable to the limits of detectability. Finally, we note that if the amount

of absorbing neutral hydrogen estimated by Grewing and Schmidt-Kaler (1968) for 3C 191 is assumed to lie in a spherical shell of radius t, then the predicted flux in emission is small, namely,

$$S_\nu \approx 1 \times 10^{-5} \text{ flux units} \left[z_{\text{abs}}^{-2} \left(\frac{t}{1 \text{ kpc}} \right)^2 \left(\frac{10 \text{ kHz}}{\Delta\nu} \right) \right] . \tag{30}$$

VI. SOME INTERESTING CANDIDATES

We list in Table 1 some quasi-stellar sources whose absorption spectra would be interesting to explore for possible redshifted 21-cm absorption lines. Typical widths that one might expect would be 10–100 kHz. All of the candidates have absorption redshifts that have been relatively accurately measured. Some of these measured redshifts are very

TABLE 1

SOME INTERESTING CANDIDATES FOR REDSHIFTED 21-CM ABSORPTION LINES

Source	S (flux units)	z_{optical}	$(1+z_{\text{radio}})\lambda_{21}$ cm (cm)	References
PKS 0237−23.....	5 2	1.3642	50	Bahcall (1968), Bahcall *et al.* (1968)
	5.0	1.5132	52	
	4 6	1 6560	56	
	4 6	1.6706	56	
	4.0	2 2015	67	
3C 191.....	5 0	1.9458	62	Bahcall *et al.* (1967)
3C 205...	6 2	1 5382	53	Schmidt (1968)
PKS 1229 − 02.....	2.8	0 395	29	Kinman and Burbidge (1967)
PKS 1116 + 12.....	2.9	1.949	62	Bahcall *et al.* (1966)
3C 270 1.....	5.6	1.498	52	Schmidt (1966)

different from the emission-line redshift (e.g., PKS 0237 − 23 and PKS 1116 + 12), and some are very close to the emission-line redshift (3C 191, 3C 205, PKS 1229 − 02, and 3C 270.1). In two cases (3C 205 and PKS 1229 − 02), the absorption redshift is slightly larger than the emission-line redshift. In general, any quasi-stellar source whose spectrum shows absorption lines due to Mg II or Si II would be promising to study because of the implied presence of relatively easily ionized material.

Table 1 also contains estimates of the monochromatic flux density S_ν at the wavelength of redshifted 21-cm radiation. The best-determined optical redshifts are unfortunately not precisely enough determined for the most advantageous use of narrow-band radio receivers. The optical redshifts listed in Table 1 are typically uncertain by 1–4 $\times$ 10^{-4}. The radio lines from an H I region could have typical fractional widths, $\Delta\nu/\nu$, of only $1 \times 10^{-5}\, T_2^{1/2}$ (cf. eq. [7]). It would be useful to take photographic spectra, with the highest possible dispersion, of some of the quasi-stellar radio sources with absorption lines in order to obtain the most accurate redshifts for use in radio work.

In order to provide predicted redshifts as accurate as possible for the radio lines, it is also helpful to realize that almost all of the optical redshifts given in Table 1 were determined by dividing an observed optical wavelength by a laboratory rest wavelength appropriate to vacuum. Thus

$$\begin{aligned} (1 + z_{\text{opt}}) &= (\lambda_{\text{observed}}^{(\text{air})}/\lambda_{\text{laboratory vacuum}}) \\ &\cong \frac{n(4000 \text{ Å})}{n \text{ (radio)}} (1 + z_{\text{radio}}) \end{aligned} \tag{31}$$

where n is the index of refraction. The relation $(1 + z_{\text{radio}}) \approx 0.999\ (1 + z_{\text{opt}})$ has been used in calculating the values of $(1 + z_{\text{radio}})\lambda_{21\ \text{cm}}$ given in Table 1. For the most accurately determined redshifts listed in Table 1, this correction may be significant.

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