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The proton–proton reaction, solar neutrinos, and a relativistic field theoretic model of the deuteron

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Abstract

In a series of recent papers, Ivanov et al. and Oberhummer et al. have calculated the rate for the $p + p \to d + e^+ + \nu_e$ reaction with a zero-range four-fermion effective interaction and find a result 2.9 times higher than the standard value calculated from non-relativistic potential theory. Their procedure is shown to give a wrong answer because their assumed interaction disagrees with low-energy pp scattering data. © 1997 Elsevier Science B.V.

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In a series of recent papers, Ivanov et al. [1] and Oberhummer et al. [2] have calculated the rate for the $p + p \to d + e^+ + \nu_e$ reaction with a simplified model for the deuteron, and find a result 2.9 times higher than the current best estimate, which is generally believed to be uncertain by only a few percent [3,4]. If correct, the Ivanov et al. result would have important implications for stellar-evolution theory and for solar-neutrino research, a fact that has been stressed in several recent articles by Oberhummer et al. [2].

The calculation of Ivanov et al. contradicts the results obtained by a long history of previous researchers (see e.g. Refs. [5–7,3]) beginning with Bethe and Critchfield in 1938. All previous authors find values for the $p + p \to d + e^+ + \nu_e$ cross section which agree with each other to within a few percent after accounting for small differences in the experimentally determined input parameters. We show here that the result of Ivanov

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et al. is incorrect because their assumed nuclear interactions do not fit the experimental data from low-energy *pp* scattering.

We begin by recalling that low-energy *pp* scattering experiments determine the *pp* scattering length, $a_p = -7.8196(26)$ fm, and effective range, $\rho_p = 2.790(14)$ fm [8].

For the *pp* interaction, Ivanov et al. postulate an effective Lagrangian,

$$\mathcal{L}_{\rm eff}^{pp} = -\frac{g_{\pi NN}^2 C^2(v)}{2m_\pi^2} [\bar{p}\gamma_5 p] [\bar{p}\gamma_5 p] , \tag{1}$$

where $g_{\pi NN}$ is a pion–nucleon–nucleon coupling, and $C(v)$ is the standard Gamow factor introduced in Eq. (1) on an ad hoc basis to partially describe the Coulomb repulsion, with v being the relative velocity in the two-proton system. At low energies, Eq. (1) gives rise to a delta-function potential between the two protons. This delta function has an effective range $\rho_p = 0$, which is incorrect. We show below that this erroneous effective range introduces an order-unity error in the cross section for the $p + p \to d + e^+ + \nu_e$ reaction and, by itself, invalidates the calculation of Ivanov et al.

The scattering length does not appear directly in the effective Lagrangian, Eq. (1), but is introduced in the one-loop, long-wavelength approximation that Ivanov et al. use in their calculations. They chose a value $a_p = -17.1$ fm [9], which has been corrected for electromagnetic effects [10]. In the standard calculations [3,5–7], the Coulomb repulsion is included explicitly in the proton–proton interaction and the nuclear interaction is then fit self-consistently to the empirically determined scattering length, -7.8196 fm. Either choice of a_p is an approximation using the Ivanov et al. procedure since they only remove the asymptotic part of the Coulomb correction, $C(v)$. Their treatment of the proton-proton Coulomb repulsion introduces an additional error in their final result which is difficult to estimate.

How do we know that the calculated rate for the *pp* reaction cannot be wrong by a large factor? Salpeter [6], and later Bahcall and May [7], showed that the rate for the $p + p \to d + e^+ + \nu_e$ reaction can be determined accurately given the measured *pp* scattering length and effective range, the deuteron binding wave number γ, the asymptotic *D*- to *S*-state ratio η_d, and the effective range ρ_d. This "effective-range approximation" makes no assumptions about the details of the nuclear interactions, except that they must yield the measured values for these quantities. The success of the effective-range approximation relies on the fact that the matrix element for the reaction is proportional to the overlap of the initial *pp* wave function and the final deuteron wave function and that most of the overlap comes from radii large compared with the range of the nuclear forces. The deuteron wave function is constrained by the measured static deuteron parameters, and the *pp* wave function is constrained by low-energy *pp*-scattering data. The effective-range approximation has been shown to agree to a few percent with numerical calculations of the matrix element for a wide variety of assumed nuclear interactions [3,7].

We can use the effective-range approximation to illustrate the effect of invalid input data on the $p + p \to d + e^+ + \nu_e$ cross section. If, as in the Ivanov et al. Lagrangian, the *pp* effective range were zero (with all other parameters fixed at the experimentally determined values), the cross section would be about 1.5 times higher than the canonical value. If we additionally assumed the scattering length were twice the measured value, then the cross section would be 3 times larger than the standard result. This numerical exercise shows that order-unity errors in a_p or ρ_p will lead to order-unity errors in the *pp* reaction rate.

In conclusion, the discrepant results of Ivanov et al. are due (at least in part) to the fact that their assumed *pp* interaction disagrees with low energy scattering data. Our previous results for a broad range of nuclear interactions show [3,7] that any calculation which is consistent with the measured low-energy scattering of the *pp* system and the measured properties of the deuteron will yield a cross section that differs from the current best-estimate by no more than a few percent. [3]

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[3] We do not discuss the difficult questions associated with the general procedure adopted by Ivanov et al. [1], in which nucleons and light nuclei are treated as fundamental particles in a relativistic field theory. This program is complicated because there is no small parameter that makes perturbative calculations reliable and because the effective theory should be derived from QCD. Degl'Innocenti et al. have also recently pointed out that an enhancement of the *pp* reaction rate by a factor of 2.9 would be in gross disagreement with helioseismological data [11].