Helioseismological Implications of Recent Solar Abundance Determinations

John N. Bahcall

Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540

Sarbani Basu

Department of Astronomy, Yale University, New Haven, CT 06520-8101

Marc Pinsonneault

Department of Astronomy, Ohio State University, Columbus, OH 43210

and

Aldo M. Serenelli

Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540

ABSTRACT

We show that standard solar models are in good agreement with the helioseismologically determined sound speed and density as a function of solar radius, the depth of the convective zone, and the surface helium abundance, as long as those models do not incorporate the most recent heavy element abundance determinations. However, sophisticated new analyses of the solar atmosphere infer lower abundances of the lighter metals (like C, N, O, Ne, and Ar) than the previously widely used surface abundances. We show that solar models that include the lower heavy element abundances disagree with the solar profiles of sound speed and density as well as the depth of the convective zone and the helium abundance. The disagreements for models with the new abundances range from factors of several to many times the quoted uncertainties in the helioseismological measurements. The disagreements are at relatively low temperatures and do not significantly affect solar neutrino emission. If errors in the calculated OPAL opacities are solely responsible for the disagreements, then the corrections in the opacity must extend from 2×10^6K ($R = 0.7 R_\odot$) to 5×10^6K ($R = 0.4 R_\odot$), with opacity increases of order 10%.

Subject headings:

1. INTRODUCTION

Why are precision tests of solar models important? The Sun is a laboratory in which the predictions of stellar evolution theory can be compared with uniquely accurate and detailed measurements. Stellar evolution calculations are used throughout astronomy to classify, date, and interpret the spectra of individual stars and of galaxies. Comparisons, discussed in this paper, between helioseismological measurements and solar model calculations suggest that at least one of the ingredients of stellar evolution calculations is not known as precisely as previously believed. We shall see that there are reasons for questioning the accuracy of the most sophisticated and detailed determinations of stellar abundances, the recent measurements of the solar metal abundances. Alternatively, unexpectedly large changes could be required in the radiative opacity. However, we shall also see that the disagreement between helioseismological measurements and solar model predictions (with the new metal abundances) occur at relatively low temperatures and therefore do not affect significantly the predicted solar neutrino fluxes.

Helioseismology provides sensitive and powerful tests of the theory of stellar evolution. In addition to measuring the depth of the solar surface convection zone and the surface helium abundance, inversions of seismic data are used to measure to high precision the speed of sound as a function of depth in the star for almost the entire solar interior. The density distribution can also be determined, although with an order of magnitude less precision than for the sound speed.

A number of investigators have made comparisons of seismic data with solar models and have confirmed that the standard solar mixture of Grevesse & Noels (1993) and the updated mixture of Grevesse & Sauval (1998) yield solar models in good agreement with the data (e.g., Bahcall & Pinsonneault 1995; Christensen-Dalsgaard, et al. 1996; Bahcall, Pinsonneault & Basu 2001; Christen-Dalsgaard 2002; Couvidat, Turck-Chieze, & Kosovichev 2003; Sackmann & Boothroyd 2003; Richard, Théado, & Vauclair 2004, and references therein.) As early as 1988, Bahcall & Ulrich (1988) showed that detailed solar models computed with the accurate physics and the numerical precision required for solar neutrino predictions yielded results in good agreement with the then-available helioseismological data.

In a series of papers that preceded the epochal and definitive measurements of the SNO and Super-Kamiokande solar neutrino experiments (Ahmad, et al. 2001; Fukuda et al. 2001), we showed that the excellent agreement between the computed sound speeds in precise standard solar models and the precise helioseismological inversions (differences $< 0.1\%$ rms throughout the solar interior)implied that new physics was required to solve the solar neutrino problem (Bahcall, Pinsonneault, Basu, & Christensen-Dalsgaard 1997; Bahcall, Basu, & Pinsonneault 1998; Bahcall, Pinsonneault, & Basu 2001; Bahcall 2001).

New and more powerful analyses of the surface chemical composition of the Sun have recently become available. These new analyses use three-dimensional atmospheric models, take account of hydrodynamic effects, and pay special attention to uncertainties in the atomic data and the observed spectra. Lower mass fractions have been obtained in this way for C, N, O, Ne, and Ar (Allende Prieto, Lambert, & Asplund 2001; Allende Prieto, Lambert, & Asplund 2002; Asplund et al. 2004; Asplund et al. 2000; Asplund 2000). The new abundance determinations, together with the previous best-estimates for other solar surface abundances (Grevesse & Sauval 1998), imply $Z/X = 0.0176$, much less than the previous value of $Z/X = 0.0229$ (Grevesse & Sauval 1998). In fact, the recent estimates for the C, N, and O mass fractions are lower than all the abundance measurements we have used in the precision solar models in this series going back to 1971 (see, e.g., Table II of Bahcall and & Pinsonneault 1995).

Despite the great improvement in the techniques now used to determine the new element abundances, the new abundances cause the depth calculated for the solar convective zone with the aid of a standard solar model, $R_{\rm CZ} = 0.726 R_\odot$ (Bahcall & Pinsonneault 2004; Bahcall, Serenelli, & Pinsonneault 2004; Basu & Antia 2004), to be in strong disagreement with the measured depth,

$$R_{\rm CZ} = 0.713 \pm 0.001 R_\odot \,, \tag{1}$$

which is determined by helioseismological techniques (Basu & Antia 1997, 2004; Basu 1998; Kosovichev & Fedorova 1991; Christensen-Dalsgaard, Gough, & Thompson 1991; and Guzik & Cox 1993). Paradoxically, the calculated depth of the convective zone obtained using the older element abundances, $R_{\rm CZ} = 0.714 R_\odot$, agrees with the helioseismological value (Bahcall, Pinsonneault, & Basu 2001). This situation has been described as the "convective zone problem" (Bahcall, Serenelli, & Pinsonneault 2004).

Our goal here is to determine the helioseismological implications of the recent abundance determinations. We compare the helioseismologically measured depth of the solar convective zone, the sound speed and density as a function of radius, and the primordial helium abundance with the values that are obtained using a series of precise solar models. The solar models considered here incorporate the most recent and accurate nuclear and stellar data, including the equation of state and radiative opacity.

We describe in § 2 the solar models whose properties are investigated in the present paper. We then discuss in § 3 the helioseismological data and the inversion technique that we have used to obtain the measured depth of the convective zone, the sound speeds and density as a function of radius, and the initial helium abundance. We compare in § 4 the properties of our set of solar models with the solar parameters that are determined by helioseismology. We summarize and discuss our conclusions in § 5.

2. DESCRIPTION OF SOLAR MODELS

We describe in this section the basic ingredients of six solar models that we use to assess the helioseismological implications of the recent heavy element abundance determinations.

The six solar models considered in detail in this paper are:

(1) BP04: older element abundances from Grevesse, & Sauvall (1998), and best-available values for all other input parameters (including improved nuclear rates and equation of state), model originally computed by Bahcall & Pinsonneault (2004);

(2) BP04+: the same as BP04 except that recent lower estimates for heavy element abundances are incorporated, model computed by Bahcall & Pinsonneault (2004);

(3) BP04–EOS96: the same as BP04 but with the OPAL 1996 EOS (Rogers, Swenson, & Iglesias 1996) instead of the OPAL 2001 EOS (Rogers 2001, Rogers and Nayfonov 2002(48));

(4) BP04+ 21%: the same as BP04+ except that the OPAL radiative opacity is increased by 21% near the base of the convective zone, model computed by Bahcall, Serenelli,& Pinsonneault (2004);

(5) BP04+ 11%: the same as BP04+ except that the OPAL radiative opacity is increased by 11% for temperatures ranging from 2×10^6K to 5×10^6K; and

(6) BP00: our best previous-generation standard solar model, obtained by Bahcall, Pinsonneault, & Basu (2001) with older values of nuclear reaction data, an older equation of state (OPAL 1996), and the Grevesse & Sauval (1998) element abundances.

The code and techniques used in these calculations have been described in Bahcall & Pinsonneault (1992, 1995), Bahcall, Pinsonneault, & Basu (2001) and Bahcall & Ulrich (1988).

The reader may wonder why we include in this paper the results from the model BP00, when BP04 has superseded BP00 by incorporating more accurate nuclear reaction data and an improved equation of state. We include results from BP00 as well as BP04 in order to have some indication of the kind of differences that can be expected, independent of solar abundance determinations, as further improvements are made in the input data to solar models. The differences between values obtained with the BP00 and the BP04+ models may be regarded as within the expected range. We shall see in what follows that the differences in solar model results caused by adopting the new heavy element abundance determinations are much larger than the differences between the results obtained with BP00 and BP04.

We want our investigations to be as precise as possible and our inferences to be as free as possible from dependence upon the idiosyncrasies of a particular stellar evolution code. Therefore, we have recalculated the BP04, BP04+ and BP04+ 21% solar models using the Garching stellar evolution code (see, e.g., Schlattl, Weiss, & Ludwig 1997 and Schlattl 2002 for details of the code), to which the nuclear energy generation routine 'exportenergy.f'[1] has been coupled. The nuclear cross sections adopted are those used in Bahcall & Pinsonneault (2004). The models were calculated using the latest version of the OPAL equation of state (Rogers 2001), OPAL radiative opacities (see below for the composition adopted) and element diffusion for helium and metals (Thoul, Bahcall, & Loeb 1994, code available at the URL given in footnote 1). The mixing length theory for convection has been used in all the models. The Schwarzschild criterion was used to determine the location of the convective boundaries.

We have verified that the Garching stellar evolution code and the Bahcall-Pinsonneault code (which has its origins in the CalTech, UCLA, and Yale codes, see Bahcall & Ulrich 1988; Bahcall & Pinsonneault 1992, 1995; Prather 1976; Pinsonneault, Kawaler, Sofia, & Demarque 1989)) yield identical results to the accuracy of interest in all of the investigations considered in this paper.

The model, BP04, which was calculated assuming the older Grevesse & Sauval (1998) solar surface composition, has a present surface ratio of heavy element to hydrogen mass fractions of $Z/X = 0.0229$. The model BP04+, which incorporates the new determinations of the solar heavy element composition (Allende Prieto, Lambert, & Asplund 2001; Allende Prieto, Lambert, & Asplund 2002; Asplund et al. 2004; Asplund et al. 2000; Asplund 2000) has a much lower ratio of heavy elements to hydrogen, $Z/X = 0.0176$. Since new solar abundance determinations are being reported as they come available, Table 1 of Bahcall, Serenelli, & Pinsonneault (2004) reports the specific element abundances adopted in computing both BP04 and BP04+.

The model BP04+ 21% was designed to bring into agreement the calculated and the helioseismologically measured depths of the convective zone using a solar model that incorporates the recent heavy element abundance determinations. Bahcall, Serenelli, & Pinsonneault (2004) showed that a local 21% increase in the tabulated OPAL radiative opacity near the base of the convective envelope will produce a model with the base of its convective zone at $R_{\rm CZ} = 0.713 R_\odot$, in essentially perfect agreement with the measured value for the depth of the convective zone. The factor by which the opacity was increased is similar to the factor needed by Basu & Antia (2004) to construct solar envelope models with the new

[1]The routine is publicly available at http://www.sns.ias.edu/~jnb.

heavy element abundances that have the same convection zone depth, helium abundance, and density profile as the Sun. Very recently, Seaton & Badnell (2004) have shown that a detailed calculation using the methods of the Opacity Project (OP, see Seaton et al. 1994) for a six element mixture yields a Rosseland-mean opacity in the region of interest of order 5% larger than the OPAL opacity for the same mixture.

All the models assume a solar age of 4.57×10^9 yr, a present solar luminosity $L_\odot = 3.8418 \times 10^{33}$ ergs^{-1}, and a present solar radius of $R_\odot = 6.9598 \times 10^{10}$ cm. For each model, OPAL opacity tables were used that correspond to the detailed composition that was adopted.

3. HELIOSEISMOLOGICAL INVERSIONS

We summarize in this section the largely standard techniques that we use to determine the differences between the solar model characteristics and the properties of the Sun as determined by helioseismological measurements.

Helioseismological inversions generally proceed through a linearization of the equations of stellar oscillation, using their variational property, around a known reference model (see, e.g., Dziembowski, Pamyatnykh & Sienkiewicz 1990; Däppen et al. 1991; Antia & Basu 1994; Dziembowski et al. 1994). The differences between the structure of the Sun and the reference model are then related to the differences in the measured oscillation frequencies of the Sun and the model by known kernels. Non-adiabatic effects and other errors in modeling the surface layers give rise to frequency shifts which are not accounted for by the variational principle (Cox & Kidman 1984; Balmforth 1992). Since the eigenfunctions of low- and medium-degree modes are essentially independent of degree in the near-surface layers, the frequency shifts are just a function of mode frequency, divided by the mode inertia (Christensen-Dalsgaard & Berthomieu 1991; Christensen-Dalsgaard & Thompson 1997). The frequency of a deeply penetrating mode is shifted less by near-surface perturbations than that of a shallowly penetrating mode of the same frequency. In the absence of any first-principle formulation, these effects are usually taken into account in an *ad hoc* manner by including an arbitrary function of frequency in the variational formulation (Dziembowski et al. 1990). Thus, the fractional change in the frequency of a mode can be expressed in terms of the fractional changes in the structure of the model, which can be characterized, for example, by the adiabatic sound speed, c, and the density, ρ, as well as a surface term. After linearization, one obtains:

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{R_\odot} K^i_{c^2,\rho}(r)\frac{\delta c^2(r)}{c^2(r)}dr + \int_0^{R_\odot} K^i_{\rho,c^2}(r)\frac{\delta\rho(r)}{\rho(r)}dr + \frac{F_{\rm surf}(\nu_i)}{I_i} \qquad (2)$$

(e.g., Dziembowski et al. 1990). Here $\delta\nu_i$ is the difference in the frequency ν_i of the ith mode between the solar data and a reference model, i representing the pair (n, l), where n is the radial order and l the degree of the model. The kernels $K^i_{c^2,\rho}$ and K^i_{ρ,c^2} are known functions of the reference model which relate the changes in frequency to the changes in c^2 and ρ, respectively. The term involving $F_{\rm surf}$ takes into account the near-surface errors in modeling the structure and the modes, and I_i is the mode inertia of the $i^{\rm th}$ mode.

Equation (2) constitutes the inverse problem that must be solved to infer the differences in structure between the Sun and the reference model. The inversions shown in this paper have been carried out using the the Subtractive Optimally Localized Averages (SOLA) technique (Pijpers & Thompson 1992, 1994). Details of how SOLA inversions are carried out and how various parameters of the inversion are selected are given by Rabello-Soares, Basu, & Christensen-Dalsgaard (1999).

In this paper, we use helioseismic inversions to determine how similar the different solar models discussed in § 2 are to the real Sun. Each of the models described in § 2 is used as a reference model. For the helioseismological data, we use solar oscillation frequencies obtained by the Michelson Doppler Imager (MDI) on board the Solar and Heliospheric Observatory (SOHO). In particular, we use frequencies obtained from MDI data that were collected for the first 360 days of its observation (Schou et al. 1998). This data set was chosen because it was derived from a long time series when solar activity was low. The length of the time series results in reduced noise, and hence a larger number of modes for which the frequencies can be determined reliably. Mode-sets derived from longer data sets are available, but they only consist of low degree modes (e.g., Bertello et al. 2000). Also, a longer time series would have meant adding observations from periods of increasing solar activity, which would have changed the frequencies. It is a well established fact that solar frequencies increase with solar activity. However, it is also known that the increase occurs as an increase in the surface term in Eq. 2, and hence does not change inversion results (Basu 2002).

We invert for both the sound-speed differences and the density differences between the solar models and the Sun.

4. COMPARISONS BETWEEN SOLAR MODELS AND OBSERVATIONS

In this section, we compare solar parameters determined from helioseismological measurements with the values obtained from the six solar models that are discussed in § 2. We first describe in § 4.1 the satisfactory agreement that is obtained for the models BP00 and BP04 (as well as the model BP04–EOS96) which incorporate the older Grevesse & Sauval (1998) heavy element abundances. We then consider in § 4.2 and § 4.3 the discrepant results that are obtained when, respectively, the solar models BP04+ and BP04+ 21%, are compared with the helioseismology data. These latter two models incorporate the recent (lower) heavy element abundance determinations. Finally, we study in § 4.4 a solar model, BP04+11, with the low metal abundances and a constant increase in opacity of 11% from the base of the convective zone to about $R = 0.4R_\odot$.

Table 1 summarizes the principal results from the solar model calculations that can be compared with the helioseismological inversions. We show in the table results for each of the six solar models discussed in § 2.

Figure 1 shows the fractional differences between the sound speeds as a function of solar radius that are computed for each of the solar models and the sound speeds determined from helioseismology. This figure shows clearly the good agreement between the sound speed profiles predicted by models with the older (1998) abundances and the values obtained from helioseismology. Figure 1 also shows that solar models computed using the recent heavy element abundance determinations disagree with the helioseismological sound speed profiles.

Figure 2 shows for the density profiles a similar trend as Figure 1 shows for the sound speeds. The density profiles of solar models computed using the older heavy element abundances agree with the helioseismological data better than solar models which use the newer heavy element abundances (see also column 4 of Table 1). Since it is well known that sound speed determinations are more accurate and more robust than density determinations, we do not discuss further the density profiles other than to remark that they are consistent with all of the other comparisons we make between solar model predictions and helioseismological measurements.

4.1. Comparisons for models BP00 and BP04: 1998 element abundances

The third column of Table 1 presents the fractional rms differences between each solar model (used as a reference model, see § 3) and the helioseismologically determined sound speeds. We see that the BP00 and the BP04 solar models, both of which are computed using the older Grevesse & Sauval (1998) heavy element abundances, are in good agreement with the solar sound speeds. The rms agreement with the solar sound speeds is about 0.1% for both BP00 and BP04. Figure 1 shows the agreement between the sound speeds predicted by the BP00 solar model (dark line) and the BP04 solar model (dashed line).

The fifth column of Table 1 shows that the calculated depth of the convective zone for the BP00 and the BP04 models is in satisfactory agreement with the the measured value of the depth of the convective zone given in equation(1), $0.713R_\odot$.

Table 1: Solar model predictions versus helioseismological determinations. The table presents for comparison with helioseismological measurements the results of a series of four solar models discussed in § 2. The successive columns give the model designation, the adopted present heavy element to hydrogen mass ratio at the solar surface, the rms fractional difference between the solar model sound speeds and the helioseismologically-determined sound speeds, the rms fractional difference for the density, the radius of the convective zone, and the present surface helium abundance. For consistency, all the results reported in this table were obtained with the Graching stellar evolution code.

MODEL	Z/X	$\sqrt{< (c - c_{\odot})^2 / c^2 >}$	$\sqrt{< (\rho - \rho_{\odot})^2 / \rho^2 >}$	$R_{\rm CZ}/{\rm R}_{\odot}$	$Y_{\rm surf}$
BP00	0.0229	0.0010	0.005	0.7141	0.243
BP04	0.0229	0.0014	0.011	0.7146	0.243
BP04–EOS96	0.0229	0.0013	0.012	0.7148	0.243
BP04+	0.0176	0.0046	0.037	0.7259	0.238
BP04+ 21%	0.0176	0.0029	0.027	0.7133	0.239
BP04+ 11%	0.0176	0.0014	0.013	0.7162	0.243

The surface helium abundance of the Sun has recently been redetermined by Basu and Antia (2004). They find

$$Y_{\rm helioseismology} = 0.2485 \pm 0.0034. \qquad (3)$$

The interpretation of the error given in equation (3) is not simple since systematic uncertainties are dominant. The fifth column of Table 1 shows that the present-day surface helium abundance obtained from models BP00 and BP04 may be slightly lower than is obtained from helioseismology, but the statistical significance of this difference is uncertain.

For completeness, we have computed a model that is identical to BP04 except that instead of using the 2001 OPAL equation of state, as was done in deriving the model BP04, we use the older 1996 OPAL equation of state. The results are given in the third row of Table 1.

Improvements in the equation of state between 1996 and 2001 are reflected in Table 1 by the slightly different values that are found for BP04 (row two) and BP04–EOS96 (row three). We see that the improvement in the equation of state does not affect significantly the agreement of the solar model results with the measured helioseismological values. We conclude that plausible changes in the equation of state are unlikely to explain the discrepancy between solar model predictions and helioseismological measurements when the lower metal abundances are used.

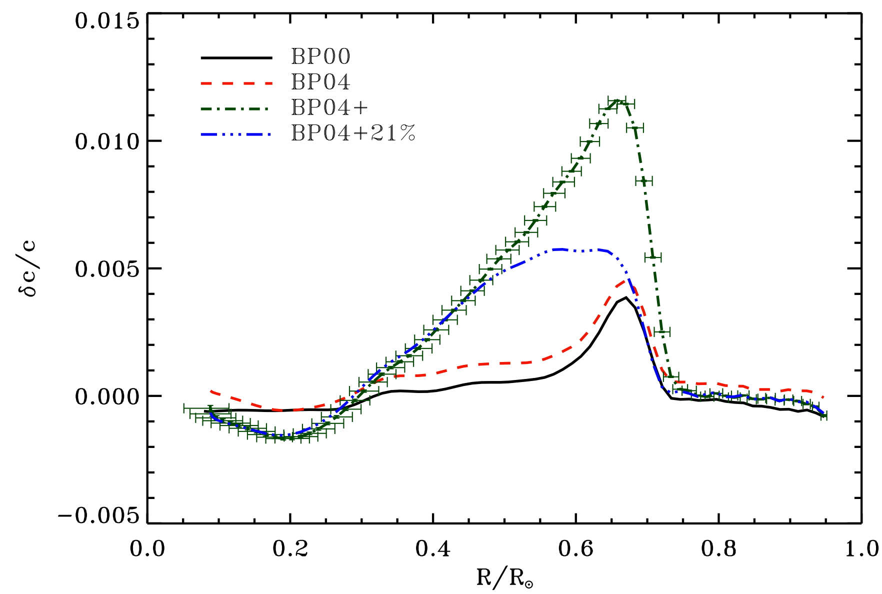

Fig. 1.— Relative sound-speed differences, $\delta c/c = (c_{\odot} - c_{\rm model})/c_{\rm model}$, between solar models and helioseismological results from MDI data. The vertical error bars show the 1σ error in the inversion due to statistical errors in the data. The horizontal error bars are a measure of the resolution of the inversions, defined as the distance between the first and third quartile points of the averaging kernels (approximately the half-width in radius of the measurement in regions of good resolution).

As discussed in Basu, Pinsonneault, and Bahcall (2001), the effect of mixing in the radiative zone of the Sun would be in the direction to reconcile the meteoritic and solar photospheric lithium abundances and to bring the computed surface helium slightly closer to the measured value. Such models have a somewhat shallower solar surface convection zone and the overall agreement with the sound speed data is comparable, or slightly less good, than models without extra mixing.

4.2. Comparisons for model BP04+ : new heavy element abundances

Figure 1 shows the dramatic lack of agreement between the helioseismological sound speeds and the values predicted by the BP04+ solar model, which uses the new heavy element abundance determinations (Allende Prieto, Lambert, & Asplund 2001; Allende Prieto, Lambert, & Asplund 2002; Asplund et al. 2004; Asplund et al. 2000; Asplund 2000). The

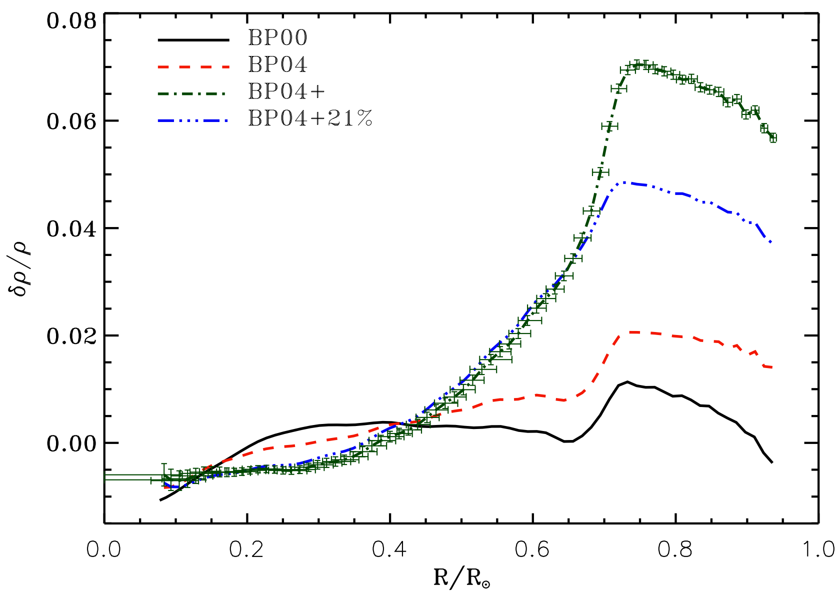

Fig. 2.— Relative density differences, $\delta\rho/\rho = (\rho_\odot - \rho_{\rm model})/\rho_{\rm model}$, between solar models and helioseismological results from MDI data. The vertical error bars show the 1σ error in the inversion due to statistical errors in the data. The horizontal error bars are a measure of the resolution of the inversions, defined as the distance between the first and third quartile points of the averaging kernels (approximately the half-width in radius of the measurement in regions of good resolution). For the density, the resolution near the center of the Sun is particularly poor.

biggest discrepancy is in the vicinity of the base of the convective zone, near $0.7R_\odot$. However, there is a significant discrepancy between BP04+ and the helioseismological values all the way into about $0.3R_\odot$.

Table 1 summarizes the magnitude of this discrepancy. For the solar model BP04+, the rms discrepancy in the sound speeds is more than a factor of three worse than for the BP04 model (and more than a factor of five worse than for the BP00 model). Furthermore, the depth of the convective zone, $0.726R_\odot$, given in column 6 of Table 1 is inconsistent with the measured value of $0.713R_\odot$.Finally, the surface helium abundance given in column 6, $Y = 0.238$, is lower than the measured value given in equation (3).

We conclude that the solar model BP04+, which is constructed using the most recent heavy element abundance estimates, is inconsistent with helioseismological measurements.

4.3. Comparisons for BP04+ 21%: enhanced opacity new abundances

The comparison between the predictions of the model BP04+ 21% and the helioseismological data is very instructive. This solar model was investigated in Bahcall, Serenelli, & Pinsonneault (2004) because the 21% increase in the radiative opacity relative to the standard OPAL opacity was found to be sufficient to resolve the discrepancy in the calculated depth of the convective zone that was obtained with BP04+ model (with no enhanced opacity).

The BP04+ 21% model was constructed with exactly the same input data as for the BP04+ model, including the recent heavy element abundance determinations, but in addition BP04+ 21% has the radiative opacity increased artificially by 21% near the base of the convective zone. The precise form of the opacity increase was postulated to be of the form obtained by multiplying the OPAL opacity in the vicinity of the convective envelope boundary by a Lorentzian function $f(T)$. Specifically, the multiplicative factor $f(T)$ was taken to be

$$f(T) = 1 + \frac{\alpha\gamma^2}{((T - T_0)^2 + \gamma^2)} \,. \tag{4}$$

Here T is the temperature in the solar model. The perturbed opacity is $\kappa_{\rm perturbed} = \kappa_0 f(T)$, where κ_0 is the unperturbed radiative opacity, α is the amplitude of the perturbation, and γ is the width of the perturbation (defined as the point where the perturbation drops to $\alpha/2$). The temperature at the base of the CZ is $T \approx 2.18 \times 10^6$K, which was used for T_0 in equation (4). The BP04+ 21% solar model was calculated for a width of the opacity perturbation $\gamma = 0.2 \times 10^6$ K $\approx 0.1 T_0$. This value of γ corresponds to a width in the solar radius of only $\Delta R = 0.02 R_\odot$.

Figure 1 shows two things about the BP04+ 21% solar model. First, the 21% increase in the opacity near the base of the solar convective zone indeed improves significantly the agreement with the measured sound speeds over what is obtained with the model BP04+. Second, the improved agreement is limited to the region near the base of the convective zone and there remains a significant disagreement down to radii of order $0.4 R_\odot$ ($T = 5 \times 10^6$K). Of course, different assumed forms of the factor $f(T)$ lead to different estimates of how much opacity change is required to construct a model with the correct depth of the convection zone (see, e.g., Basu & Antia 2004).

In summary, Figure 1 indicates that the radiative opacity would have to be changed in a broad range of temperatures (radii) in order to resolve the discrepancies between helioseismological measurements and solar model predictions made using the new heavy element abundances. The relatively low value for the surface helium abundance, $Y = 0.239$ obtained with BP04+ 21% (see Table 1), may also reflect the need for an opacity correction that

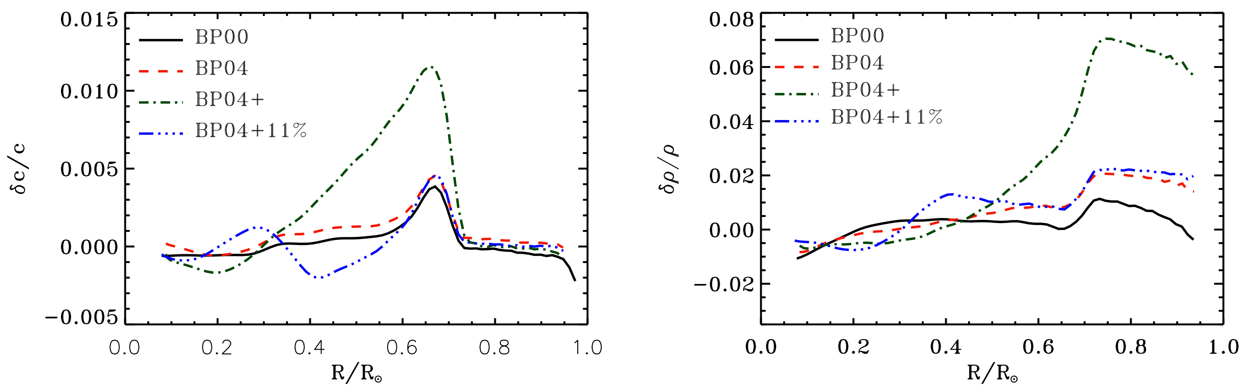

Fig. 3.— A model with an 11% opacity increase. The figure shows the relative sound speed and density differences (left and right panels respectively) between solar models and helioseismological results from MDI data. The BP04+11% model has the same characteristics as BP04+ (i. e., low metal abundances) except that the radiative opacities have been increased by a constant 11% factor from the base of the convective zone down to 5×10^6K ($R = 0.4R_\odot$).

extends down to $\sim 5 \times 10^6$K.

4.4. Comparisons for BP04+ 11%

Motivated by the results of § 4.3, we have computed a variety of solar models assuming the correctness of the recently determined low metal abundances but with different assumed opacity changes. We have studied the helioseismological properties of these models. The reader will immediately recognize that one can in principle consider an infinite number of such 'low-metal, higher-opacity' models, with prescriptions for changing the opacity of varying complexity and artificiality . We acknowledge that there is limited utility in computing such models without a physical basis for the assumed opacity changes.

However, we have found a relatively simple prescription for changing the opacity, while adopting the low metal abundances, that yields reasonable agreement with the observed helioseismological properties. We present the results for this model here not out of any conviction that the assumed opacity law is correct, but rather to illustrate the general quality of the fit to the helioseismological data that is possible and to indicate approximately how much the opacity would have to be shifted in order to obtain reasonably good agreement with the helioseismological measurements.

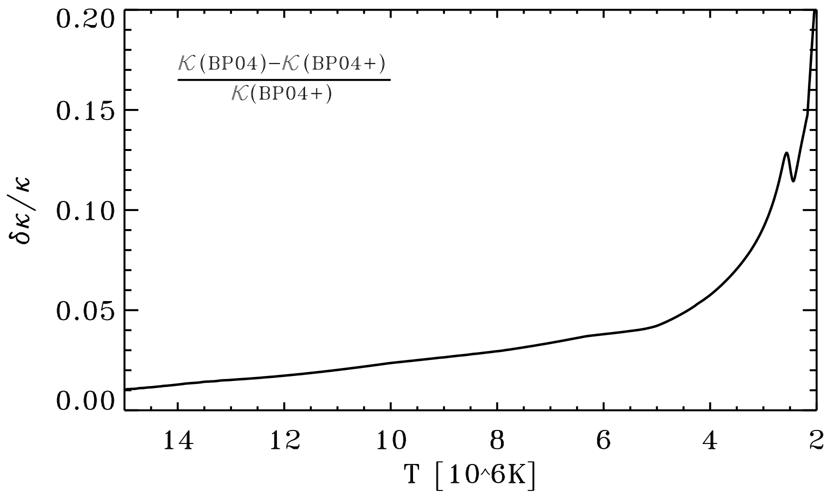

Fig. 4.— Opacity difference between BP04 and BP04+ solar models. The figure shows the fractional opacity difference between the two solar models BP04 (higher metal abundances) and BP04+ (lower metal abundances) as a function of the temperature in the BP04 solar model.

The results of § 4.3 indicate that the opacity must be changed over a relatively broad range of temperatures if we adopt the lower metal abundances. For simplicity, we assumed a constant 11% increase above the OPAL opacity from 2×10^6K ($R = 0.7 R_\odot$) down to 5×10^6K ($R = 0.4 R_\odot$), where the opacity increase was smoothly turned off (half-width of turn off is 2×10^5K). We denote this model by BP04+11%.

We are sure that the prescription of a constant opacity increase that is implemented in BP04+11% is too simple to represent the improvements in the radiative opacity that are likely to result from detailed quantum mechanical calculations of the solar mixture of hydrogen, helium, and heavy elements. But, we shall see that this model with a constant opacity increase fits the data reasonably well and is a crude approximation to what might guess is required by comparing (see Figure 4 below) the opacities in the BP04 model (successful in describing the helioseismological data) and the BP04+ model (unsuccessful in describing the helioseismological data).

Figure 3 and Table 1 show that the BP04+11% solar model fits the helioseismological data with an accuracy that is comparable to our best-fitting solar models, BP00 and BP04. We conclude that an increase in the opacity of the order of 10% in the range 2×10^6K to 5×10^6K would resolve the discrepancy between the predictions of solar models computed with the new lower metal abundances and the helioseismological measurements.

Figure 4 shows the fractional opacity difference between the BP04 solar model (higher metal abundances, fits well helioseismology data) and the BP04+ solar model (lower metal abundances, fits poorly helioseismology data) as a function of the temperature in the BP04 solar model. The fractional opacity difference is less than or of order 1% in the central regions of the model ($R < 0.2R_\odot$ and $T < 9 \times 10^6$ K), the regions in which the solar neutrinos are produced. However, in the outer regions near the base of the convective zone, the fractional difference is as large as 15% [2].

We have evolved solar models with larger opacity increases near the base of the convective zone and smaller increases further in, but we have not obtained by this procedure a substantial improvement in the agreement with helioseismological data over what is achieved with the BP04+11% model.

Basu & Antia (2004) discussed the characteristics of a solar envelope model which invoked, in order to satisfy helioseismic constraints, a 19% increase in radiative opacity relative to the tabulated OPAL opacities. We have evolved a full solar model with the same opacity increase and heavy element abundance as the Basu & Antia (2004) model (i.e. a 19% increase in opacity from the base of the convection zone to a temperature of 5×10^6K and the heavy element to hydrogen mass ratio of Z/X=0.0171). We find, as expected from the previous discussion of BP04+11% and from Figure 3, that the 19% increase in opacity is too large to provide a good fit to the helioseismological data. The depth of the convection zone for the evolved model is $R = 0.708R_\odot$ and the rms fractional sound speed discrepancy is $\delta c/c = 0.0033$. The reason for the difference in our conclusion and the Basu & Antia (2004) result almost certainly lies in the fact that the Basu & Antia envelope model was forced to have abundance profiles near the base of the convective zone that are different from what we find in our stellar evolution models, while at the same time being silent about the helioseismological properties in the radiative interior. The Basu & Antia envelope model was forced to have heavy-element and helium profiles in agreement with the helioseismological determinations near the base of the convective zone. For standard solar models, the heavy element and helium profiles are different from that of the Sun near the base of the convective zone (Basu & Antia 1994; Bahcall, Pinsonneault, Basu, & Christensen-Dalsgaard 1997; Antia

[2]In principle, it might be more informative to compare opacities in BP04 and BP04+ at the same density and chemical composition as well as temperature. We have computed the difference in opacities taking into account in an approximate way the change in density, at a constant temperature, between BP04 and BP04+. The resulting opacity difference is very similar in shape to what is shown in Figure 4 for the opacity difference at the same temperature but slightly different densities. Including the density dependence, makes very little difference near the base of the convective zone but increases the fractional opacity difference by about 40% of its value at the highest temperature ($T = 5 \times 10^6$ K) at which an opacity perturbation was introduced into the solar model BP04+11%.

& Chitre 1998), probably because of turbulent mixing not included in the standard models (Elliott & Gough 1998). Over the radiative interior of the Sun, $R = 0.0$ to $R = 0.7R_\odot$ standard solar models like BP00 or BP04 are, as we have seen, in excellent agreement with the helioseismological data.

5. CONCLUSIONS

We summarize and discuss in this section our five principal conclusions. The main quantitative results of our studies are given in Table 1 and in Figure 1 and Figure 3.

(1) Larger heavy element abundances yield satisfactory solar models. Standard solar models constructed with the older (i. e., higher) heavy element abundances (Grevesse & Sauval 1998) are in good agreement with the helioseismological data. We compare in § 4 and especially in Table 1 and Figure 1 the predictions of the solar models BP00 (vintage 2000 nuclear reaction rates and equation of state) and BP04 (our best current solar model) with the helioseismological data. The solar sound speeds, depth of the convective zone, and surface abundance of helium determined from helioseismology are all in agreement with the values obtained from these two solar models that were computed using the older element abundances. We find similar results for the solar model BP04–EOS96, which is the same as the BP04 model except that BP04–EOS96 was computed using the 1996 version of the OPAL equation of state.

We interpret the differences between the predictions of the models BP00, BP04, and BP04–EOS96 as indicating the expected range of characteristic parameters that can occur with typical improvements in the input data to the solar models.

(2) Standard models with less heavy elements disagree with helioseismology. A solar model constructed with the new heavy element abundances, BP04+, is inconsistent with the helioseismologically measured sound speeds, the depth of the convective zone, and the surface helium abundance. For BP04+, the rms fractional difference between the solar and the model sound speeds is more than three times the difference obtained with the model BP04 and more than five times the difference obtained with the model BP00 (see Table 1)[3]. The depth of the convective zone in the BP04+ solar model

[3]If we take the difference between the rms fractional discrepancies for BP04 (0.14%) and BP00 (0.10%) (see Table 1) as some measure of the uncertainty in solar model predictions of the sound speed, then the discrepancy for the BP04+ model (0.46%) differs from the average discrepancy for BP04 and BP00 (0.12%) by about eight times the uncertainty.

differs from the helioseismological value by about 13 times the error quoted in the helioseismological measurement (cf. Equation 1 and Table 1). Moreover, the surface helium abundance obtained with the model BP04+ differs from the measured helioseismological value by about three times the quoted measurement error (cf. equation 3 and Table 1).

We conclude that solar models constructed with the newer and lower heavy element abundances are inconsistent with helioseismological data unless some significant change is made in the currently standard input data for solar models.

(3) Increasing the opacity near the base of the CZ helps, but is not enough. If we do adopt the newer heavy element abundances (Allende Prieto, Lambert, & Asplund 2001; Allende Prieto, Lambert, & Asplund 2002; Asplund et al. 2004; Asplund et al. 2000; Asplund 2000) and also increase the radiative opacity by 21% near the base of the solar convective zone, then the agreement between the solar model and the helioseismological determinations improves. However, the improvement is limited, like the assumed change in the radiative opacity, to regions near the base of the convective zone. These results can be seen clearly in Figure 1 (see also Table 1). The solar model BP04+ 21% was determined uniquely in Bahcall, Serenelli, & Pinsonneault (2004) by increasing the opacity in a narrow region near the base of the convective zone by just the amount required to make the depth calculated with new heavy element abundance determinations agree with the measured depth.

(4) A 11% increase in opacity over a broader range is okay. Suppose that we hypothesize that a change in the OPAL radiative opacity is required to explain the reason why solar models constructed with the newer heavy element abundances are in conflict with helioseismology measurements. Then we see from Figure 1 and Figure 4 that the opacity change must extend from about 2.2×10^6K at the base of the convective zone ($R = 0.71 R_\odot$) all the way down to about 5×10^6K ($R = 0.4 R_\odot$).

Figure 3 and Table 1 show the results obtained for the solar model BP04+ 11% , which was computed using the new metal abundances and an opacity increased by 11% over the OPAL opacity in the region from 2×10^6K to 5×10^6K. This model BP04+ 11% yields results that agree with the helioseismological measurements about as well as the BP00 and BP04 models.

(5) The predicted solar neutrino fluxes are not significantly affected. The disagreements between the results of solar model calculations with low metal abundances and the helioseismological measurements occur at relatively low temperatures and do not significantly affect solar neutrino emission. The differences in the predicted solar neutrino fluxes for the most different solar models considered in this paper, BP04 and

BP04+, are all within the 1σ quoted theoretical errors (compare column 2 and column 3 of Table 1 of Bahcall and Pinsonneault 2004).

For the BP04 and BP04+11% solar models, the differences in predicted neutrino fluxes are slightly smaller, especially for the most important neutrino sources: 1% ($p-p$ neutrinos), 2% (^{7}Be neutrinos), and 6% (^{8}B neutrinos). The fractional differences for the CNO neutrinos represent larger percentages ($\sim 35\%$), but are within the large 1σ uncertainties determined in Bahcall and Pinsonneault (2004).

In this paper, we have only considered changes in the radiative opacity (or the equation of state) as a way to reconcile the new (lower) heavy element abundance determinations with helioseismological measurements. With this assumption, we have seen that we require an increase in the tabulated OPAL opacities of order 11% over a relatively broad range in temperatures. It would be very useful to study whether such a change in opacities is consistent with other astronomical data.

There are additional sources of potential change in the solar model input data, most importantly the uncertainties in the measurements of the heavy element abundance and the uncertainties in the calculation of the heavy element diffusion coefficients. The recent heavy element abundance determinations have quoted uncertainties of order 0.05 dex (12%) (see Allende Prieto, Lambert, & Asplund 2001; Allende Prieto, Lambert, & Asplund 2002; Asplund et al. 2004; Asplund et al. 2000; Asplund 2000). The heavy element diffusion coefficients are uncertain by about 15% (see Thoul, Bahcall, and Loeb 1994).

It may well be that the correct reconciliation of abundance determinations will involve modest adjustments relative to the present standard values of all of the factors mentioned above, namely, the abundances themselves, the diffusion coefficients, and the radiative opacity. The increase of the radiative opacity by 11% obtained in this paper with the help of the model BP04+ 11% may be regarded as a plausible upper limit to the opacity correction that is required since it assumes no change in any of the other input parameters.

Why have we not constructed and explored even more solar models with a variety of hypothetical changes in the radiative opacity, diffusion coefficients, and heavy element abundances? The reason is that for the opacity changes by themselves there is an infinity of conceivable corrections, with different amplitudes and shapes. Moreover, one can assume whatever changes one wants, within the quoted uncertainties, for the diffusion coefficients and the heavy element abundances. Improved calculations of the radiative opacity (see, e. g., Seaton and Badnell 2004 for recent refinements) will determine what, if any, significant refinements are implied by more accurate calculations. Once those calculations are available it will be appropriate to make new solar models to incorporate the newly calculated opacities.

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