What Do We (Not) Know Theoretically about Solar Neutrino Fluxes?

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Solar model predictions of ^{8}B and p-p neutrinos agree with the experimentally determined fluxes (including oscillations): $\phi(pp)_{\rm measured} = (1.02 \pm 00.02 \pm 0.01)\phi(pp)_{\rm theory}$ and $\phi(^8{\rm B})_{\rm measured} = (0.88 \pm 0.04 \pm 0.23)\phi(^8{\rm B})_{\rm theory}$, 1σ experimental and theoretical uncertainties, respectively. We use improved input data for nuclear fusion reactions, the equation of state, and the chemical composition of the Sun. The solar composition is the dominant uncertainty in calculating the ^{8}B and CNO neutrino fluxes; the cross section for the $^3{\rm He}(^4{\rm He}, \gamma)^7{\rm Be}$ reaction is the most important uncertainty for the calculated ^{7}Be neutrino flux.

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This Letter is part of a series that spans more than 40 years [1]. The goals of this series are to provide increasingly more precise theoretical calculations of the solar neutrino fluxes and detection rates and to make increasingly more comprehensive evaluations of the uncertainties in the predictions. We describe here two steps forward (improved accuracy of the equation of state of the solar interior and some of the nuclear fusion data) and one step backward (increased systematic uncertainties in the determination of the surface composition of the Sun).

Using recent improvements in input data, we calculate the best estimates, and especially the uncertainties, in the solar model predictions of solar neutrino fluxes. We compare the calculated neutrino fluxes with their measured values. We stress the need for more accurate measurements of the surface composition of the Sun and of specific nuclear reaction rates.

Table I presents, in the second (third) column, labeled BP04 (BP04+), our best solar model calculations for the neutrino fluxes. The uncertainties are given in column 2. BP04+ was calculated with new input data for the equation of state [4], nuclear physics [5,6], and solar composition [7]. BP04, our currently preferred model, is the same as BP04+ except that BP04 does not include the most recent analyses of the solar surface composition [7], which conflict with helioseismological measurements. The error estimates, which are the same for BP04, BP04 +, and ^{14}N (see Table I), include the recent composition analyses.

For the BP04 solar model, the base (mass) of the convective zone is $0.715R_\odot$ ($0.024M_\odot$), the surface heavy element to hydrogen ratio by mass, $Z/X = 0.0229$, the surface helium abundance is 0.243, and 1.6% of the luminosity is from CNO reactions. The central temperature, helium abundance, and Z/X are, respectively, 15.72×10^6 K, 0.640, and 0.0583. All of these values are in the acceptable range as determined by helioseismology. However, for BP04+, the base of the convective zone (CZ) is $R_{\rm CZ}/R_\odot = 0.726$, which conflicts with the measured value of 0.713 ± 0.001 (or ± 0.003; see Ref. [8]). By examining a series of models, we have determined that the reason for the too-shallow CZ in the BP04+ model is the lower heavy element abundance, $Z/X = 0.0176$. Therefore, we prefer BP04.

The measurements from different solar neutrino experiments [9] and the KamLAND reactor data [10] can be combined in a global analysis to obtain the best empirical values for the p-p, ^{8}B, and ^{7}Be solar neutrino fluxes. We use the fluxes from the global analysis of Ref. [11], which allows all the solar neutrino fluxes to be free parameters subject only to the luminosity constraint (i.e., energy conservation). Comparing the measured values with the theoretical predictions, we find for BP04:

$$\phi(pp)_{\rm measured} = (1.02 \pm 0.02 \pm 0.01)\phi(pp)_{\rm theory}, \quad (1)$$

$$\phi(^8{\rm B})_{\rm measured} = (0.88 \pm 0.04 \pm 0.23)\phi(^8{\rm B})_{\rm theory}, \quad (2)$$

$$\phi(^7{\rm Be})_{\rm measured} = (0.91^{+0.24}_{-0.62} \pm 0.11)\phi(^7{\rm Be})_{\rm theory}. \quad (3)$$

In Eqs. (1)–(3), the 1σ experimental uncertainties are given before the 1σ theoretical uncertainties.

The measured and theoretical values for the fluxes agree within their combined 1σ uncertainties. The measurement error of the ^{8}B neutrino flux is smaller than the uncertainty in the theoretical calculation, but the opposite is true for the p-p and ^{7}Be neutrino fluxes.

Column 4 of Table I presents the fluxes calculated using our previous best solar model, BP00 [2]. The BP04 best estimate neutrino fluxes and their uncertainties have not changed markedly from their BP00 values despite refinements in input parameters. The only exception is the CNO flux uncertainties that have almost doubled due to the larger systematic uncertainty in the surface chemical composition estimated in this Letter.

We describe improvements in the input data relative to BP00. Quantities that are not discussed here are the same

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TABLE I. Predicted solar neutrino fluxes from solar models. The table presents the predicted fluxes, in units of $10^{10}(pp)$, $10^9(^7\mathrm{Be})$, $10^8(pep, {}^{13}\mathrm{N}, {}^{15}\mathrm{O})$, $10^6(^8\mathrm{B}, {}^{17}\mathrm{F})$, and $10^3(hep)$ $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. Columns 2–4 show BP04, BP04 +, and our previous best model BP00 [2]. Columns 5–7 present the calculated fluxes for solar models that differ from BP00 by an improvement in one set of input data: nuclear fusion cross sections (column 5), equation of state for the solar interior (column 6), and surface chemical composition for the Sun (column 7). Column 8 uses the same input data as for BP04 except for a recent report of the $^{14}\mathrm{N} + p$ fusion cross section. References to the improved input data are given in the text. We use OPAL radiative opacities calculated for each chemical composition. The last two rows ignore neutrino oscillations and present for the chlorine and gallium solar neutrino experiments the capture rates in SNU (1 SNU equals 10^{-36} events per target atom per sec). Because of oscillations, the measured rates are smaller: 2.6 ± 0.2 and 69 ± 4, respectively. We use the neutrino absorption cross sections and their uncertainties that are given in Ref. [3].

Source	BP04	BP04+	BP00	Nucl.	EOS	Comp.	^{14}N
pp	$5.94(1 \pm 0.01)$	5.99	5.95	5.94	5.95	6.00	5.98
pep	$1.40(1 \pm 0.02)$	1.42	1.40	1.40	1.40	1.42	1.42
hep	$7.88(1 \pm 0.16)$	8.04	9.24	7.88	9.23	9.44	7.93
^{7}Be	$4.86(1 \pm 0.12)$	4.65	4.77	4.84	4.79	4.56	4.86
^{8}B	$5.79(1 \pm 0.23)$	5.26	5.05	5.77	5.08	4.62	5.74
^{13}N	$5.71(1^{+0.37}_{-0.35})$	4.06	5.48	5.69	5.51	3.88	3.23
^{15}O	$5.03(1^{+0.43}_{-0.39})$	3.54	4.80	5.01	4.82	3.36	2.54
^{17}F	$5.91(1^{+0.44}_{-0.44})$	3.97	5.63	5.88	5.66	3.77	5.85
Cl	$8.5^{+1.8}_{-1.8}$	7.7	7.6	8.5	7.6	6.9	8.2
Ga	131^{+12}_{-10}	126	128	130	129	123	127

as for BP00. Each class of improvement is represented by a separate column, columns 5–7, in Table I.

Column 5 contains the fluxes computed for a solar model that is identical to BP00 except that we have used improved values for direct measurements of the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ cross section, $S_{20\,\mathrm{keV}}(^7\mathrm{Be} + p) = 20.6 \pm 0.8$ eV b [5], and the calculated p-p, $S_0(pp) = 3.94(1 \pm 0.004) \times 10^{-25}$ MeV b, and hep, $S_0(hep) = (8.6 \pm 1.3) \times 10^{-20}$ keV b, cross sections [6]. The reactions that produce the ^{8}B and hep neutrinos are rare; changes in their production cross sections affect, respectively, only the ^{8}B and hep fluxes. The 15% increase in the calculated ^{8}B neutrino flux, which is primarily due to a more accurate cross section for $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, is the only significant change in the best estimate fluxes.

The fluxes in column 6 were calculated using a refined equation of state [4]. Solar neutrino calculations are insensitive to the present level of uncertainties in the equation of state.

The most important changes in the astronomical data since BP00 result from new analyses of the surface chemical composition of the Sun. The input chemical composition affects the radiative opacity and hence the physical characteristics of the solar model, as well as, to a lesser extent, the nuclear reaction rates. New values for C, N, O, Ne, and Ar have been derived [7] using three-dimensional rather than one-dimensional atmospheric models, including hydrodynamical effects, and paying particular attention to uncertainties in atomic data and observational spectra. The new abundance estimates, together with the previous best estimates for other solar surface abundances [12], imply $Z/X = 0.0176$, much less than the previous value of $Z/X = 0.0229$ [12]. Column 7 gives the fluxes calculated for this new composition mixture. The largest change in the neutrino fluxes for the p-p chain is the 9% decrease in the predicted ^{8}B neutrino flux. The N and O fluxes are decreased by much more, ~35%, because they reflect directly the inferred C and O abundances.

The CNO nuclear reaction rates are less well determined than the rates for the more important (in the Sun) p-p reactions [13]. The rate for $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ is poorly known, but important for calculating CNO neutrino fluxes. Extrapolating to the low energies relevant for solar fusion introduces a large uncertainty. Column 8 gives the neutrino fluxes calculated with input data identical to BP04 except for the cross section factor $S_0(^{14}\mathrm{N} + p) = 1.77 \pm 0.2$ keV b that is about half the current best estimate; this value assumes a particular R-matrix fit to the experimental data [14]. The p-p cycle fluxes are changed by only ~1%, but the ^{13}N and ^{15}O neutrino fluxes are reduced by 40%–50% relative to the BP04 predictions. CNO nuclear reactions contribute 1.6% of the solar luminosity in the BP04 model and only 0.8% in the model with a reduced $S_0(^{14}\mathrm{N} + p)$.

Table II shows the individual contributions to the flux uncertainties. Columns 2–5 present the fractional uncertainties from the nuclear reactions whose measurement errors are most important for calculating neutrino fluxes. Unless stated otherwise, we have used throughout this Letter the uncertainties estimated in Ref. [13] for nuclear cross sections.

The measured rate of the ^{3}He-^{3}He reaction, which after the inception of this series [1] changed by a factor of 4, and the measured rate of the $^7\mathrm{Be} + p$ reaction, which for most of this series has been the dominant uncertainty in predicting the ^{8}B neutrino flux, are by now very well determined. If the published systematic uncertainties for the ^{3}He-^{3}He and ^{7}Be + p reactions are correct, then their uncertainties no longer contribute in a crucial way to the calculated theoretical uncertainties (see columns 2 and 4 of Table II).

The most important nuclear physics uncertainty in calculating solar neutrino fluxes is now the rate of the ^{3}He-^{4}He reaction (column 3 of Table II). The systematic uncertainty in the the rate of $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$ reaction (see Ref. [13]) causes an 8% uncertainty in the prediction of both the ^{7}Be and the ^{8}B solar neutrino fluxes.

For $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$, we have continued to use in Table II the uncertainty given in Ref. [13], although the recent reevaluation in Ref. [14] suggests that the uncertainty could be somewhat larger (see column 8 of Table I).

The uncertainties due to the calculated radiative opacity and element diffusion, as well as the measured solar luminosity (columns 6–8 of Table II), are all moderate, non-negligible but not dominant. For the ^{8}B and CNO neutrino fluxes, the uncertainties that are due to the radiative opacity, diffusion coefficient, and solar luminosity are all in the range 2% to 6%.

The surface composition of the Sun is the most problematic and important source of uncertainties. Systematic errors dominate: the effects of line blending, departures from local thermodynamic equilibrium, and details of the model of the solar atmosphere. We assume that the uncertainty in all important abundances is approximately the same. We have defined previously the 3σ range of Z/X as the spread over all modern determinations (see Refs. [1,2,15]), which now implies $\Delta(Z/X)/(Z/X) = 0.15(1\sigma)$, 2.5 times larger than the uncertainty adopted in BP00. The most recent uncertainty quoted for oxygen, the most abundant heavy element in the Sun, is similar: 12% [7].

Heavier elements like Fe affect the radiative opacity and hence the neutrino fluxes more strongly than the relatively light elements [2]. This is the reason why the difference between the fluxes calculated with BP04 and BP04+ (or between BP00 and Comp.; see Table I) is less than would be expected for the 23% decrease in Z/X. The abundances that have changed significantly since BP00 (C, N, O, Ne, Ar) are all for lighter species for which meteoritic data are not available.

The dominant uncertainty listed in Table II for the ^{8}B and CNO neutrinos is the chemical composition, represented by Z/X(see column 9). The uncertainty ranges from 20% for the ^{8}B neutrino flux to ~35% for the CNO neutrino fluxes. Since the publication of BP00, the best published estimate for Z/X decreased by 4.3σ (BP00 uncertainty) and the estimated uncertainty due to Z/X increased for ^{8}B (^{15}O) neutrinos by a factor of 2.5 (2.8). Over the past three decades, the changes have almost always been toward a smaller Z/X. The monotonicity is surprising since different sources of improvements have caused successive changes. Nevertheless, since the changes are monotonic, the uncertainty estimated from the historical record is large.

We list below our principal conclusions and their implications. First, the experimentally determined values for the p-p, ^{7}Be, and ^{8}B solar neutrino fluxes are in agreement with the values predicted by the standard solar model. More precise measurements of the p-p and ^{7}Be fluxes will test critically the theory of energy generation in the solar interior. Second, recent precise measurements or improved calculations of nuclear reaction parameters, the equation of state, and the surface chemical composition of the Sun have refined the input data to the solar model calculations but have not changed the calculated neutrino fluxes outside the previously quoted theoretical 1σ uncertainties (see columns 3–6 of Table I and Ref. [2]). Third, the rate of the reaction $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$ is the largest nuclear physics contributor to the uncertainties in the solar model predictions of the neutrino fluxes in the p-p chain. For the important ^{7}Be neutrino flux that can be measured in the BOREXINO [16] and KamLAND [10] detectors, there is currently an 8% uncertainty due to the cross section for $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$. Fourth, the cross section for the reaction $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ is the largest nuclear physics contributor to the uncertainties in the calculated CNO neutrino fluxes. It is important to measure this cross

TABLE II. Principal sources of uncertainties in calculating solar neutrino fluxes. Columns 2–5 present the fractional uncertainties in the neutrino fluxes from laboratory measurements of, respectively, the ^{3}He-^{3}He, ^{3}He-^{4}He, p-^{7}Be, and p-^{14}N nuclear fusion reactions. The last four columns, 6–9, give, respectively, the fractional uncertainties due to the calculated radiative opacity, the calculated rate of element diffusion, the measured solar luminosity, and the measured heavy element to hydrogen ratio.

Source	3–3	3–4	1–7	1–14	Opac.	Diff.	$L_\odot$	Z/X
pp	0.002	0.005	0.000	0.002	0.003	0.003	0.003	0.010
pep	0.003	0.007	0.000	0.002	0.005	0.004	0.003	0.020
hep	0.024	0.007	0.000	0.001	0.011	0.007	0.000	0.026
^{7}Be	0.023	0.080	0.000	0.000	0.028	0.018	0.014	0.080
^{8}B	0.021	0.075	0.038	0.001	0.052	0.040	0.028	0.200
^{13}N	0.001	0.004	0.000	0.118	0.033	0.051	0.021	0.332
^{15}O	0.001	0.004	0.000	0.143	0.041	0.055	0.024	0.375
^{17}F	0.001	0.004	0.000	0.001	0.043	0.057	0.026	0.391

section more accurately in order to understand well the energy production in stars heavier than the Sun. Neutrino oscillation studies can use the fluxes from both the low ^{14}N model and BP04 (see Table I); the results for oscillation parameters should be essentially identical. Fifth, the largest uncertainty in calculating all the solar neutrino fluxes is now the uncertainty in the measured surface composition of the Sun (see Table II). Unfortunately, the principal uncertainties in inferring the composition are systematic. Sixth, the recent reanalyses [7] of the solar chemical composition imply a lower surface heavy element abundance and, consequently, a base of the convective zone that conflicts with helioseismological measurements [8]. For this reason, we have not used the most recent low heavy element abundances in our preferred model, BP04. The low heavy element abundances lead in BP04+ to a slightly better agreement with the ^{8}B neutrino measurement, but the improvement between best estimates is smaller than the 1σ uncertainty.

We make three recommendations for future work, based on the known dependences of neutrino fluxes on input parameters [15]. First, the low energy cross section of the $^{3}\text{He}(^{4}\text{He}, \gamma)^{7}\text{Be}$ reaction should be measured to better than ±5% (1σ) (a factor of 2 improvement) in order that the uncertainty in this reaction not limit the interpretation of future ^{7}Be solar neutrino experiments. Second, the uncertainty in the surface heavy element abundances (particularly elements like iron that contribute most significantly to the radiative opacity [2]) should be reduced to less than ±0.02 dex (a factor of 3 improvement) in order that the calculational uncertainty from the composition not exceed the current error (± 4% [9–11]) in the empirical determination of the ^{8}B neutrino flux. Third, the uncertainty in the low energy extrapolation of the rate of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction must be $\leq 25\%$ in order that the p-p flux can be used for a precision measurement of the neutrino mixing angle θ_{12} [11] and an accurate test of stellar evolution theory.

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