# 1 Problem sets for dS/CFT (Maldacena)

## 1.1 Review of the two point function computation in $dS$.

Consider a massless scalar field in $dS_4$ with metric

$$ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2} \tag{1}$$

a) Write the wave equation and solve it. Show that for each Fourier mode you get the solutions $f \sim (1 + i|k|\eta)e^{-i|k|\eta}$ and its complex conjugate.

b) Write $\phi = a^\dagger f + a f^\dagger$. Demand that $[a, a^\dagger] = 1$, use the canonical commutation relations for $\phi$ and $\pi_\phi$ and normalize $f$ properly.

c) Define the Bunch Davies vacuum through $a|BD\rangle = 0$. Understand why this is a reasonable definition. Then compute

$$\langle BD|\phi_{\vec{k}}(\eta)\phi_{-\vec{k}}(\eta')|BD\rangle$$

d)** After defining this for each fourier mode, could you figure out how it looks once you sum over all Fourier modes?. Can you write the answer in a de Sitter invariant fashion ? Do you get a divergence?, why?

## 1.2 Interactions

Add a $\phi^3$ interaction to the above problem and compute the three point function,

$$\langle BD|\phi_{\vec{k}_1}(\eta)\phi_{\vec{k}_2}(\eta)\phi_{\vec{k}_3}(\eta)|BD\rangle$$

Understand the Keldysh contour necessary for the computation. How do you kill off the oscillatory early time pieces ? Understand that you need to rotate the contour into the imaginary direction.

If you want a simpler problem, then consider a conformally coupled scalar field and add a $\phi^4$ interaction. In this case the propagators are simpler and the computation is similar to the flat space one.

## 1.3 Direct computation of the wavefunction

a) Consider the action for the scalar field $S = \frac{1}{2}\int(\nabla\phi)^2$ in dS. Think about it in the flat slicing.

b) Compute the action with fixed boundary condition at some time $\eta_c$, and $\phi_b = \phi(\eta_c)$. At early times, put the positive frequency boundary condition $\phi \to e^{i|k|\eta}$.

c) Evaluate $e^{iS_{classical}}$ for the above solution.

d) Do the same in Euclidean AdS space $ds^2 = \frac{dz^2+dx^2}{z^2}$. Again put boundary conditions and evaluate the action. Understand the relation to c)

e) Compute the derivative of the wavefunctions with respect to the boundary conditions $\frac{\delta}{\delta\phi_b(\vec{k})}\frac{\delta}{\delta\phi_b(-\vec{k})}$ $e^{iS}|_{\phi_b=0}$. Can this be interpreted as the correlation function of a conformal field theory? Could you go to position space? Are there any IR divergencies in this case?

d) Understand why the Green's function

$$\langle 0|T\phi(\eta)\phi(\eta')|BD\rangle = f^*(\eta)f(\eta') - f(\eta)f(\eta') \quad , \qquad 0 > \eta > \eta' \tag{2}$$

is the right one to use in the perturbative computation of the wavefunction. You could also try to normalize it correctly.

f)** You could attempt to write this answer in terms of proper distances in $AdS$. Namely, first go back to position space (don't forget the $k$ dependent normalization factors) and write it in terms of proper distances. Does it have a singularity at the antipodal point?. What is the corresponding function in $AdS$ space, and what is the antipotal point in this case.

### 1.4 Action for $AdS_4$

Compute the on shell action for $AdS_4$ for an $S^3$ boundary. Compute the action

$$S_{Euclidean} = \frac{R_{AdS}^2}{16\pi G_N}\left[-\int_{\Sigma_4}\sqrt{g}(R+6) - 2\int_{\partial\Sigma_4} K\right] \tag{3}$$

where $K$ is the extrinsic curvature term, $K = \frac{1}{2}h^{ab}\partial_n h_{ab}$ where $h$ is the metric of the boundary and $n$ the normal direction. Write the AdS metric as

$$ds^2 = d\rho^2 + \sinh^2\rho d\Omega_3^2 \tag{4}$$

and evaluate the action as a function of the cutoff $\rho_c$. Discard the divergent terms and focus on the finite ones. Write down the final answer for $\Psi = Z \sim e^{-S_E}$.

### 1.5 Action for $dS_4$

Do the same for $dS_4$ with the Hartle-Hawking analytic continuation. Consider the metric

$$ds^2 = -d\tau^2 + \cosh^2\tau d\Omega_3^2 \tag{5}$$

Evaluate now $\Psi \sim e^{iS}$. Compare with the answer in the previous problem.

## 2 Exercises on Inflation (Creminelli)

1. Using symmetry arguments, calculate the tilt of the spectrum of a scalar with small mass, $m^2 \ll H^2$, in a fixed de Sitter background.

2. Using symmetry arguments, show that the $n$-point function of $\zeta$ in Fourier space in a generic model of inflation is of the form

$$\langle\zeta_{\vec{k}_1}\ldots\zeta_{\vec{k}_n}\rangle = (2\pi)^3\delta(\sum\vec{k}_i)F(k_i)\ , \tag{6}$$

   where $F$ is an homogeneous function of the $k$'s of degree $-3(n-1)$.

3. Calculate the equal time 2-point function of a massless scalar in a fixed de Sitter background in real space. What is the physical meaning of the IR divergence?

4. Photons are massless, but they are not produced during inflation. Why?

5. A Goldstone boson, like an axion, with decay constant $f_a$ lives during inflation and therefore gets quantum perturbations. Estimate the corrections you expect to the free field theory calculation from self interactions, i.e. the deviation from gaussianity. What happens if $H > 4\pi f_a$?

6. Consider a massless scalar $\varphi$ in de Sitter space with an interaction $\frac{M}{6}\varphi^3$. Calculate the 3-point function $\langle \varphi_{\vec{k}_1} \varphi_{\vec{k}_2} \varphi_{\vec{k}_3} \rangle$.