The large-scale structure of the Universe

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Research over the past 25 years has led to the view that the rich tapestry of present-day cosmic structure arose during the first instants of creation, where weak ripples were imposed on the otherwise uniform and rapidly expanding primordial soup. Over 14 billion years of evolution, these ripples have been amplified to enormous proportions by gravitational forces, producing ever-growing concentrations of dark matter in which ordinary gases cool, condense and fragment to make galaxies. This process can be faithfully mimicked in large computer simulations, and tested by observations that probe the history of the Universe starting from just 400,000 years after the Big Bang.

The past two and a half decades have seen enormous advances in the study of cosmic structure, both in our knowledge of how it is manifest in the large-scale matter distribution, and in our understanding of its origin. A new generation of galaxy surveys — the 2-degree Field Galaxy Redshift Survey, or 2dFGRS[1], and the Sloan Digital Sky Survey, or SDSS[2] — have quantified the distribution of galaxies in the local Universe with a level of detail and on length scales that were unthinkable just a few years ago. Surveys of quasar absorption and of gravitational lensing have produced qualitatively new data on the distributions of diffuse intergalactic gas and of dark matter. At the same time, observations of the cosmic microwave background radiation, by showing us the Universe when it was only about 400,000 years old, have vindicated bold theoretical ideas put forward in the 1980s regarding the contents of the Universe and the mechanism that initially generated structure shortly after the Big Bang. The critical link between the early, near-uniform Universe and the rich structure seen at more recent times has been provided by direct numerical simulation. This has made use of the unremitting increase in the power of modern computers to create ever more realistic virtual universes: simulations of the growth of cosmic structure that show how astrophysical processes have produced galaxies and larger structures from the primordial soup. Together, these advances have led to the emergence of a 'standard model of cosmology' which, although seemingly implausible, has nevertheless been singularly successful.

Figure 1 strikingly illustrates how well this standard model can fit nearby structure. The observational wedge plots at the top and at the left show subregions of the SDSS and 2dFGRS, illustrating the large volume they cover in comparison to the ground-breaking Center for Astrophysics (CfA) galaxy redshift survey[3] carried out during the 1980s (the central small wedge). These slices through the local three-dimensional galaxy distribution reveal a tremendous richness of structure. Galaxies, groups and clusters are linked together in a pattern of sheets and filaments that is commonly known as the 'cosmic web'[4]. A handful of particularly prominent aggregations clearly stand out in these images, the largest containing of the order of 10,000 galaxies and extending for several hundred million light years. The corresponding wedge plots at the right and at the bottom show similarly constructed surveys of a virtual universe, the result of a simulation of the growth of structure and of the formation of galaxies in the current standard model of cosmology. The examples shown were chosen among a set of random 'mock surveys' to have large structures in similar positions to the real surveys. The similarity of structure between simulation and observation is striking, and is supported by a quantitative comparison of clustering[5]. Here we review what we can learn from this excellent match.

The early 1980s produced two audacious ideas that transformed a speculative and notoriously uncertain subject into one of the most rapidly developing branches of physics. The first was the proposal that the ubiquitous dark matter that dominates large-scale gravitational forces consists of a new (and still unidentified) weakly interacting elementary particle. Because these particles are required to have small random velocities at early times, they were dubbed 'cold dark matter' or CDM. (Hot dark matter is also possible, for example a neutrino with a mass of a few tens of electron volts. Early cosmological simulations showed, however, that the galaxy distribution in a universe dominated by such particles would not resemble that observed[6].) The second idea is 'cosmic inflation'[7], the proposal that the Universe grew exponentially for many doubling times perhaps $\sim 10^{-35}$ seconds after the Big Bang, driven by the vacuum energy density of an effective scalar field that rolls slowly from a false to the true vacuum. Quantum fluctuations in this 'inflaton' field are blown up to macroscopic scales and converted into genuine ripples in the cosmic energy density. These weak seed fluctuations grow under the influence of gravity and eventually produce galaxies and the cosmic web. Simple models of inflation predict the statistical properties of these primordial density fluctuations: their Fourier components should have random and independent phases and a near-scale-invariant power spectrum[8]. Inflation also predicts that the present Universe should have a flat geometry. With concrete proposals for the nature of the dark matter and for the initial fluctuation distribution, the growth of cosmic structure became, for the first time, a well-posed problem that could be tackled with the standard tools of physics.

The backbone of the cosmic web is the clumpy yet filamentary distribution of dark matter. The presence of dark matter was first inferred from the dynamics of galaxy clusters by Zwicky[9]. But it took over half a century for dark matter to become an integral part of our view of galaxies and of the Universe as a whole, and for its average density to be estimated reliably. Today, the evidence for the pervasive presence of dark matter is overwhelming and includes galactic rotation curves, the structure of galaxy groups and clusters, large-scale cosmic flows and, perhaps most directly, gravitational lensing, a phenomenon first proposed as an astronomical tool by Zwicky himself[10]. The distorted images of background galaxies as their light travels near mass concentrations reveal the presence of dark matter in the outer haloes of galaxies[11,12], in galaxy clusters[13] and in the general mass field[14].

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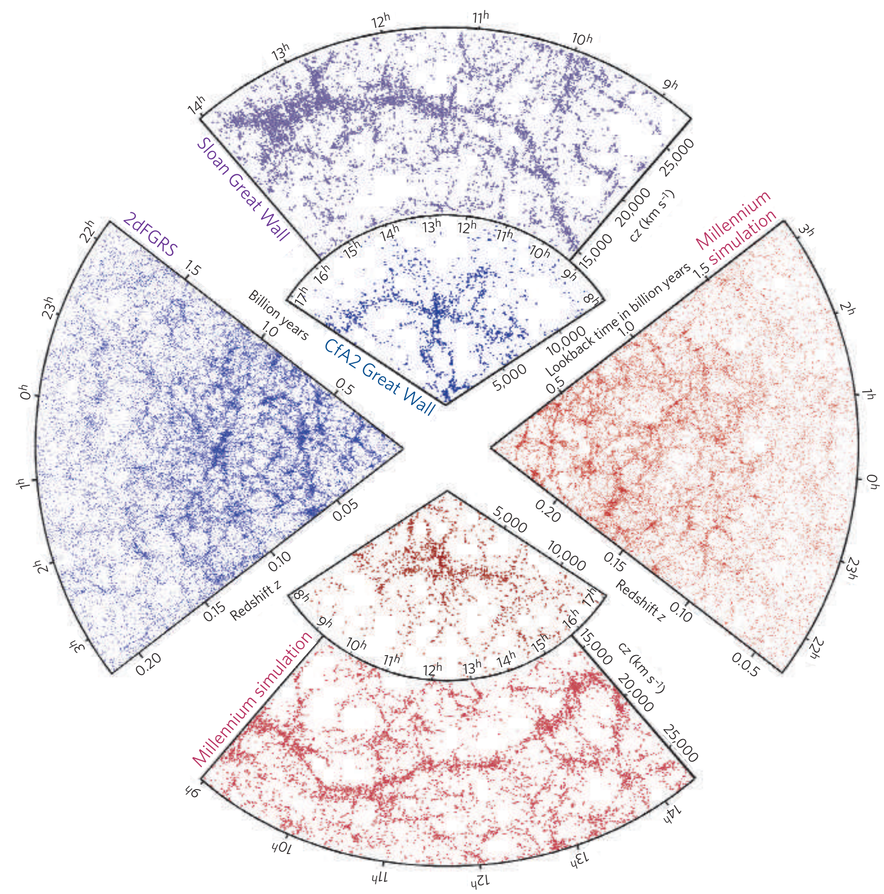

Figure 1 | The galaxy distribution obtained from spectroscopic redshift surveys and from mock catalogues constructed from cosmological simulations. The small slice at the top shows the CfA2 'Great Wall'[3], with the Coma cluster at the centre. Drawn to the same scale is a small section of the SDSS, in which an even larger 'Sloan Great Wall' has been identified[100]. This is one of the largest observed structures in the Universe, containing over 10,000 galaxies and stretching over more than 1.37 billion light years. The cone on the left shows one-half of the 2dFGRS, which determined distances to more than 220,000 galaxies in the southern sky out to a depth of 2 billion light years. The SDSS has a similar depth but a larger solid angle and currently includes over 650,000 observed redshifts in the northern sky. At the bottom and on the right, mock galaxy surveys constructed using semi-analytic techniques to simulate the formation and evolution of galaxies within the evolving dark matter distribution of the 'Millennium' simulation[5] are shown, selected with matching survey geometries and magnitude limits.

When expressed in units of the critical density required for a flat cosmic geometry, the mean density of dark matter is usually denoted by Ω_{dm}. Although a variety of dynamical tests have been used to constrain Ω_{dm}, in general such tests give ambiguous results because velocities are induced by the unseen dark matter and the relation of its distribution to that of the visible tracers of structure is uncertain. The notion of a substantial bias in the galaxy distribution relative to that of dark matter was introduced in the 1980s to account for the fact that different samples of galaxies or clusters are not directly tracing the underlying matter distribution[15–17]. Defined simply as the ratio of the clustering strengths, the 'bias function' was also invoked to reconcile low dynamical estimates for the mass-to-light ratio of clusters with the high global value required in the theoretically preferred flat, $\Omega_{dm} = 1$ universe. But because massive clusters must contain approximately the universal mix of dark matter and baryons (ordinary matter), this uncertainty is neatly bypassed by comparing the measured baryon fraction in clusters with the universal fraction under the assumption that the mean baryon density, Ω_b, is the value inferred from Big Bang nucleosynthesis[18]. Applied to the Coma cluster, this simple argument gave $\Omega_{dm} \leq 0.3$ where the inequality arises because some or all of the dark matter could be baryonic[18]. This was the first determination of $\Omega_{dm} < 1$ that could not be explained away by invoking bias. Subsequent measurements have confirmed the result[19] which also agrees with recent independent estimates based, for example, on the relatively slow evolution of the abundance of galaxy clusters[20,21] or on the detailed structure of fluctuations in the microwave background radiation[22].

The mean baryon density implied by matching Big Bang nucleosynthesis to the observed abundances of the light elements is only $\Omega_b h^2 \approx 0.02$, where h denotes the Hubble constant in units of 100 km s^{-1} Mpc^{-1}. Dynamical estimates, although subject to bias uncertainties, have long suggested that $\Omega_m = \Omega_{dm} + \Omega_b \approx 0.3$, implying that the dark matter cannot be baryonic. Plausibly it is made up of the hypothetical elementary particles postulated in the 1980s, for example axions or the lowest mass supersymmetric partner of the known particles. Such low estimates of the mean matter density Ω_m are incompatible with the flat geometry predicted by inflation unless the Universe contains an additional unclustered and dominant contribution to its energy density, for example a cosmological constant Λ such that $\Omega_m + \Omega_\Lambda \approx 1$. Two large-scale structure surveys carried out in the late 1980s, the APM (automated photographic measuring) photographic survey[23] and the QDOT redshift survey of infrared galaxies[24], showed that the power spectrum of the galaxy distribution, if it traces that of the mass on large scales, can be fitted by a simple CDM model only if the matter density is low, $\Omega_m \approx 0.3$. This independent confirmation of the dynamical arguments led many to adopt the now standard model of cosmology, ΛCDM.

It was therefore with a mixture of amazement and *déjà vu* that cosmologists greeted the discovery in 1998 of an accelerated cosmic expansion[25,26]. Two independent teams used distant type Ia supernovae to perform a classical observational test. These 'standard candles' can be observed out to redshifts beyond 1. Those at $z \geq 0.5$ are fainter than expected, apparently indicating that the cosmic expansion is currently speeding up. Within the standard Friedmann cosmology, there is only one agent that can produce an accelerating expansion: the cosmological constant first introduced by Einstein, or its possibly time- or space-dependent generalization, 'dark energy'. The supernova evidence is consistent with $\Omega_\Lambda \approx 0.7$, just the value required for the flat universe predicted by inflation.

The other key prediction of inflation, a density fluctuation field consistent with amplified quantum noise, received empirical support from the discovery by the COsmic Background Explorer (COBE) satellite in 1992 of small fluctuations in the temperature of the cosmic microwave background (CMB) radiation[27]. These reflect primordial density fluctuations, modified by damping processes in the early Universe which depend on the matter and radiation content of the Universe. More recent measurements of the CMB[28–32] culminating with those by the WMAP (Wilkinson Microwave Anisotropy Probe) satellite[22] have provided a striking confirmation of the inflationary CDM model: the measured temperature fluctuation spectrum is nearly scale-invariant on large scales and has a series of 'acoustic' peaks that reflect the coherent oscillations experienced by the photon–baryon fluid before the moment when the primordial plasma recombined and the radiation escaped. The fluctuation spectrum depends on the parameters that define the geometry and content of the Universe and the initial fluctuation distribution, so their values are constrained by the data. In practice, there are degeneracies among the parameters, and the strongest constraints come from combining the CMB data with other large-scale structure datasets. Present estimates[22,33–36] give a flat universe with $\Omega_{dm} = 0.20 \pm 0.020$, $\Omega_b = 0.042 \pm 0.002$, $\Omega_\Lambda = 0.76 \pm 0.020$, $h = 0.74 \pm 0.02$. The consistency of these values with other independent determinations and the close agreement of the CMB data with theoretical predictions formulated over 20 years earlier[37] belong amongst the most remarkable successes of modern cosmology.

The growth of large-scale structure

The microwave background radiation provides a clear picture of the young Universe, where weak ripples on an otherwise uniform sea display a pattern that convincingly supports our standard model for the cosmic mass/energy budget and for the process that initially imprinted cosmic structure. At that time there were no planets, no stars, no galaxies, none of the striking large-scale structures seen in Fig. 1. The richness of the observed astronomical world grew later in a complex and highly nonlinear process driven primarily by gravity. This evolution can be followed in detail only by direct numerical simulation. Early simulations were able to reproduce qualitatively the structure observed both in large galaxy surveys and in the intergalactic medium[16,38]. They motivated the widespread adoption of the CDM model well before it gained support from microwave background observations. Many physical processes affect galaxy formation, however, and many aspects must be treated schematically within even the largest simulations. The resulting uncertainties are best estimated by exploring a wide range of plausible descriptions and checking results against observations of many different types. The main contribution of early CDM galaxy formation modelling was perhaps the dethroning of the 'island universe' or 'monolithic collapse' paradigm and the realization that galaxy formation is a process extending from early times to the present day, rather than an event that occurred in the distant past[39].

In a ΛCDM universe, quasi-equilibrium dark matter clumps or 'haloes' grow by the collapse and hierarchical aggregation of ever more massive systems, a process described surprisingly well by the phenomenological model of Press and Schechter and its extensions[40,41]. Galaxies form at the centres of these dark haloes by the cooling and condensation of gas which fragments into stars once it becomes sufficiently dense[42]. Groups and clusters of galaxies form as haloes aggregate into larger systems. They are arranged in the 'cosmic web', the larger-scale pattern of filaments and sheets which is a nonlinear gravitational 'sharpening' of the pattern already present in the gaussian random field of initial fluctuations[4]. The first observable objects were probably massive stars collapsing in small haloes and switching on at redshifts of 50 and higher[43]. By a redshift of 15 these may have been sufficiently numerous for their radiation to re-ionize all the gas in the Universe[44]. So far they have not been observed directly, but it is one of the main goals of the next generation of low-frequency radio telescopes to observe their effects directly in the strongly redshifted 21-cm transition of neutral hydrogen.

Detailed simulations from ΛCDM initial conditions have been used to study the formation of the first luminous objects and the re-ionization of the Universe, but these still await testing against observation[44,45]. In contrast, predictions for the structure, the ionization state and the heavy element content of intergalactic gas at redshifts below 6 can be checked in detail against absorption features observed in the spectra of distant quasars which provide, in effect, a one-dimensional topographic image of the intervening large-scale structure.

As an example, Fig. 2 shows a typical high-resolution spectrum of a distant quasar at redshift $z = 3.26$. At shorter wavelengths than the Lyman α emission line of the quasar, there is a 'forest' of absorption lines of differing strength. The modern interpretation is that these features arise from Lyman α absorption by the smoothly varying distribution of foreground intergalactic hydrogen, in effect from the filaments, sheets and haloes of cosmic structure. It was a conceptual breakthrough, and an important success for the CDM paradigm, when hydrodynamical simulations showed that this interpretation could explain in detail the observed statistics of the absorption lines[38,46]. Considerable recent advances both in the quality and in the quantity of data available have made it possible to measure a variety of statistics for the Lyman α forest as a function of redshift to high precision[47–49]. Comparing with appropriately designed numerical simulations has provided strong confirmation of the underlying paradigm at a level that is remarkable, given the evidence that intergalactic gas is contaminated with galaxy ejecta in a way that the simulations do not yet adequately reproduce[36,50–52]. This approach has also helped to strengthen constraints on the paradigm's parameters, in particular on the spectrum of fluctuations produced by inflation and on the masses of neutrinos.

At lower redshift direct and quantitative measures of large-scale structure can be obtained from the weak, coherent distortions of the images of faint galaxies induced by gravitational lensing as their light travels through the intervening cosmic web[53]. The distortions depend only on the gravitational field in intergalactic space and so lensing data test predictions for the mass distribution in a way that is almost independent of the complex astrophysics that determines the observable properties of galaxies. The lensing effect is very weak, but can be measured statistically to high precision with large enough galaxy samples.

As an example, Fig. 3 shows a measure of the mean square coherent distortion of distant galaxy images within randomly placed circles on the

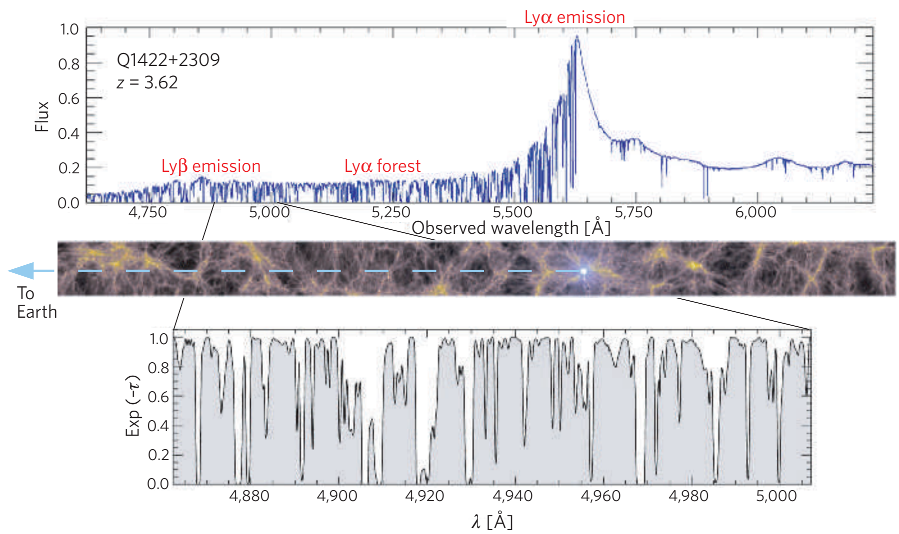

Figure 2 | The Lyman α forest as a probe of large-scale structure. The panel on the top shows a typical high-resolution spectrum of a quasar at redshift $z = 3.62$. Shortward of the redshifted Lyman α emission line at $1216(1 + z)$ Å, the spectrum shows a 'forest' of absorption lines of different strength produced by intervening neutral hydrogen gas along the line-of-sight from the quasar to the Earth. Hydrodynamical simulations reproduce the observed absorption spectra with remarkable fidelity, as illustrated by the simulated spectrum in the bottom panel, corresponding to intervening large-scale structure at $z \approx 3$. The sketch in the middle panel shows an example of the gas distribution in a simulated ΛCDM model.

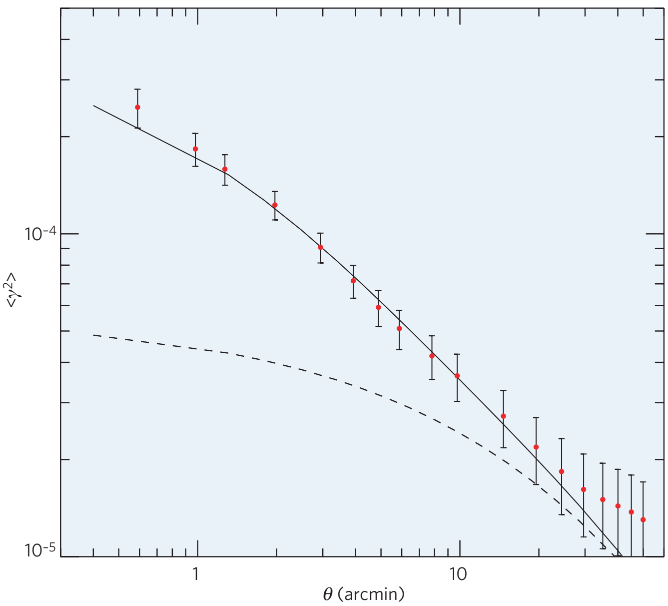

Figure 3 | Variance of the weak lensing shear as a function of top-hat smoothing scale. The data points show recent measurements from the VIRMOS survey[54]. The solid line gives the predicted signal for the nonlinear mass distribution in the standard ΛCDM model (normalized so that the linear mass overdensity in spheres of radius 8 h^{-1} Mpc is $\sigma_8 = 0.84$), and the dashed line shows a linear extrapolation based on the structure present at early times. Because the weak lensing shear depends sensitively on the nonlinear clustering of the total mass distribution, it provides a particularly powerful probe of cosmology. Figure courtesy of Ludo van Waerbeke.

sky as a function of the radius of those circles[54]. Clearly, the distortion is detected with very high significance. The two curves show the predicted signal in the standard ΛCDM model based on (i) detailed simulations of the growth of structure in the dark matter distribution, and (ii) a simple linear extrapolation from the structure present at early times. Nonlinear effects are strong because the distortions are dominated by the gravity of individual dark matter haloes. Meaningful comparison between theory and observation thus requires high-precision large-scale structure simulations, and generating these constitutes a great numerical challenge. Similar lensing measurements, but now within circles centred on observed galaxies (rather than random points), can be used to determine the average total mass surrounding galaxies as a function of radius, redshift and galaxy properties[55]. This wealth of information can only be interpreted by simulations that follow both the dark matter distribution and the formation and evolution of the galaxy population.

The Lyman α forest and gravitational lensing thus provide windows onto the large-scale structure of the Universe that complement those obtained from galaxy surveys by extending the accessible redshift range and, more importantly, by measuring the structure in the diffuse gas and in the total mass distribution rather than in the distribution of galaxies. In principle, these measures should have different (and perhaps weaker) sensitivity to the many uncertain aspects of how galaxies form. Remarkably, all three measures are consistent both with each other and with the standard model at the level that quantitative comparison is currently possible[36,54,56].

Galaxy surveys such as those illustrated in Fig. 1 contain an enormous amount of information about large-scale structure. The strength of clustering is known to depend not only on galaxy luminosity, colour, morphology, gas content, star-formation activity, type and strength of nuclear activity and halo mass, but also on the spatial scale considered and on redshift. Such dependences reflect relations between the formation histories of galaxies and their larger-scale environment. Some (for example, the dependence on halo or galaxy mass) are best thought of as deriving from the statistics of the initial conditions. Others (for example the dependence on nuclear or star-formation activity) seem more naturally associated with late-time environmental influences. Early studies attempted to describe the relation between the galaxy and mass distributions by a bias function. Recent data suggest that this concept is of limited value, except, perhaps, on the largest scales; bias estimates depend not only on scale, redshift and galaxy properties, but also on the particular measure of clustering studied. Understanding the link between the mass and galaxy distributions requires realistic simulations of the galaxy formation process throughout large and representative regions of the Universe. Given the complexity of galaxy formation, such simulations must be tuned 'by hand' to match as many of the observed properties of galaxies as possible. Only if clustering turns out to be insensitive to such tuning can we consider the portrayal of large-scale structure to be robust and realistic.

In Fig. 4, we show the time evolution of the mass and galaxy distributions in a small subregion of the largest simulation of this type yet[5]. The emergence of the cosmic web can be followed in stunning detail, producing a tight network of filaments and walls surrounding a foam of voids. This characteristic morphology was seen in the first generation of cold dark matter simulations carried out over 20 years ago[16], but the match was not perfect; the recipe adopted to relate the galaxy and mass distributions was too crude to reproduce in detail the clustering of galaxies. It has taken models like those of Fig. 4 to explain why the observed galaxy autocorrelation function is close to a power law whereas the simulated dark matter autocorrelation function shows significant features[5,57].

Simulated autocorrelation functions for dark matter and for galaxies are shown in Fig. 5 for the same times imaged in Fig. 4. The shape difference between the two is very evident, and it is remarkable that at $z = 0$ the power-law behaviour of the galaxy correlations extends all the way down to 10 kpc, the observed size of galaxies. Similar behaviour has recently been found for luminous red galaxies in the Sloan Digital Sky Survey[58]. The galaxy distribution in this simulation also reproduces the observed dependence of present-day clustering on luminosity and colour[5] as well as the observed galaxy luminosity functions, the observationally inferred formation histories of elliptical galaxies, and the bimodal colour-magnitude distribution observed for galaxies[59,60].

A striking feature of Fig. 4 is the fact that while the growth of large-scale structure is very clear in the mass distribution, the galaxy distributions appear strongly clustered at all times. This difference shows up dramatically in the autocorrelation functions plotted in Fig. 5 and has been a prediction of CDM theories since the first simulations including crude bias recipes[16]. A decade later when direct measurements of galaxy clustering at redshifts as high as $z \approx 3$–4 found "surprisingly" large amplitudes, comparable to those found in the present-day Universe[61,62], the results turned out to be in good agreement with estimates based on more detailed modelling of galaxy formation in a CDM universe[63,64]. In effect, the galaxies already outline the pattern of the cosmic web at early times, and this pattern changes relatively little with the growth of structure in the underlying dark matter distribution.

Could the standard model be wrong?

Given the broad success of the ΛCDM model, is it conceivable that it might be wrong in a significant way requiring a fundamental revision? The concordance of experimental results relying on a variety of physical effects and observed over a wide range of cosmic epochs suggests that this is unlikely. Nevertheless, it is clear that some of the most fundamental questions of cosmology (what is the dark matter? the dark energy?) remain unanswered. In addition, some of the key observational underpinnings of the model still carry worrying uncertainties. Can we use our ever-improving measurements of large-scale structure to carry out critical tests?

Perhaps the deepest reason to be suspicious of the paradigm is the apparent presence of a dark energy field that contributes ~70% of the Universe's content and has, for the past 5 billion years or so, driven an accelerated cosmic expansion. Dark energy is problematic from a field theoretical point of view[65]. The simplest scenario would ascribe a vacuum energy to quantum loop corrections at the Planck scale, hc^5/G, which is of the order of 10^{19} GeV, where gravity should unify with the other fundamental forces. This is more than 120 orders of magnitude

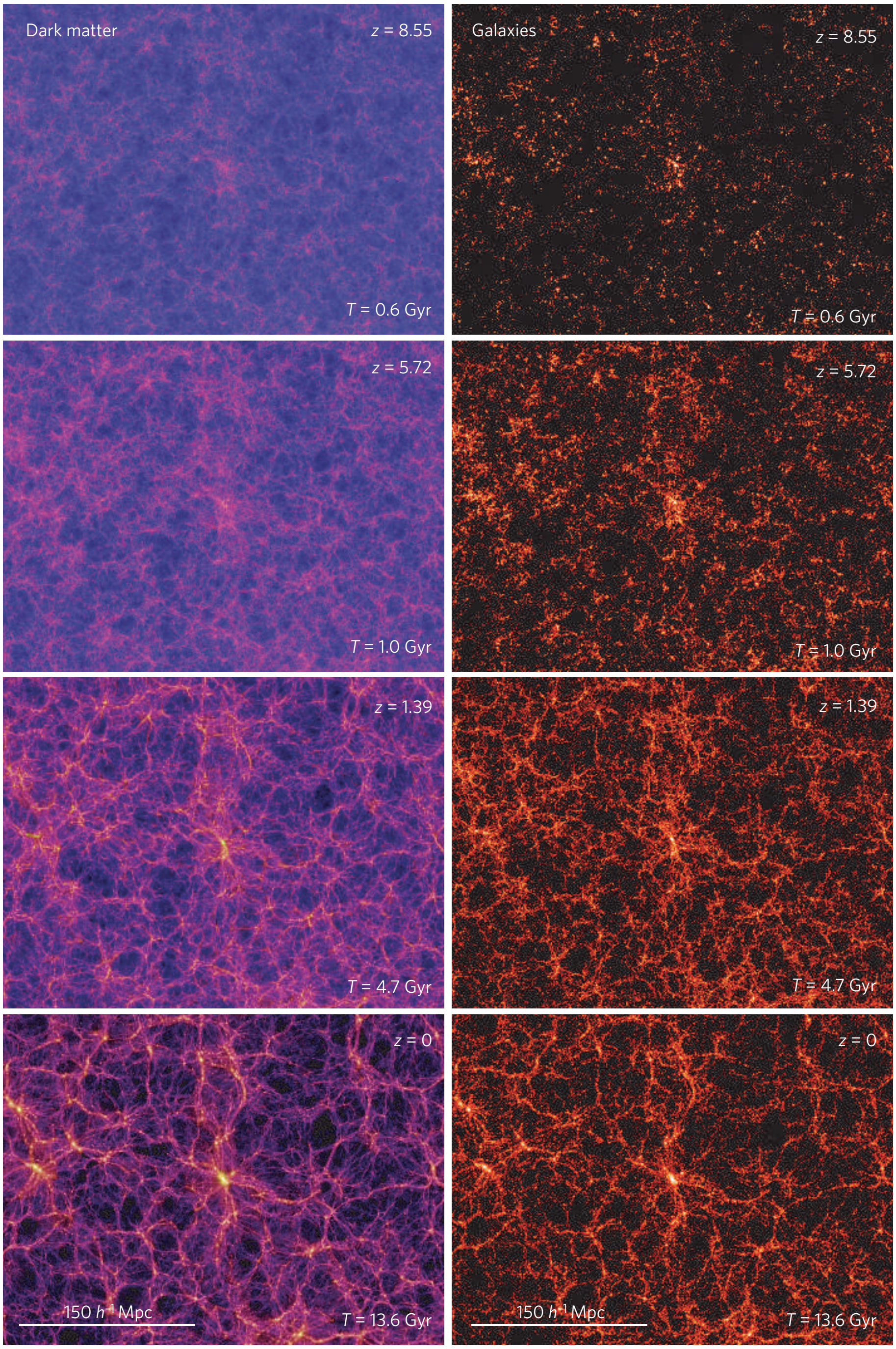

Figure 4 | Time evolution of the cosmic large-scale structure in dark matter and galaxies, obtained from cosmological simulations of the *Λ*CDM model. The panels on the left show the projected dark matter distribution in slices of thickness 15 h^{-1} Mpc, extracted at redshifts $z = 8.55$, $z = 5.72$, $z = 1.39$ and $z = 0$ from the Millennium *N*-body simulation of structure formation[5]. These epochs correspond to times of 600 million, 1 billion, 4.7 billion and 13.6 billion years after the Big Bang, respectively. The colour hue from blue to red encodes the local velocity dispersion in the dark matter, and the brightness of each pixel is a logarithmic measure of the projected density. The panels on the right show the predicted distribution of galaxies in the same region at the corresponding times obtained by applying semi-analytic techniques to simulate galaxy formation in the Millennium simulation[5]. Each galaxy is weighted by its stellar mass, and the colour scale of the images is proportional to the logarithm of the projected total stellar mass. The dark matter evolves from a smooth, nearly uniform distribution into a highly clustered state, quite unlike the galaxies, which are strongly clustered from the start.

larger than the value required by cosmology. Postulating instead a connection to the energy scale of quantum chromodynamics would still leave a discrepancy of some 40 orders of magnitude. A cosmological dark energy field that is so unnaturally small compared with these particle physics scales is a profound mystery.

The evidence for an accelerating universe provided by type Ia supernovae relies on a purely phenomenological calibration of the relation between the peak luminosity and the shape of the light curve. It is this that lets these supernovae be used as an accurate standard candle. Yet this relation is not at all understood theoretically. Modern simulations of thermonuclear explosions of white dwarfs suggest that the peak luminosity should depend on the metallicity of the progenitor star[66,67]. This could, in principle, introduce redshift-dependent systematic effects, which are not well constrained at present. Perhaps of equal concern is the observation that the decline rate of type Ia supernovae correlates with host galaxy type[68,69], in the sense that the more luminous supernovae (which decline more slowly) are preferentially found in spiral galaxies.

Interestingly, it has also been pointed out that without the evidence for accelerated expansion from type Ia supernovae, a critical density Einstein–de Sitter universe can give a good account of observations of large-scale structure provided the assumption of a single power law for the initial inflationary fluctuation spectrum is dropped, a small amount of hot dark matter is added, and the Hubble parameter is dropped to the perhaps implausibly low value $h \approx 0.45$ (ref. 70).

The CMB temperature measurements provide particularly compelling support for the paradigm. The WMAP temperature maps do, however, show puzzling anomalies that are not expected from gaussian fluctuations[71–73], as well as large-scale asymmetries that are equally unexpected in an isotropic and homogeneous space[74,75]. Although these signals could perhaps originate from foregrounds or residual systematics, it is curious that the anomalies seem well matched by anisotropic Bianchi cosmological models, although the models examined so far require unacceptable cosmological parameter values[76]. Further data releases from WMAP and future CMB missions such as PLANCK will shed light on these peculiarities of the current datasets. Perhaps the anomalous effects will go away; or they could be the first signs that the standard model needs substantial revision.

The unknown nature of the dark matter is another source of concern. Is the dark matter really 'cold' and non-interacting, and is it really dark? Does it exist at all? Until the posited elementary particles are discovered, we will not have definitive answers to these questions. Already there are hints of more complicated possibilities. It has been suggested, for instance, that the γ-ray excess flux recently detected in the direction of the Galactic Centre[77] might be due to self-annihilating dark matter particles[78], an idea that is, in principle, plausible for a range of dark matter candidates in supersymmetric field theories. Alternative theories of gravity, most notably modified newtonian dynamics (MOND)[79] have been proposed to do away with the need for dark matter altogether. Although MOND can explain the rotation curves of galaxies, on other scales the theory does not seem to fare so well. For example, although it can account for the total mass in galaxy clusters, MOND requires the presence of large amounts of unseen material within the central few kiloparsecs of the cluster cores[80]. It has yet to be demonstrated convincingly that MOND can reproduce observed large-scale structure starting from the initial conditions imaged in the CMB and so pass the test illustrated in Fig. 1.

At present the strongest challenge to ΛCDM arises not from large-scale structure, but from the small-scale structure within individual galaxies. It is a real possibility that the model could be falsified by measurements of the distribution and kinematics of matter within galaxies, and some astronomers argue that this has, in fact, already happened. The internal structure of dark matter haloes predicted by the ΛCDM model can be calculated quite precisely from high-resolution simulations. These predict the survival of a large number of self-bound substructures which orbit within haloes[81,82], as well as a universal halo density profile which is cusped in the middle, corresponding to a steeply rising rotation curve[83]. Unfortunately, the effects of galaxy formation within a dark matter halo are difficult to calculate, accounting, in part, for the lively debate that continues to rage over whether the measured rotation curves of dwarf and low surface brightness galaxies are in conflict with the theory[84,85]. The second contentious issue on galaxy scales, the small number of observed satellites, may have been resolved by identifying astrophysical processes that could have rendered most of the surviving subhaloes invisible[86,87]. Gravitational lensing measurements may offer a test of this explanation[88]. Lensing also allows independent determinations of halo density profiles, a method that has in fact led to new challenges for ΛCDM. Recent results on cluster scales favour steeper inner mass profiles than expected, but the significance of this discrepancy is unclear because of uncertainties originating in halo triaxiality and projection effects[89].

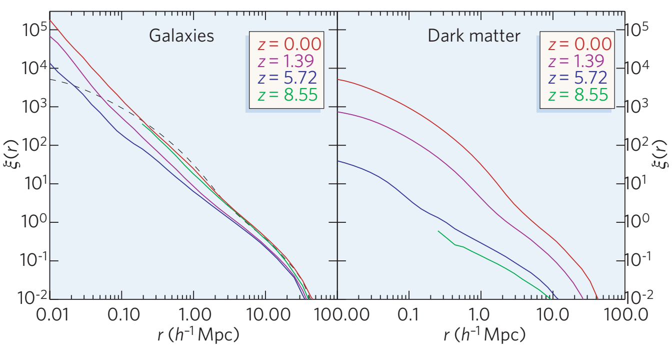

Figure 5 | Two-point correlation function of galaxies and dark matter at different epochs, in the Millennium simulation of structure formation[5]. The panel on the left gives the I-band galaxy correlation function ξ (selected according to $M_I - 5\log h < -20$ in the rest-frame) at redshifts $z = 8.55$, $z = 5.72$, $z = 1.39$ and $z = 0$ (corresponding to the epochs depicted in Fig. 4). The panel on the right shows the dark matter correlation functions at the same epochs. For comparison, the present-day dark matter correlation function is also drawn as a dashed line in the left panel. At $z = 8.55$, only data for $r > 200\ h^{-1}$ kpc are shown because the finite numerical resolution of the simulation precludes an accurate representation of the mass distribution on smaller scales than this at early times. The galaxy correlation function has a near power-law behaviour over several orders of magnitude and has almost equal strength at $z = 8.55$ and $z = 0$. By contrast, the dark matter correlation function grows by a large factor over this time span, and has a different shape from the galaxy correlation function.

Future tests of large-scale structure and cosmology

Very few of the important questions in cosmology and large-scale structure can be regarded as closed. The recent history of the subject provides a vivid reminder of how new theoretical insights and/or new observational datasets can quickly overturn conventional wisdom in rapidly advancing fields of science. At the present time, the two outstanding questions are the identity of the dark matter and the nature of the dark energy.

There is every reason to be optimistic about the prospects of detecting cold dark matter particles from the halo of our Galaxy, either directly in laboratory searches or indirectly through particle annihilation radiation. Additionally, if cold dark matter is indeed a supersymmetric particle, evidence for its existence may be forthcoming from experiments at CERN's large-hadron collider[90].

Unravelling the nature of the dark energy is a much more daunting task. A strategy that has gained momentum in recent years is to set tighter empirical constraints on the amount of dark energy and on its possible time evolution. Large projects such as the Joint Dark Energy Mission, currently at an early design phase by NASA, are being planned to measure the equation of state parameter, $w = P/(\rho c^2)$, of the dark energy, where P is the 'dark pressure' of the vacuum, and its time evolution, $w' = \mathrm{d}w/\mathrm{d}z$. The hope is that such empirical constraints will clarify the nature of the dark energy and perhaps point to a field-theoretical explanation. The range of possibilities is large. We might find that the dark energy interacts with the dark matter, or that the dark energy is not a field at all but rather a manifestation of some nonlinear effect within general relativity or one of its extensions.

Progress towards constraining dark energy is likely to come both from refinements of classical cosmological probes and from entirely new ways to study large-scale structure. Examples in the first category include measuring the abundance of galaxy clusters as a function of cosmic time. This probes the growth of the mass fluctuation spectrum and the variation of the cosmological volume element[91]. Extending such measurements to redshifts $z \geq 1$ may set useful constraints on the dark energy equation of state, provided systematic effects can be kept under control. Also promising are observations of high-redshift type Ia supernovae for much larger samples than have been accumulated so far. Again, it will be crucial to control systematic effects. The PLANCK satellite mission and subsequent polarization-optimized experiments will make definitive measurements of the CMB and perhaps unlock some of its last secrets.

Examples of new tests of the large-scale structure include weak lensing tomography and the study of baryon oscillations in the matter distribution at late times. The physical mechanism that generated acoustic peaks in the CMB temperature power spectrum also imprinted an oscillatory feature in the linear power spectrum of the dark matter[92]. The Virgo consortium's Millennium simulation, illustrated in Fig. 1 and Fig. 4, demonstrated that the oscillations survive the destructive influence of nonlinear gravitational evolution even to the present day, albeit in distorted form[5]. Most importantly, this simulation also demonstrated that these 'baryon wiggles' should be visible in suitably selected galaxy samples. Early indications suggest that the baryon oscillations in the galaxy distribution have, in fact, been detected in the 2dFGRS and SDSS[93–95], although at comparatively low statistical significance.

A recent study using Virgo's earlier Hubble volume simulations showed that the baryon wiggles should also be detectable in galaxy cluster samples[96]. The length scale of the wiggles is a 'standard ruler' which, when observed at different redshifts, constrains the geometry and expansion history of the Universe and thus the dark energy equation of state. An example of what may be possible in the future is illustrated

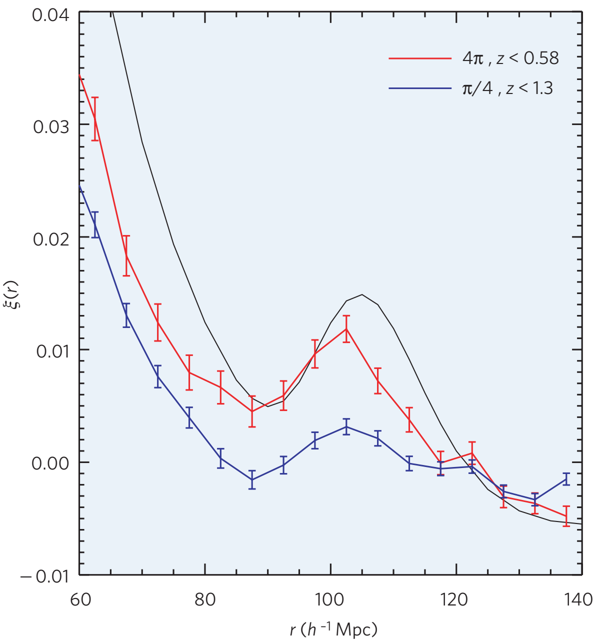

Figure 6 | The large-scale autocorrelation function of rich clusters. The two curves give the correlation function of clusters with X-ray temperature $kT > 5$ keV in light-cones constructed from the Hubble volume ΛCDM simulation[101]. The red line shows results for 124,000 clusters in a spherical light-cone out to $z = 0.58$, and the blue line shows results for 190,000 clusters in a light-cone of opening angle $\pi/2$ extending out to $z = 1.3$. The error bars are Poisson errors. The black line shows the results of linear theory scaled by the bias appropriate for the $z = 0.58$ sample. Nonlinear effects are responsible for the slight displacement of the position of the bump in the simulations relative to the position given by linear theory. Figure courtesy of Raul Angulo.

in Fig. 6, which shows the autocorrelation function of galaxy clusters in light-cones constructed from the Hubble volume ΛCDM simulation. The bump visible at a separation of ~100 h^{-1} Mpc is the baryon feature that translates into a series of peaks when Fourier-transformed to give the power spectrum. New generations of galaxy and cluster surveys will target these oscillations and use them to constrain the evolution of dark energy.

In the more distant future, there are hopes that one day we will be able to probe the inflationary epoch directly by detecting the predicted background of gravitational waves[97,98]. Not only would this provide strong evidence that inflation really happened but it would also rule out certain cosmological models inspired by string theory in which the collision of branes leads to the formation of our Universe. These predict a very weak gravitational wave background[99].

In the meantime, astrophysical studies of large-scale structure will continue to grow and to diversify, focusing on new issues such as the nature and evolution of nonlinear structure during the first billion years where we currently have no direct observations. No doubt new observations will continue to surprise us. Today, through the joint mysteries of dark matter and dark energy, cosmology arguably poses some of the most fundamental and exciting challenges of contemporary science. ■

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