Stars + Galaxies: Back of the Envelope Properties

David Spergel

Free-fall time

$$\ddot{r} = -\frac{GM}{r^2} \tag{1}$$

$$\frac{r}{t^2} = -\frac{GM}{r^2} \tag{2}$$

$$t_{free-fall} \simeq \frac{r^3}{GM} \simeq \frac{1}{\sqrt{G\rho}} \tag{3}$$

Free-fall time for neutron star is milliseconds (characteristic timescale for gravitational waves)

Free-fall time for the Sun is 10^3s (characteristic timescale for gravitational waves)

Characteristic time for universe = Hubble Time

Cosmology

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho$$

$$t^{-2} \propto G\rho$$

Kelvin-Helmholtz Time

- Time scale to radiate gravitational energy

$$U = GM^2/R$$

$$t = GM^2/RL$$

30 million years for the Sun

Timescale for proto-star evolution

Einstein Time

- Time scale to radiate gravitational energy

$$U = Mc^2$$

$$t = Mc^2/L$$

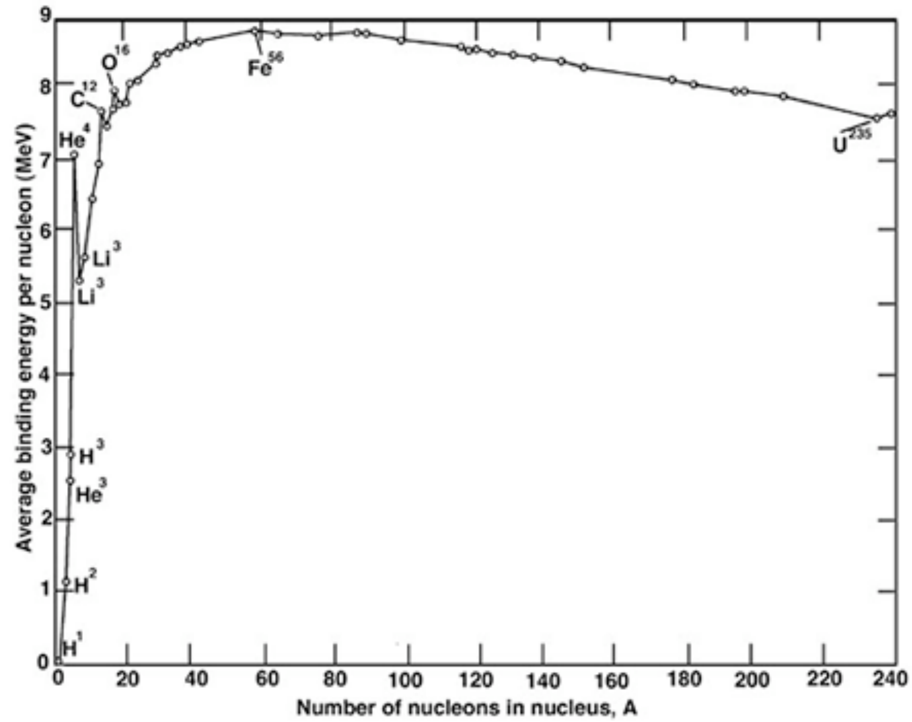
10^{13} years for the Sun

Nuclear Energy Timescale

$$U = \varepsilon \; Mc^2$$

$$t = \varepsilon \; Mc^2/L$$

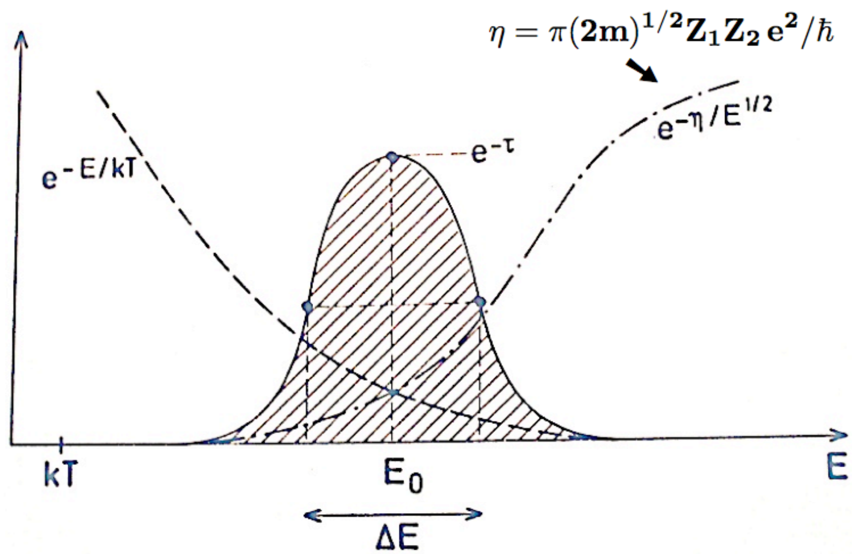
- Helium burning is 7 MeV nucleon
- Sun doesn't use all of the available fuel
- Lifetime ~ 10^{10} years



Nuclear Burning

$$R = \int f(v)\sigma(v)dv$$

$$\propto \int v^2 \exp\left(-\frac{m_n v^2}{2kT}\right)\frac{S(E)}{E}\exp\left(\frac{Z_1 Z_2 e^2}{hv}\right)$$



Stellar Structure

- Hydrostatic Equilibrium:
- Mass Conservation:
- Thermal Conduction:
- Equation of State:
- Energy Production:

Hydrostatic Equilibrium

$$\frac{dp}{dR} = -\frac{GM(r)}{R^2}\rho$$

Using $\bar{\rho} \simeq M_*/R_*^3$, this implies

$$\frac{p_*}{R_*} = \frac{GM_*}{R_*^2}M_*R_*^3 = \frac{GM_*^2}{R_*^5}$$

Using ideal gas law, $p = \rho kT/\mu$,

$$kT = \frac{GM\mu}{R}$$

Mass-Luminosity Relation

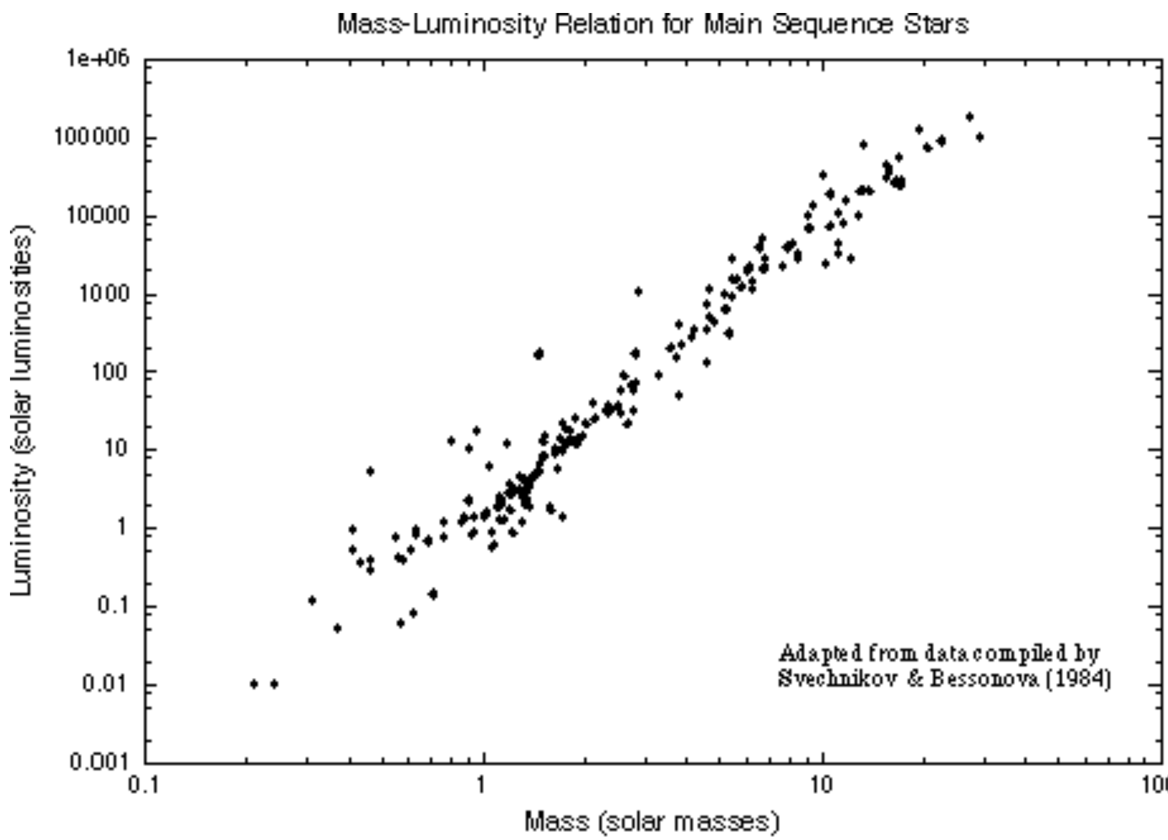
$$\frac{dT}{dR} = -\frac{l(r)}{4\pi r^2}\frac{3}{16}\frac{\kappa\rho}{\sigma_B T^3}$$

Using $\bar{\rho} \simeq M_*/R_*^3$, this implies

$$L = R^2 \frac{T_*^3}{\rho}\frac{T_*}{R_*}$$

$$L \propto M_*^3$$

Mass Luminosity Relation



Stellar Lifetimes

$$t = \frac{\epsilon M c^2}{L}$$

$$t \propto M^{-2}$$

Massive Stars live short brilliant lives!

Stellar Populations

Mass function:

$$\frac{dn}{dM} \propto M^{-2.35}$$

The lowest mass stars dominate the mass of a stellar population

$$L(t) = \int_0^{M_{max}(t)} M^{-2.35} L(M) dM = M_{max}^{2.15}(t) \propto t^{-1.07}$$

The most massive stars dominate the luminosity of a population

Radii and Temperature

$$R \propto M^{0.9}$$

$$L \propto T^4 R^2$$

$$T \propto \frac{L^{0.25}}{R^{0.5}} \propto M^{0.4}$$

Spiral Arms

Later Stages of Stellar Evolution

- Red Giant Branch (RGB)
  - Degenerate Core of Helium
  - Envelope burning Hydrogen
- Helium Flash
- Horizontal Branch
  - Core burning of Helium to Carbon
- Asymptotic Giant Branch (AGB)
  - Degenerate Core of Carbon
  - Envelope burning Helium

Fuel Consumption Thereom

The contribution by any Post Main Sequence evolutionary phase to the total luminosity of a simple stellar population is proportional to the amount of nuclear fuel burned in that phase

$$\frac{t_{HB}}{t_{MS}} = \frac{L_{MS}}{L_{HB}}\frac{U_{HB}}{U_{MS}} \simeq \frac{L_{MS}}{L_{HB}}\frac{E_{He\rightarrow C}}{E_{H\rightarrow He}}$$

Degeneracy Pressure

- As a star burns H -> He, it leaves behind a degenerate gas supported by electron degeneracy pressure
- Nuclear burning cycles are alternated by period of rapid gravitational collapse
- Chandrasekhar Mass (maximum mass supported by degeneracy pressure) (followed by flashes)

Chandrasekhar Mass

$$E_G = -\frac{GM^2}{R}$$

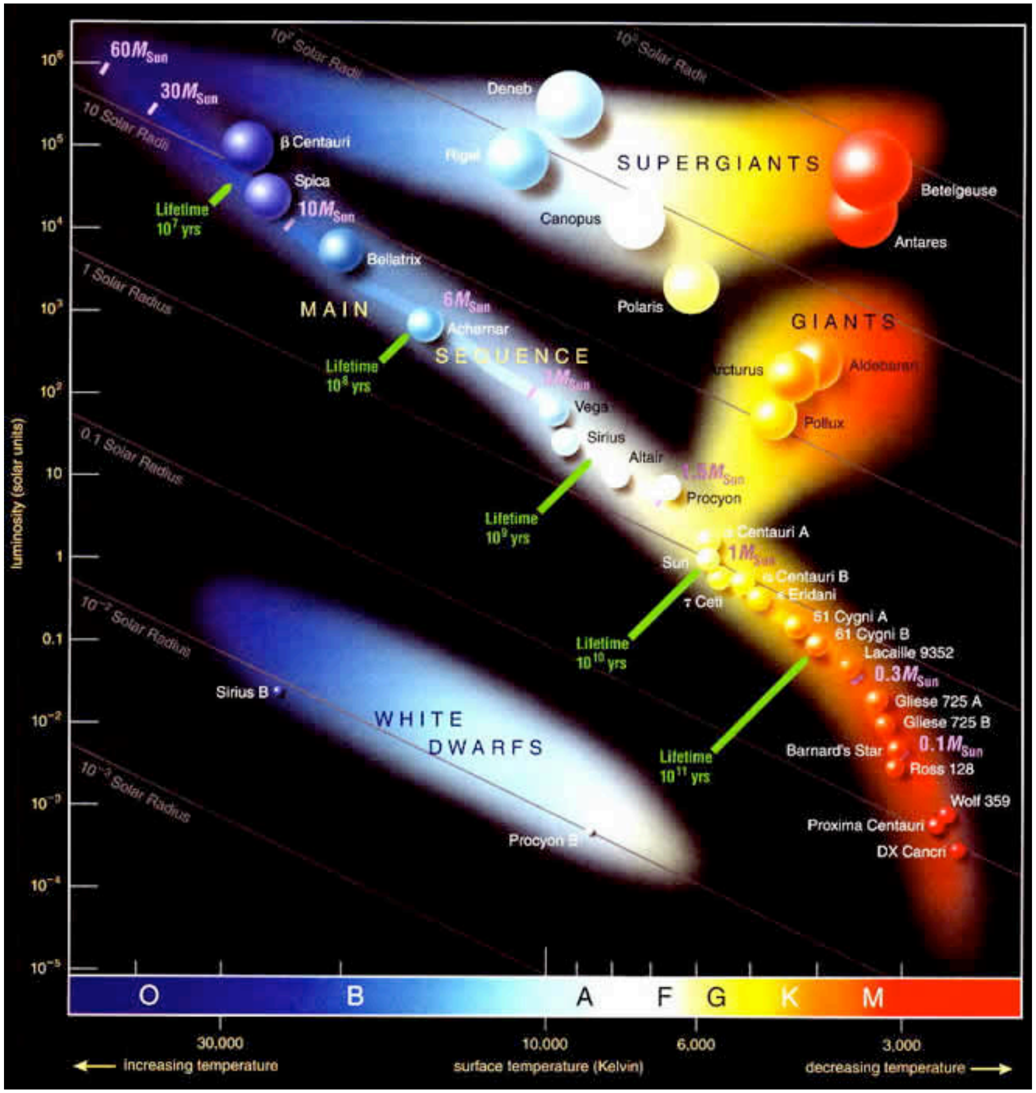
When the electrons become relativistic, their total Fermi energy is approximately,

$$E_F = Ncp_F = Nc\left(\frac{\hbar}{\Delta x}\right) = \frac{N^{4/3}\hbar c}{R} = \frac{M^{4/3}\hbar c}{m_p^{4/3}R}$$

Equating the two:

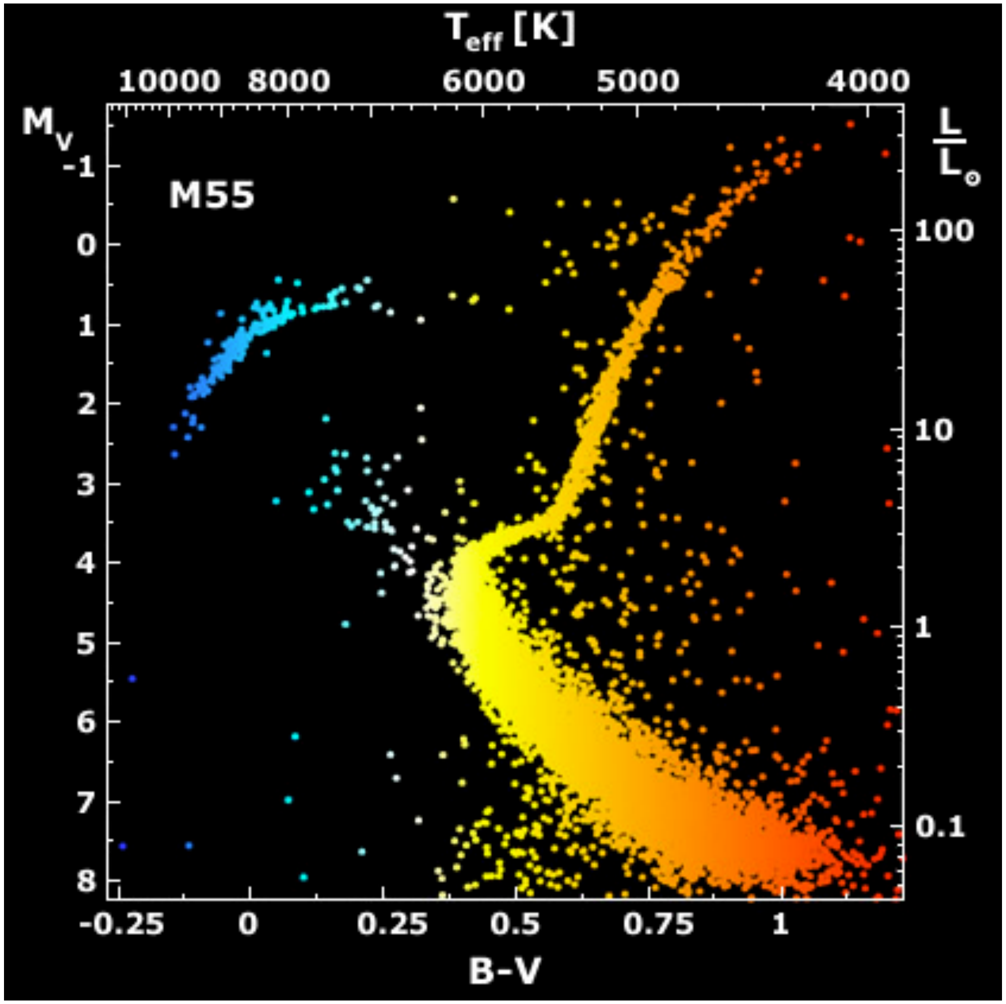
$$M_{ch} = \left(\frac{\hbar c}{G}\right)^{3/2}\frac{1}{m_p^2} = \frac{M_{Pl}^3}{m_p^2}$$

HR Diagram

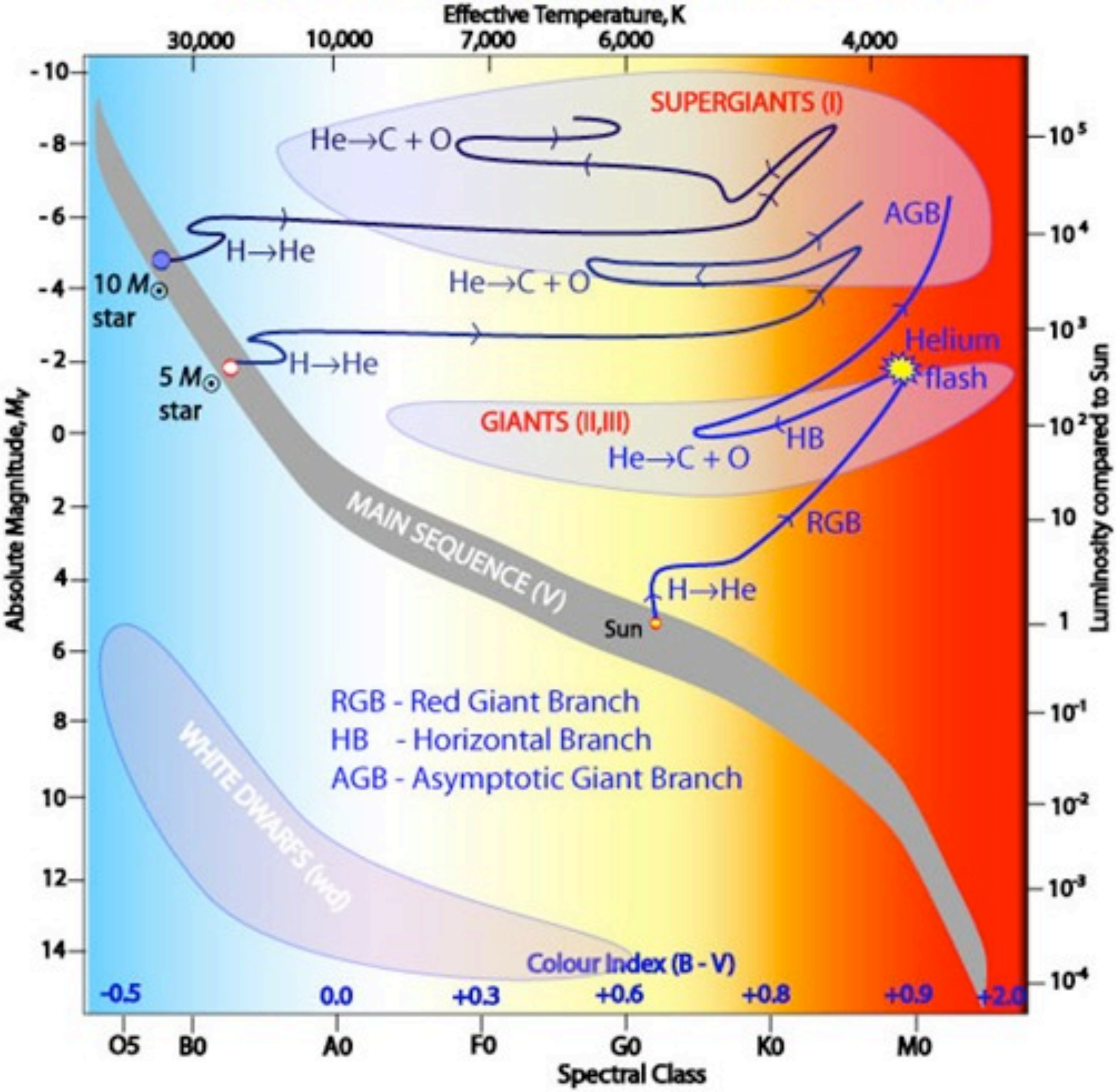


SUPERGIANTS
GIANTS
MAIN
SEQUENCE
WHITE
DWARFS
Deneb
Rigel
β Centauri
Spica
Bellatrix
Achernar
Canopus
Polaris
Betelgeuse
Antares
Arcturus
Aldebaran
Pollux
Vega
Sirius
Altair
Procyon
Sun
τ Ceti
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Sirius B
Procyon B
Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^10 yrs
Lifetime 10^11 yrs
luminosity (solar units)
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
increasing temperature
surface temperature (Kelvin)
decreasing temperature

Globular Cluster HR Diagram



Evolutionary Tracks off the Main Sequence



Stellar Models

- MESA (Paxton et al. 2011, ApJS, 192, 3)
- mesa.sourceforge.net
- Can stably evolve stars through Helium flash, RGB and HB to White Dwarf

Dust and Gas

- Stars form in Molecular Clouds
- These clouds contain copious amounts of dust that absorb starlight (in the optical, UV and near IR) and reemit in the IR
- Dust grains are micron size and composed primarily of carbon and silicates

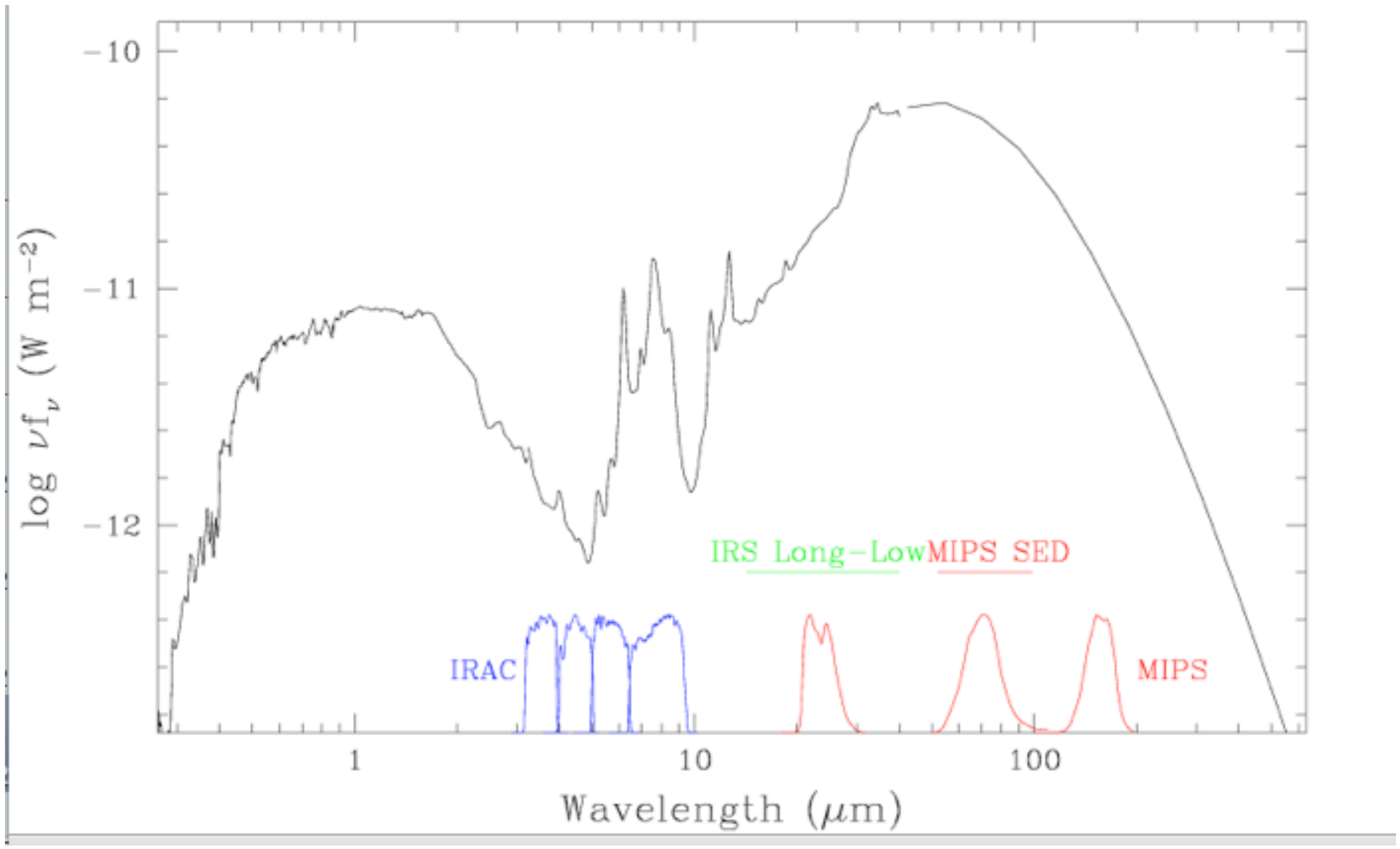
Dust Emission

- Electric Dipole Limit (Size << λ)

$$\sigma_{abs} \propto \lambda^{-2}$$

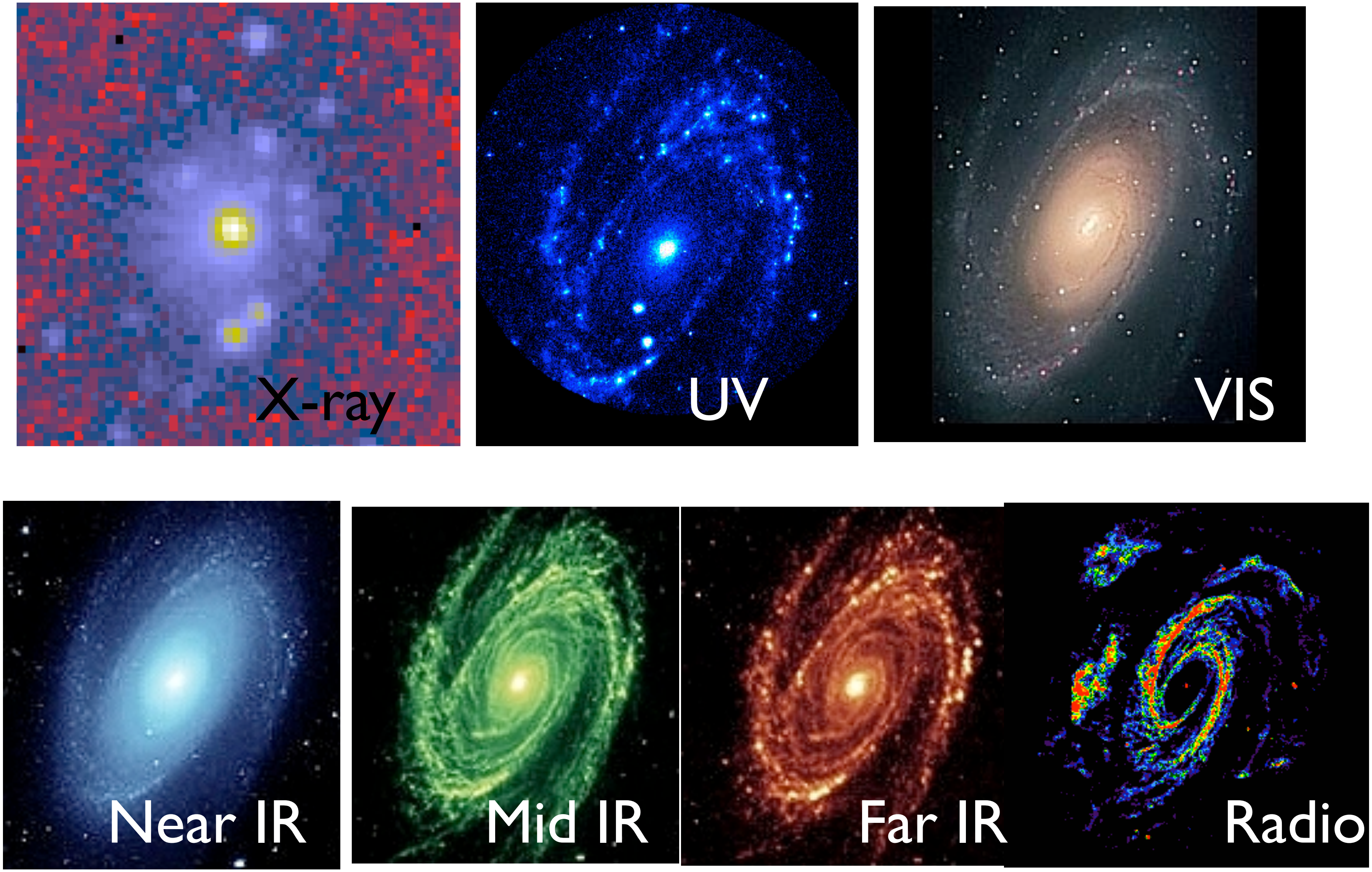
$$F_\nu \propto \nu^2 B_\nu(T) \propto \nu^4$$

Galaxy Spectrum



Other Emission Processes

- Radio:
  - Free-free emission
  - Synchrotron emission
  - Radio emission scales with synchrotron



M81 D=3 Mpc

Galaxy Properties

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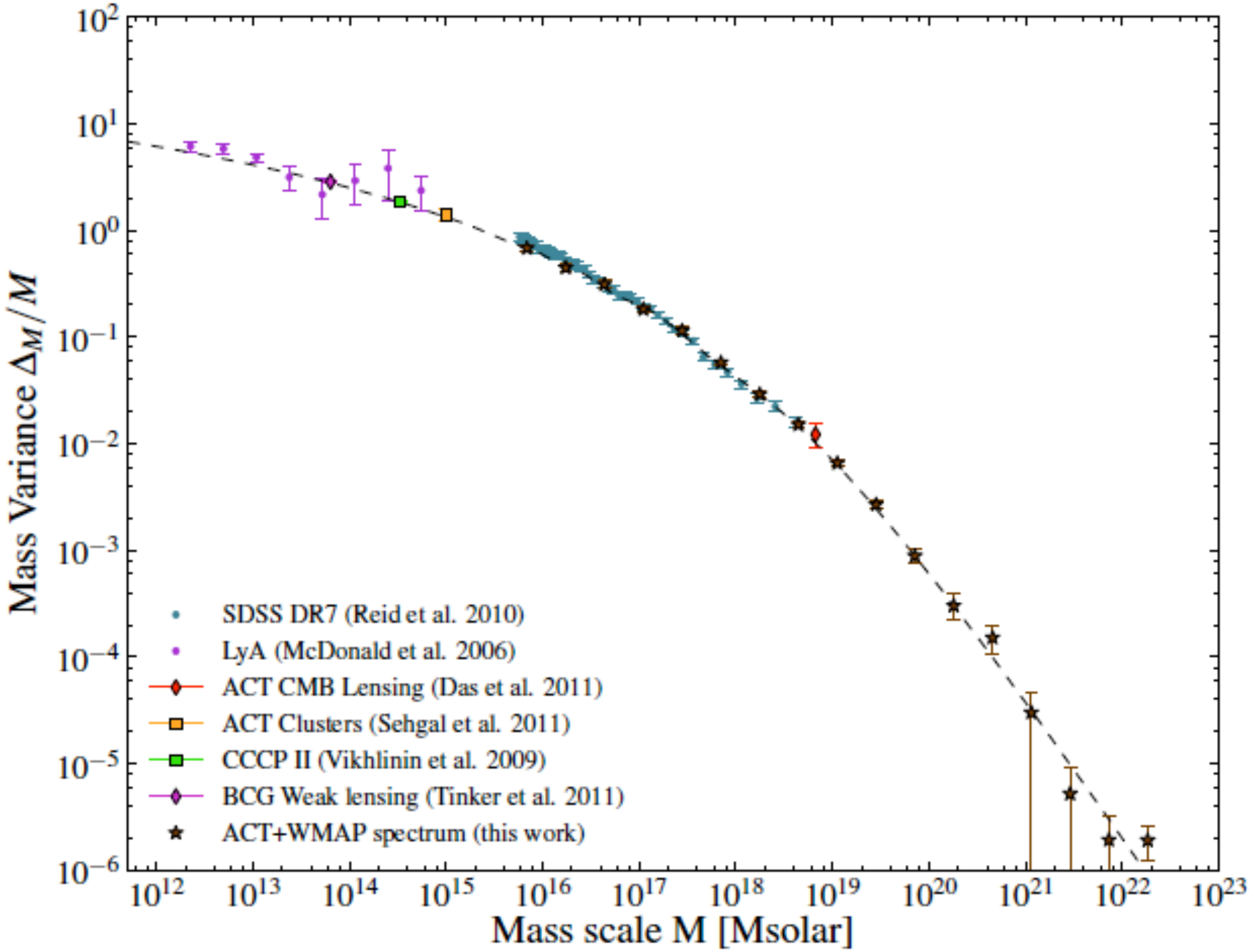


Mass Variance Δ_M/M
Mass scale M [Msolar]
SDSS DR7 (Reid et al. 2010)
LyA (McDonald et al. 2006)
ACT CMB Lensing (Das et al. 2011)
ACT Clusters (Sehgal et al. 2011)
CCCP II (Vikhlinin et al. 2009)
BCG Weak lensing (Tinker et al. 2011)
ACT+WMAP spectrum (this work)

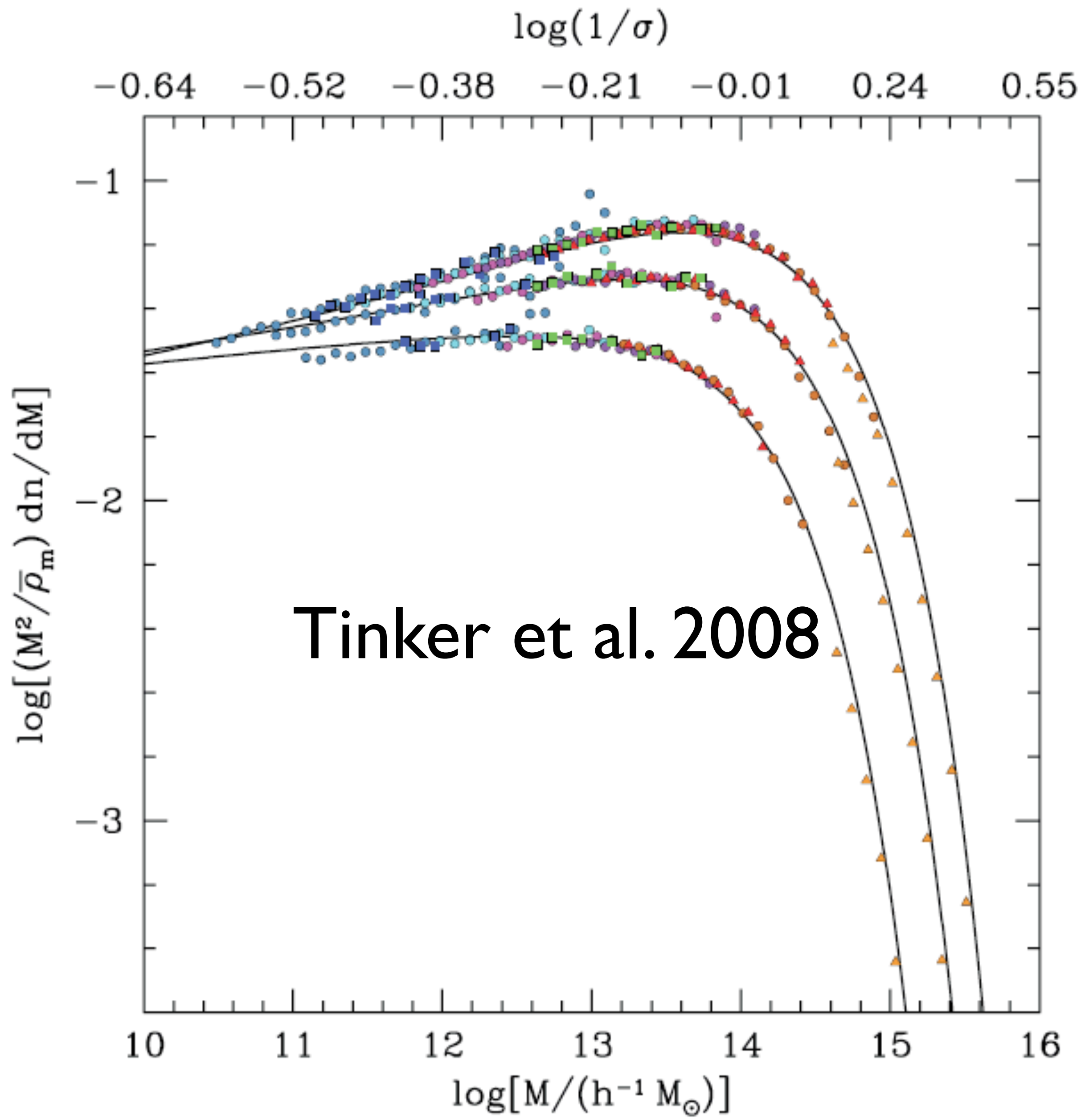


FIG. 5.—Measured mass functions for all WMAP1 simulations, plotted as $(M^2/\bar{\rho}_m)\, dn/dM$ against $\log M$. The solid curves are the best-fit functions from Table 2. The three sets of points show results for $\Delta = 200$, 800, and 3200 (*from*

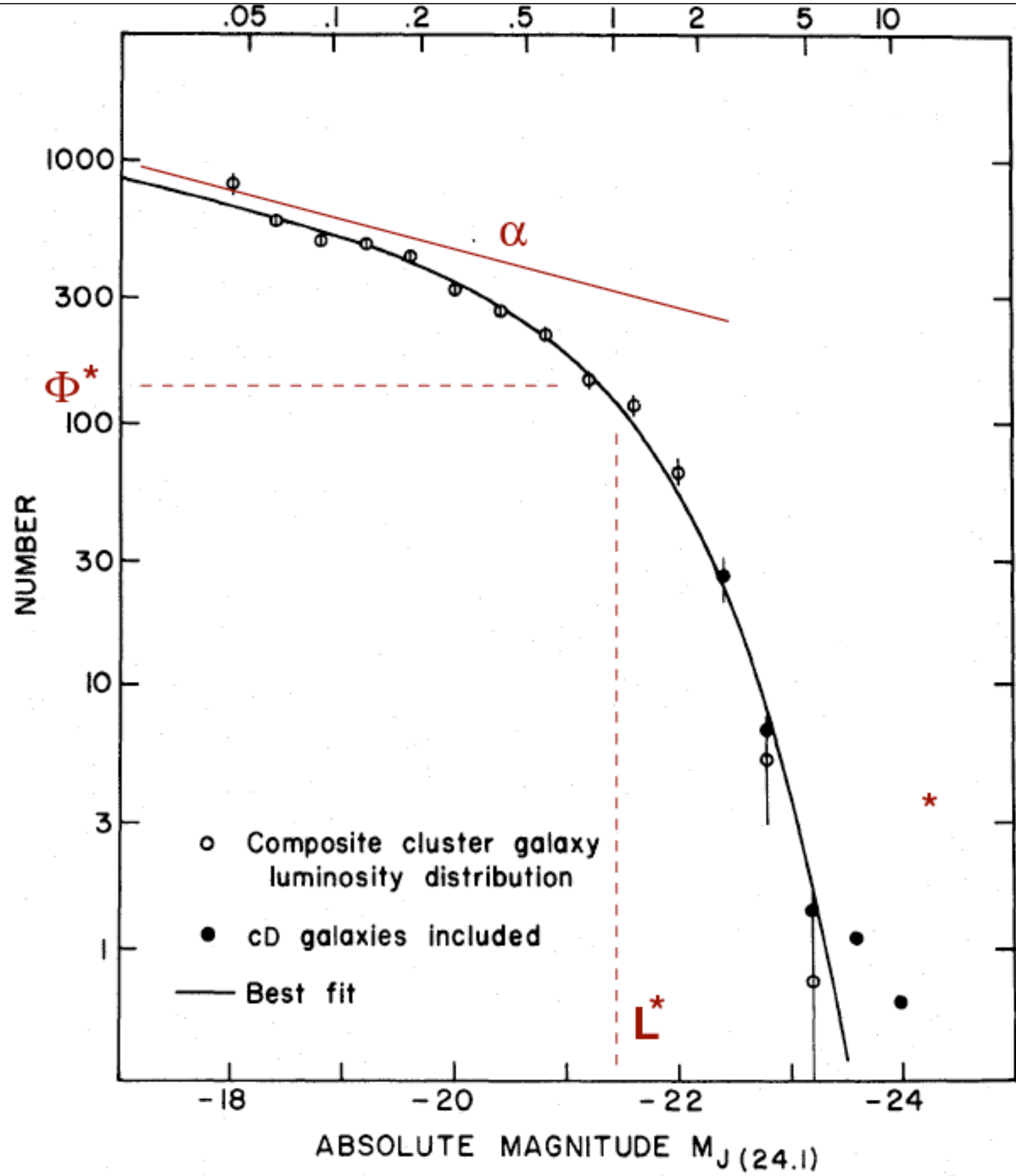
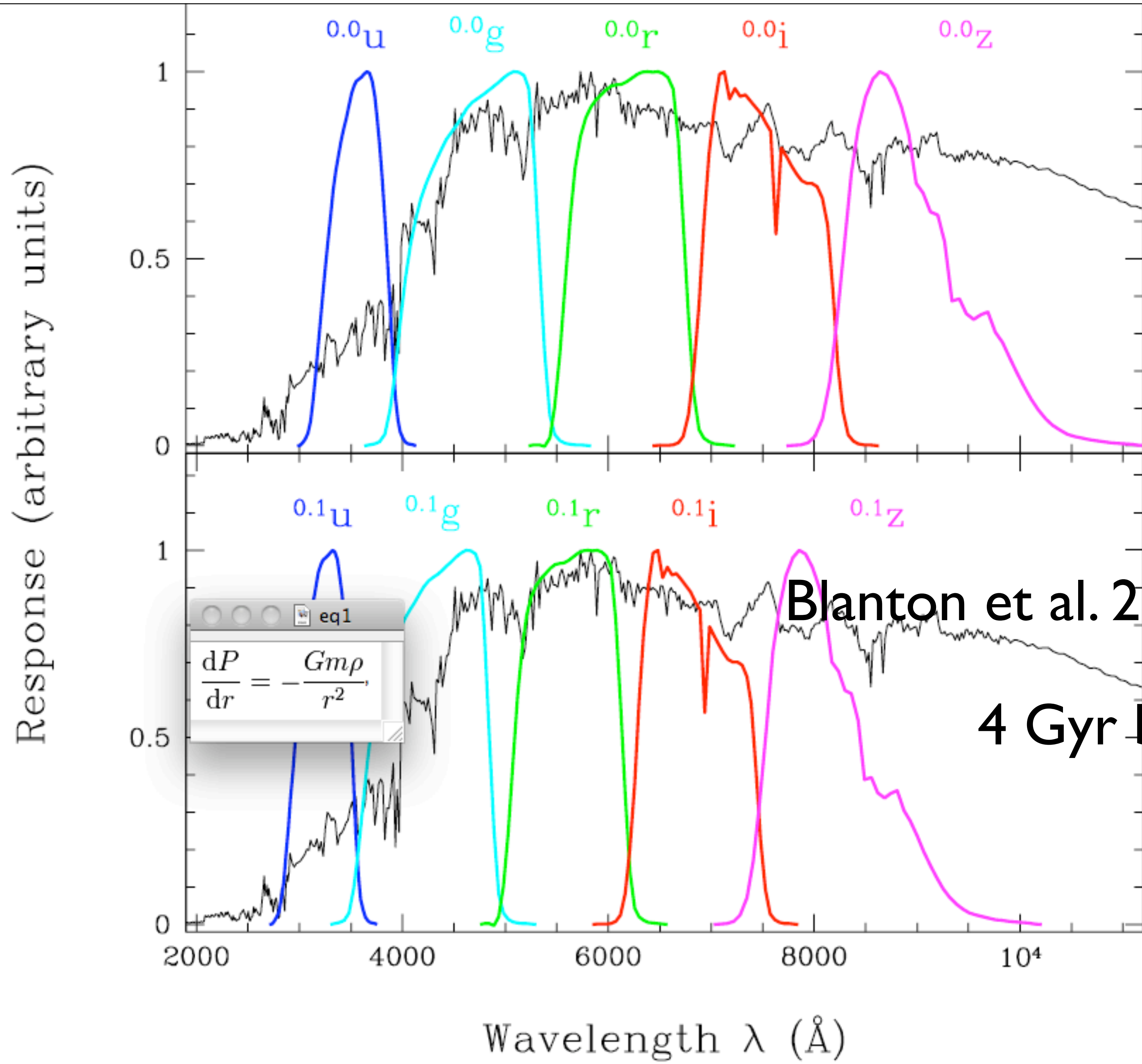


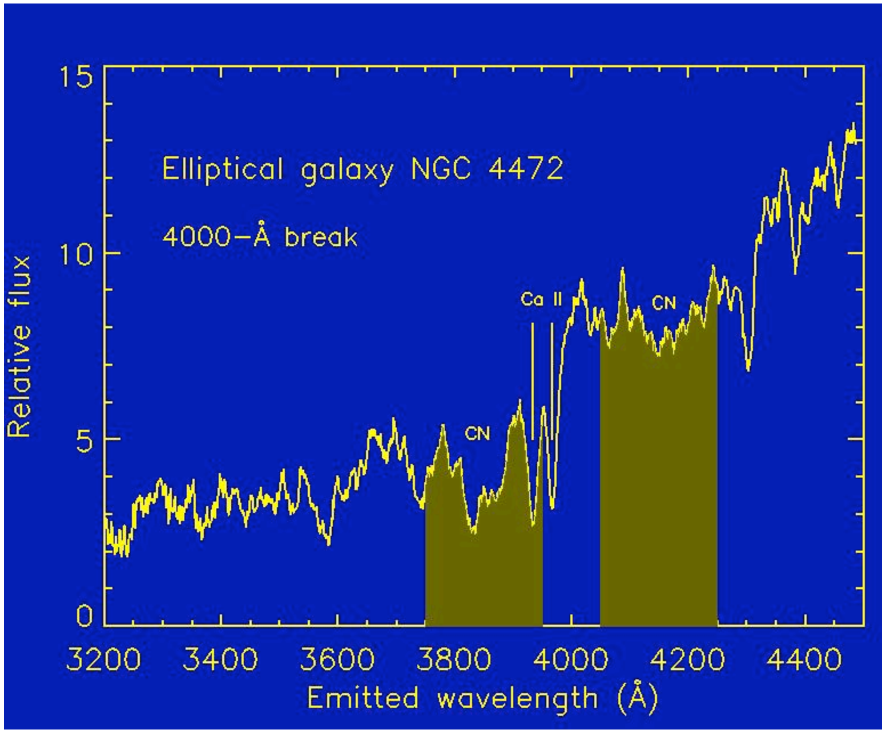
FIG. 2.—Best fit of analytic expression to observed composite cluster galaxy luminosity distribution. Filled circles show the effect of including cD galaxies in composite.

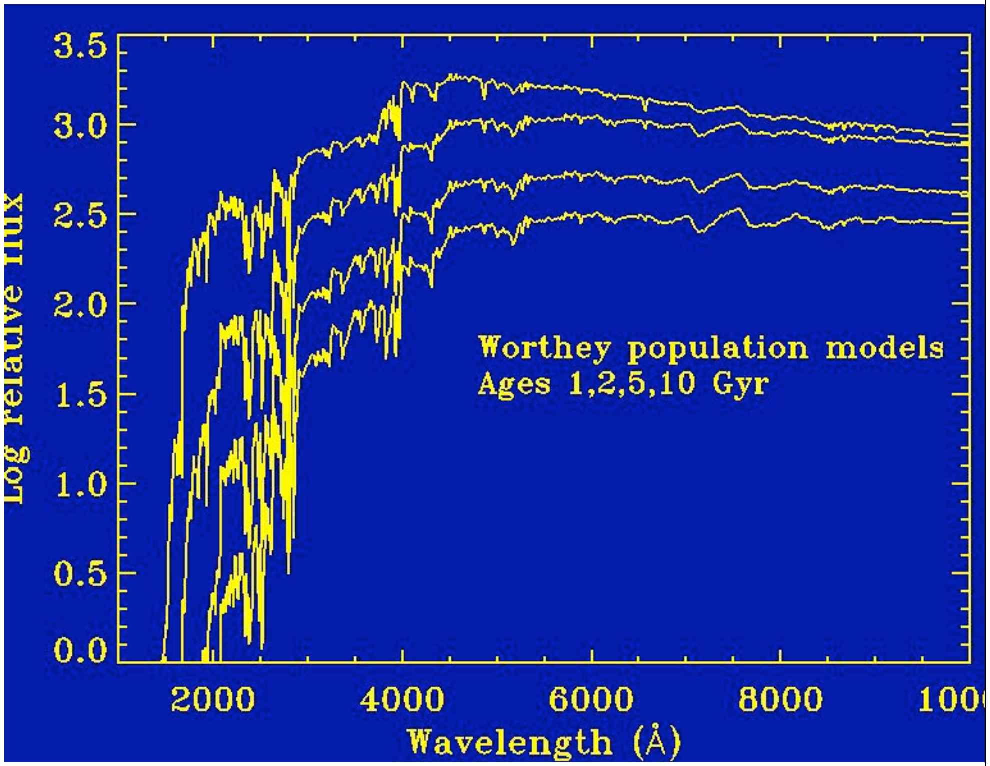


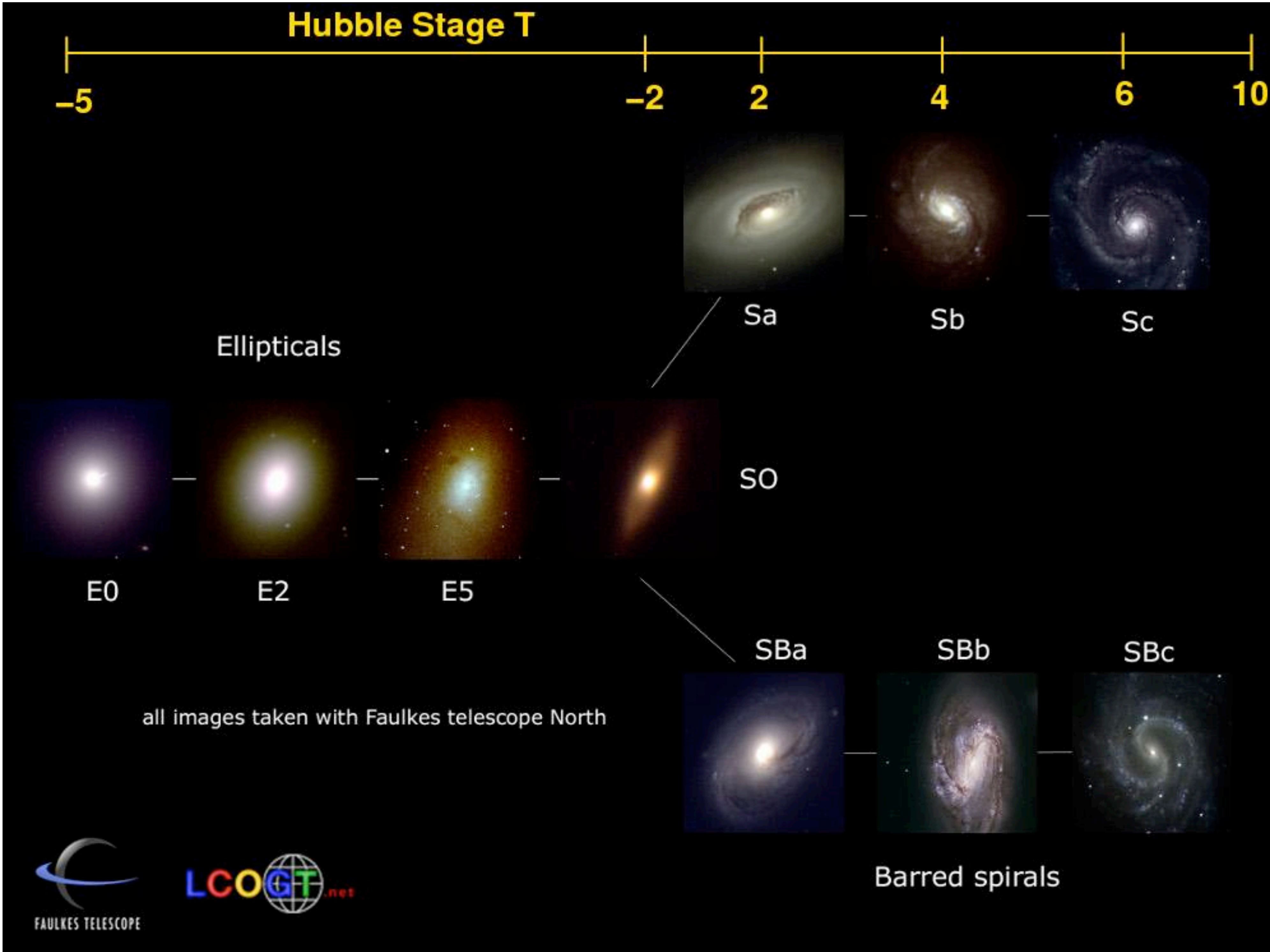
Blanton et al. 2002

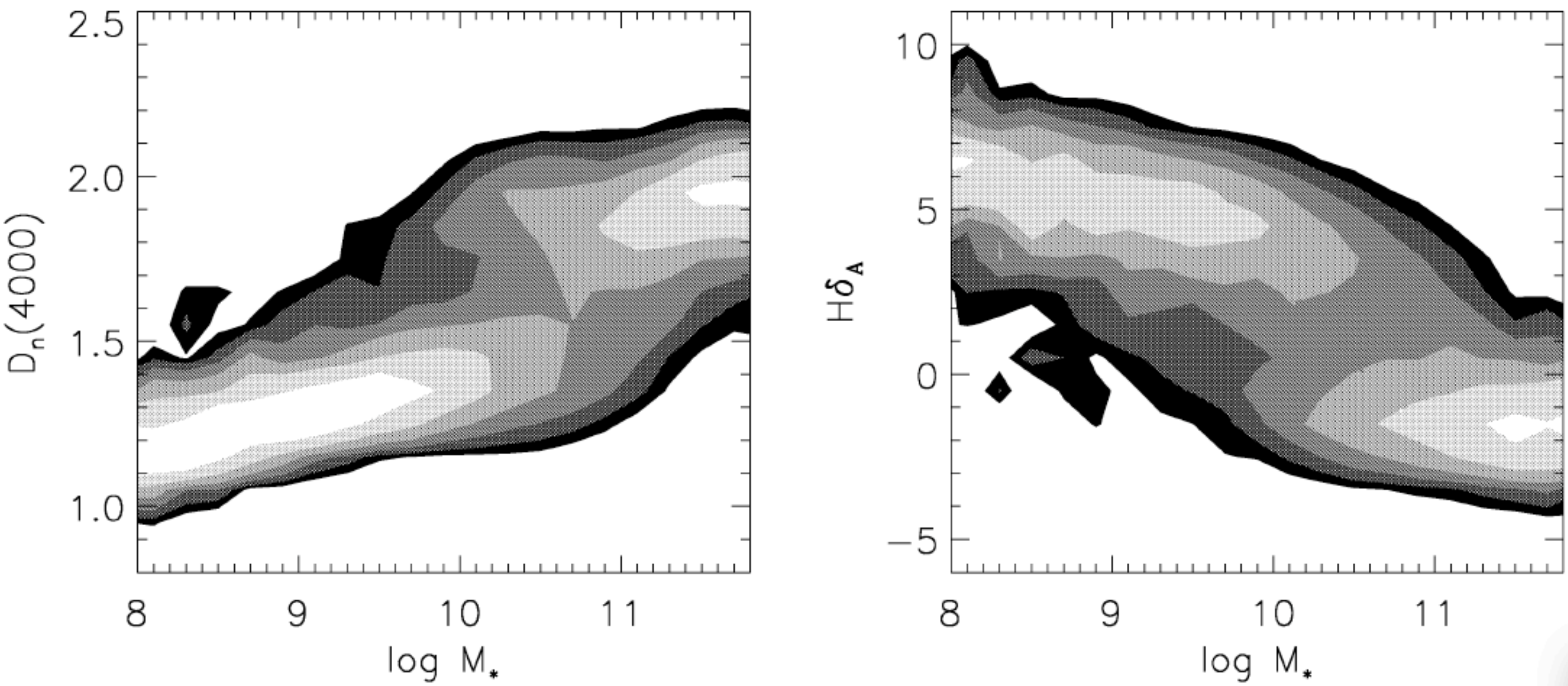
4 Gyr burst

4000 Angstrom Break



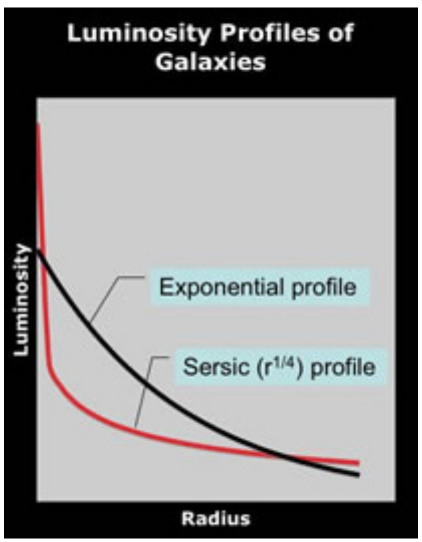



Hubble Stage T
–5
–2
2
4
6
10
Sa
Sb
Sc
Ellipticals
SO
E0
E2
E5
SBa
SBb
SBc
all images taken with Faulkes telescope North
FAULKES TELESCOPE
LCOGT.net
Barred spirals



Kauffmann et al. 2003

Galaxy Morphology



Spiral Galaxies

- Two parameter family: Luminosity and Surface Brightness
- Exponential distribution of stars
- Tully-Fisher

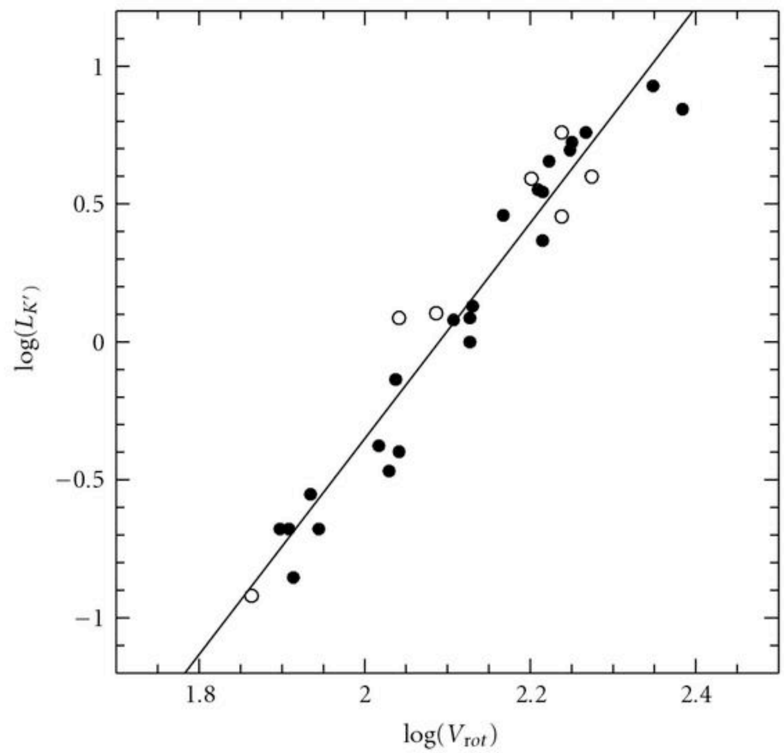
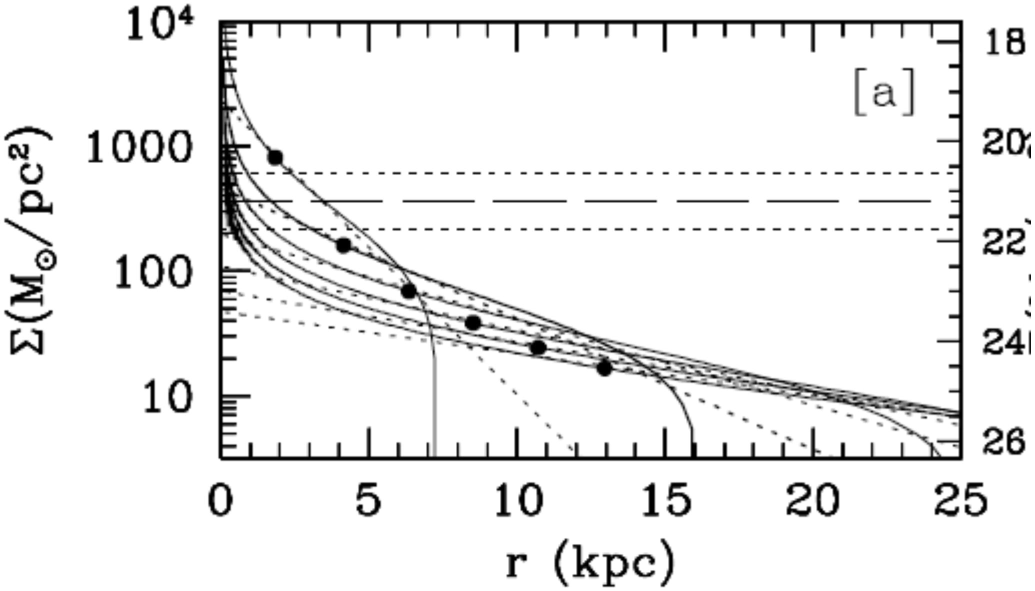


Figure 2: The near-infrared Tully-Fisher relation of Ursa Major spirals [5]. The rotation velocity is the asymptotically constant value. The line is a least-square fit to the data and has a slope of 3.9 ± 0.2.

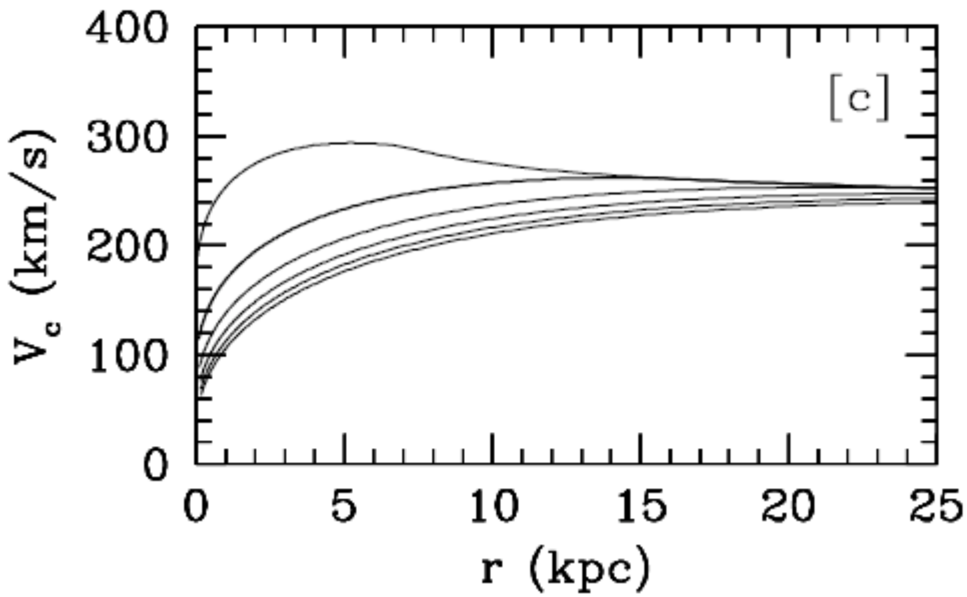
Sanders and Verheijen 1998

Spiral Galaxy Formation

- Tidal torque generates solid body rotation in gas
- Gas cools and collapses to form a disk conserving angular momentum



[a]
Σ(M⊙/pc²)
r (kpc)



[c]
Vc (km/s)
r (kpc)

Elliptical Galaxies

- R_e effective radius
- I_e mean surface brightness within eff. radius
- σ_0 velocity dispersion

$$\log\ D_n = \log R_e + 0.8 \log I_e$$

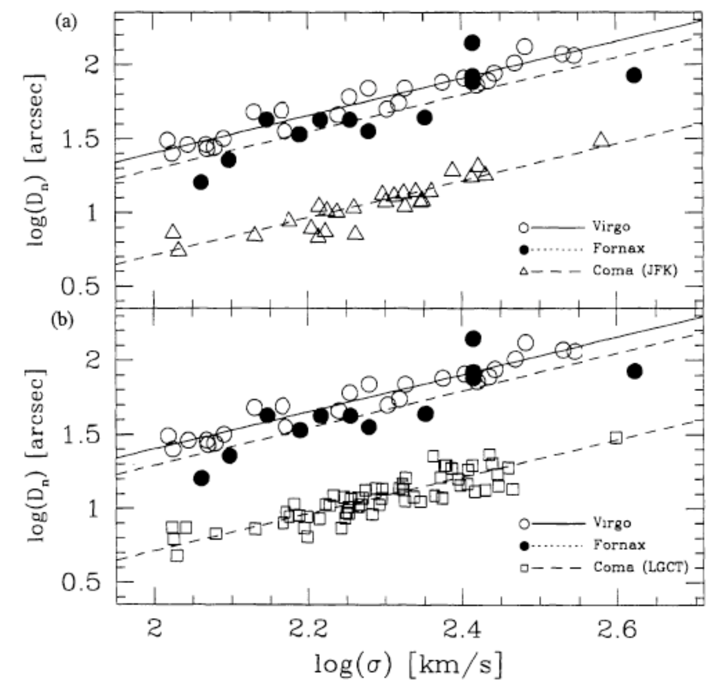
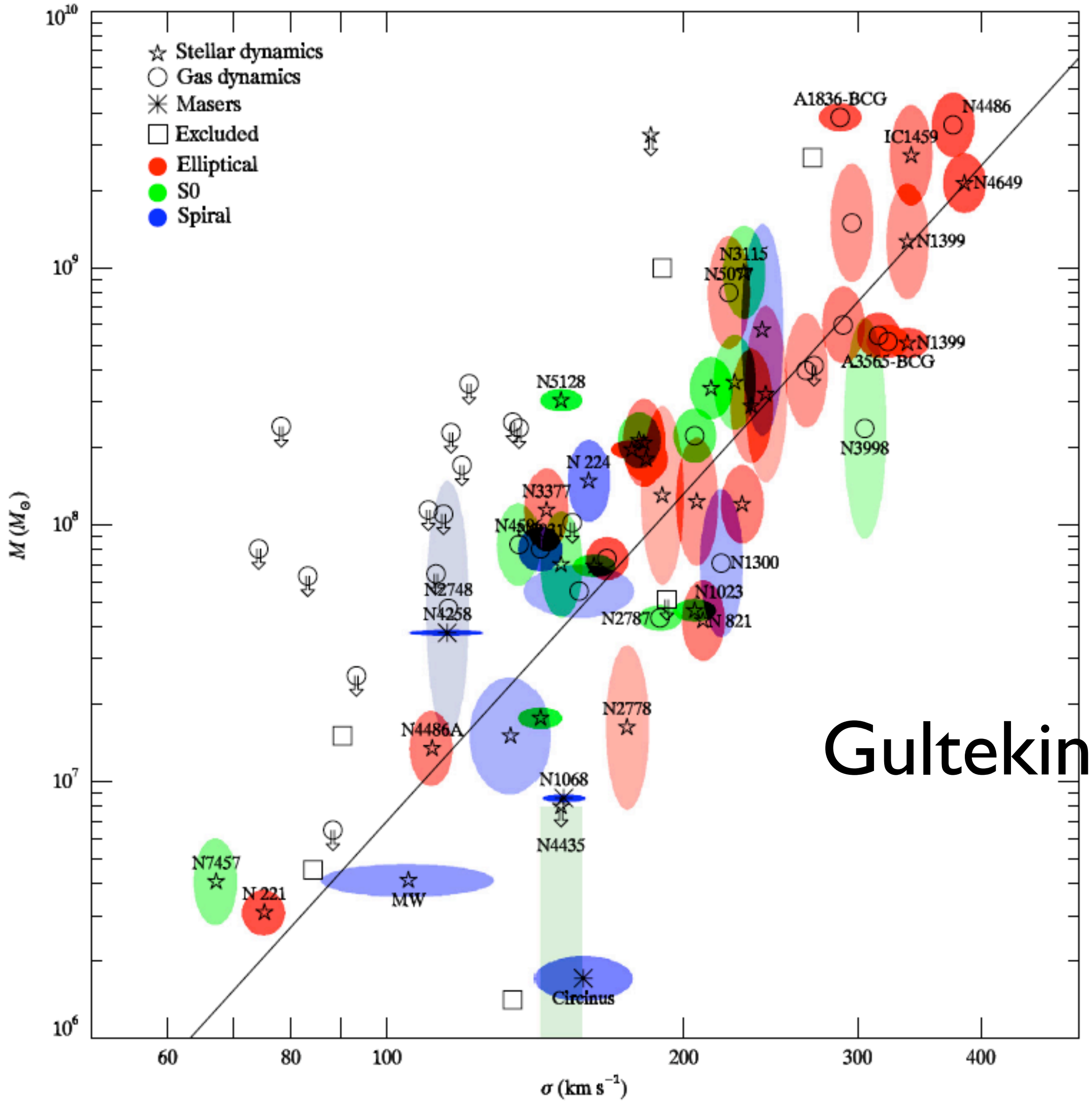


Figure 3. (a) The $\log D_n - \log\sigma$ relation for the Virgo, Fornax and Coma samples using the data from LGCT for the latter cluster. (b) same as panel (a) with the data of JFK. The solid, dotted and dashed lines give the fits to the Virgo, Fornax and Coma data points respectively.

Black Hole Scaling Relation



Gultekin et al. 2009