Exercises Group 6

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Creminelli 1

The metric for de Sitter space is given by

$$ds^2_{\text{ds}} = \frac{1}{H^2\eta^2}\left(-d\eta^2 + dx^2\right) \tag{1}$$

We recognize three symmetries of the metric:
1. Translational symmetry
2. Rotational symmetry: $\zeta(k) = \zeta(|k|)$
3. Scale symmetry $x \to \lambda x$, $k \to \frac{k}{\lambda}$, $\eta \to \lambda\eta$

And so the general form of the two point correlation function is given by

$$\langle \delta\phi^{\dagger}_{\vec{k}} \delta\phi_{\vec{k'}} \rangle = \frac{1}{k^3}(2\pi)^3\delta^3(\vec{k} - \vec{k'})F(k\eta) \tag{2}$$

where the $\frac{1}{k^3}$ derives form the scale invariance and the δ from the translational invariance. We know that in a dS_4 metric the equation of motion for a massless scalar field has a solution $f \sim (1 + i|k|\eta)e^{-i|k|\eta}$ and so $F(k\eta) = (1 + k^2\eta^2)$. This solution in the long wavelength regime ($k \to 0$) becomes scale (coordinate) invariant.
When we consider the mass we have a breaking of the conformal invariance a so a modification of the two point correlation function. Starting from the equation of motion for a massive scalar field we see how the mass tilt the power spectrum.

$$\phi'' - \frac{2}{\eta}\phi' + \frac{1}{H^2\eta^2}(k^2 + m^2)\phi = 0 \tag{3}$$

When i consider the long wavelength limit it becomes

$$\phi'' - \frac{2}{\eta}\phi' + \frac{m^2}{H^2\eta^2}\phi = 0 \tag{4}$$

So if I consider a power law solution for the scalar field ($\phi \sim (k\eta)^p$):

$$p(p-1)(k\eta)^{p-2} - 2p(k\eta)^{p-2} + \frac{m^2}{H^2}(k\eta)^{p-2} = 0 \tag{5}$$

from which

$$p = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \tag{6}$$

In the limit $\frac{m^2}{H^2} = 0$there are two solutions: $p = 0$ (for the frozen mode) and $p = 3$ (for the decaying mode).
When $\frac{m^2}{H^2} << 1$ expanding we have:

$$p = \frac{1}{3}\frac{m^2}{H^2} \tag{7}$$

and the two point correlation function (2) becomes

$$\langle \delta\phi^\dagger_{\vec{k}} \delta\phi_{\vec{k'}} \rangle = \frac{1}{k^3}(2\pi)^3 \delta^3(\vec{k} - \vec{k'}) \frac{1}{k^{-2p}} \tag{8}$$

So the tilt of the spectrum is given by

$$n_s - 1 = \frac{d\ln(P_\phi(k)k^3)}{d\ln k} = 2p \tag{9}$$

with $p \simeq \frac{1}{3}\frac{m^2}{H^2}$

Maldacena 1.4

For an AdS_4 with an S^3 boundary we have the following metric:

$$ds^2 = d\rho^2 + \sinh^2 \rho d\Omega_3^2 \tag{10}$$

For the computation of the action it is necessary to calculate the extrinsic curvature term $K = \frac{1}{2} h^{ab} \partial_n h_{ab}$ where h is the metric of the S^3 boundary and the normal direction n coincides with $\rho (\partial_n = -\partial_\rho)$. So we have:

$$K = 3 \coth \rho \tag{11}$$

For an Euclidean space the Ricci tensor is $R = -12$ and so we have

$$S_{Euclid} = \frac{R_{AdS}^2}{16\pi G_N} \left[6 \int_0^{\rho_c} d\rho \sinh^3 \rho d\Omega_3^2 + 6 \coth \rho_c \sinh^3 \rho_c \int d\Omega_3^2 \right] \tag{12}$$

For the boundary I have integrate for a fixed ρ. So the action becomes

$$S_{Euclid} = \frac{2\pi^2 R_{AdS}^2}{16\pi G_N} \left[2 \cosh^3 \rho_c - 6 \cosh \rho_c + 4 - 6 \sinh^2 \rho_c \cosh \rho_c \right] \tag{13}$$

To discard the divergent term I expand the hyperbolic function as $\cosh \rho = \frac{e^\rho + e^{-\rho}}{2}$ and $\sinh \rho = \frac{e^\rho - e^{-\rho}}{2}$ and so expanding

$$S_{Euclid} = \frac{2\pi^2 R_{AdS}^2}{16\pi G_N} \left[e^{-3\rho} - 3e^{-\rho} + 4 \right] \tag{14}$$

Finally the partition function is

$$\Psi = Z \sim e^{-S_E} = e^{\frac{\pi R_{AdS}^2}{8G_N} \left[3e^{-\rho} - e^{-3\rho} - 4 \right]} \tag{15}$$

Silverstein 2

a) To express to compactification is possible to write the complete metric as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + R(x)^2 \tilde{g}_{IJ} dx^I dx^J \tag{16}$$

where the last part is referred to the metric of the $D-4$ dimensional manifold X of linear size R. Starting from the Einstein action

$$S = \int d^D x \sqrt{-g^{(D)}} R^{(D)} \tag{17}$$

to solve the problem is necessary to rewrite the Ricci scalar in terms of the new metric. Starting from the Christoffel symbol

$$\Gamma^M_{NA} = \frac{1}{2} g^{MS} \big(\partial_A g_{SN} + \partial_N g_{SA} - \partial_S g_{NA} \big) \tag{18}$$

we have

$$\Gamma^\mu_{\nu\lambda} = \Gamma^{\mu(4)}_{\nu\lambda} \qquad \Gamma^I_{J\mu} = \frac{\dot{R}}{R} \delta^I_J \tag{19}$$

$$\Gamma^I_{JK} = \Gamma^{I(4)}_{JK} \qquad \Gamma^\mu_{IJ} = -\frac{1}{2} g^{\mu\nu} \partial_\nu R^2 \tilde{g}_{IJ} \tag{20}$$

So the Ricci scalar is given by

$$R = g^{\mu\nu} R_{\mu\nu} + g\mu J R_{\mu J} + g^{IJ} R_{IJ} = g^{\mu\nu} R_{\mu\nu} + \tilde{g}^{IJ} R_{IJ} R^{-2} \tag{21}$$

The two Ricci tensor are

$$R_{\mu\nu} = R^{(4)}_{\mu\nu} - (D-4)\partial_\mu \partial_\nu \log R + \Gamma^\alpha_{\mu\nu} (D-4) \partial_\alpha \log R - (D-4) \partial_\mu \log R \partial_\nu \log R \tag{22}$$

$$R_{IJ} = R^{(D-4)}_{IJ} - \frac{1}{2} \tilde{g}_{IJ} \partial_\alpha (g^{\alpha\beta} \partial_\beta R^2) - \frac{1}{2} \tilde{g}_{IJ} \Gamma^\alpha_{\alpha\beta} g^{\beta\lambda} \partial_\lambda R^2 + \frac{1}{2} \tilde{g}_{IJ} \partial_\alpha \log R g^{\alpha\lambda} \partial_\lambda R^2 \tag{23}$$

And the action becomes

$$S = M_{Pl}^{D-2} \int \sqrt{-g^{(4)}} A_{S^{(D-4)}} R(x)^{D-4} \left[R^{(4)} + R(x)^{-2} R^{(D-4)} + (D-4)(D-5)(\partial_\mu \log R(x))^2 \right] \tag{24}$$

and for the scalar field

$$S = \int \sqrt{-g^{(4)}} A_{S^{(D-4)'}} e^{(D-4)\sigma} \left[R^{(4)} + e^{-2\sigma} R^{(D-4)} + (D-4)(D-5)(\partial_\mu \sigma)^2 \right] \tag{25}$$

I redefine the field σ as

$$\sigma \to \frac{\sigma}{M_{Pl}} \frac{1}{\sqrt{(D-4)(D-5)}} \tag{26}$$

So now $R = e^\sigma = e^{\frac{\sigma}{M_{Pl}} \frac{1}{\sqrt{(D-4)(D-5)}}}$ and the volume is

$$V_X = A_{S^{(D-4)}} e^{\sqrt{\frac{D-4}{D-5}} \frac{\sigma}{M_{Pl}}} \tag{27}$$

and $c_X = \sqrt{\frac{D-4}{D-5}}$ cannot be small.

c)Given the terms

$$|dC_p + B \wedge dC_{p-2}|^2 \tag{28}$$

for B we have the following gauge transformation:$B \to B + d\Lambda_1$. So to respect the same symmetry C_p have to transform as:

$$C_p \to C_p - d\Lambda_1 \wedge C_{p-2} \qquad dC_p \to dC_p - d\Lambda_1 \wedge dC_{p-2} \tag{29}$$

Is trivial to show that under these transformations the "kinetic term" (28) is invariant.

d) For an $AdS_5 \times X_5$:

$$ds^2 = \frac{r^2}{R^2} g_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + ds^2_{X_5} \tag{30}$$

In D-dimension the action is

$$S^D = M_{Pl}^{D-2} \int d^D x \sqrt{-g^D} R^{(D)} \tag{31}$$

Starting from the Christoffel symbols (18) we see that they have the form

$$\Gamma^\mu_{\nu\rho} = \frac{1}{2} g^{\mu\alpha} (\partial_\nu g_{\alpha\rho} + \partial_\rho g_{\alpha\nu} - \partial_\alpha g_{\rho\nu}) \tag{32}$$

from which

$$\Gamma^\mu_{\nu\rho} = \Gamma^{\mu(4)}_{\nu\rho} + (\propto \partial r^2) \tag{33}$$

and the same form for the Riemann tensor and the Ricci tensor; so

$$R = g^{\mu\nu} R_{\mu\nu} = \frac{R^2}{r^2} g^{\mu\nu} R^{(4)}_{\mu\nu} + ...(\propto \partial r^2)... = \frac{R^2}{r^2} R^{(4)} + ...(\propto \partial r^2)... \tag{34}$$

So the action in 10 dimension is

$$S^{(10)} = M^8 Vol(X) \int d^4 x \sqrt{-g} \int_0^{r_{UV}} dr \frac{r^3}{R^3} (\frac{R^2}{r^2} R^{(4)} + ...) \tag{35}$$

From which the four-dimensional Planck mass is

$$M_{Pl}^2 = M^8 Vol(X) \frac{r_{UV}^2}{2R} \tag{36}$$

The Lyth bound is defined as

$$\frac{\Delta\phi}{M_{Pl}} > \left(\frac{r}{4\pi}\right)^{1/2} \tag{37}$$

where r is the tensor-scalar ratio. M is related to α' by $M^8 = \frac{2}{\alpha'^4}$. Finally we have

$$r < \frac{4\pi\alpha'^2 R}{Vol(X)} \tag{38}$$

<u>Susskind 0</u>

General 5D metric:

$$ds_5^2 = -dT^2 + dX^2 = -dT^2 + dX_0^2 + \underbrace{dX_1^2 + dX_2^2 + dX_3^2}_{r^2_{\text{general}}} \tag{39}$$

From which

$$ds_5^2 = -dT^2 + dX_0^2 + r^2_{\text{general}} \tag{40}$$

We can express r_{general} as a spherical radius rather than a cartesian radius, so

$$\rightarrow ds_5^2 = -dT^2 + dX_0^2 + dr^2_{\text{general}} + r^2_{\text{general}} d\Omega_2^2 \tag{41}$$

The metric of flat slicing foliations is

$$ds_4^2 = R^2\left[-d\tau^2 + e^{2\tau}dx^2\right] \tag{42}$$

Expressing the space-like part in spherical coordinate $dx^2 = dr^2_{\text{flat}} + r^2_{\text{flat}} d\Omega_2^2$ we find

$$ds_4^2 = R^2\left[-d\tau^2 + e^{2\tau}\left(dr^2_{\text{flat}} + r^2_{\text{flat}} d\Omega_2^2\right)\right] \tag{43}$$

By comparison of the general ds_5^2 de Sitter metric and the flat metric ds_4^2 we see that $d\Omega_2^2$ coefficients should not change, thus

$$\boxed{r_{\text{general}} = Re^{\tau} r_{\text{flat}}} \tag{44}$$

So we have reduced the general 5D metric to

$$ds_5^2 = -dT^2 + dX_0^2 + R^2 e^{2\tau} \left[dr_{\text{flat}}^2 + r_{\text{flat}}^2 d\Omega_2^2 \right] \tag{45}$$

Now we have to obtain ds_4^2 from ds_5^2 and so we need to map $(T, X_0) \to (\tau, r)$. By comparison of the metrics we see that we must require

$$-dT^2 + dX_0^2 = -R^2 d\tau^2 \tag{46}$$

Therefore we obtain the mapping

$$T = R\sinh(\tau) \qquad X_0 = R\cosh(\tau) \tag{47}$$

which can be also seen from the hyperboloid geometry. Upon substitution we obtain the desired form:

$$ds_4^2 = R^2 \left[-d\tau^2 + e^{2\tau} \left(dr_{\text{flat}}^2 + r_{\text{flat}}^2 d\Omega_2^2 \right) \right] \tag{48}$$

Zaldarriaga 6

Baryons account for between 5 and 10 percent of the total energy density in the Universe but their effect even if small is not negligible and can be seen on the power spectrum and in particular on the matter transfer function. To estimate this effect we start from the Boltzmann equations for the photons, the dark matter and the baryons:

$$\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi} \tag{49}$$

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = -\frac{k}{3}\Phi + \dot{\tau}\big[\Theta_1 - \frac{iv_b}{3}\big] \tag{50}$$

$$\dot{\delta}_{DM} + ikv_{DM} + 3\dot{\Phi} = 0 \tag{51}$$

$$\dot{v}_{DM} + aHv_{DM} - ik\Phi = 0 \tag{52}$$

$$\dot{\delta}_b + ikv_b + 3\dot{\Phi} = 0 \tag{53}$$

$$\dot{v}_b + aHv_b - ik\Phi = \frac{\dot{\tau}}{R}\Bigg[3i\Theta_1 + v_b\Bigg] \tag{54}$$

where the multipole bigger than 1 are been neglected(this assumption is good before recombination since optically thick to photons) and is good in the tightly coupled limit ($\dot{\tau} = -n_e\sigma_T a \gg 1$). In this regime from the last equation

$$v_b = -3i\Theta_1 + \frac{R}{\dot{\tau}}\Bigg[\dot{v}_b + aHv_b - ik\Phi\Bigg] \tag{55}$$

we see that the second term is suppressed and so

$$v_b = -3i\Theta_1 \tag{56}$$

and

$$\dot{\delta}_b = -ikv_b - 3\dot{\Phi} = -3\dot{\Phi} - 3k\Theta_1 = 3\Theta_0 \tag{57}$$

From which $\delta_b = 3\Theta_0$ and so photon and baryon move together without anisotropic stress given by Θ_2. We can think this as a photon-baryon fluid. Then we can study the evolution of the photon.
+Starting from the Einstein equation

$$k^2\Phi + 3aH\big(\dot{\Phi} + aH\Phi\big) = 4\pi Ga^2\big(\rho_m\delta + 4\rho_r\Theta_0\big) \tag{58}$$

$$k^2\Phi = 4\pi G a^2\left[\rho_m\delta + 4\rho_r\Theta_0 + \frac{3aH}{k}\big(i\rho_m v + 4\rho_r\Theta_1\big)\right] \tag{59}$$

From the Dodelson we have that, in terms of the new variable $y = \frac{\rho_m}{\rho_r} = \frac{a}{a_{eq}}$,

$$\Phi = \frac{3}{2Q^2y^2}\left[y\delta + 4\Theta_0 + 3\frac{(y+1)^{1/2}}{Qy}(iyv + 4\Theta_1)\right] \tag{60}$$

where $Q = \sqrt{2}\frac{k}{k_{eq}}$. Considering scales subhorizon($Q^{-1} \leq y \leq 1$), before recombination($a < a_{eq}$) and very small($k\eta \gg 1$) we have

$$\Phi = \frac{3}{2Q^2y^2}4\Theta_0 = \frac{6}{(k\eta)^2}\Theta_0 \tag{61}$$

Taking the solution for the potential for $k\eta \gg 1$

$$\Phi = 3\Phi(0)\frac{\sin(k\eta/\sqrt{3}) - k\eta/\sqrt{3}\cos(k\eta/\sqrt{3})}{\big(k\eta/\sqrt{3}\big)^3} \simeq -9\Phi(0)\frac{\cos(k\eta/\sqrt{3})}{(k\eta)^2} \tag{62}$$

the temperature perturbation becomes

$$\Theta_0 \sim -\frac{3}{2}\Phi(0)\cos\frac{k\eta}{\sqrt{3}} \tag{63}$$

and the photon dipole

$$\Theta_1 = -\frac{\dot{\Theta}_0}{k} \sim -\frac{\sqrt{3}}{2}\Phi(0)\sin\frac{k\eta}{\sqrt{3}} \tag{64}$$

These describe the oscillations of the temperature perturbation and of the dipole.
Since in this regime $\Theta_0 = \frac{\delta_b}{3}$ and $\Theta_1 = \frac{iv_b}{3}$ we obtain for the baryon

$$\delta_b = -\frac{9}{2}\Phi(0)\cos\frac{k\eta}{\sqrt{3}} \tag{65}$$

$$v_b = \frac{3\sqrt{3}}{2}\Phi(0)i\sin\frac{k\eta}{\sqrt{3}} \tag{66}$$

Now we have to study the effect of these solutions on the transfer function. To do this we consider the Boltzmann equations for the dark matter(DM) and baryon perturbation within the horizon and after recombination in the matter dominated era. We know that in this era

$$\Omega_{DM}(a) + \Omega_b(a) = 1 \tag{67}$$

To solve the equation we take a linear combination of the baryon and DM equations. We define the dotal density and velocity

$$\delta_m = \Omega_{DM}(a)\delta_{DM} + \Omega_b(a)\delta_b \tag{68}$$

$$v_m = \Omega_{DM}(a)v_{DM} + \Omega_b(a)v_b \tag{69}$$

For these quantities holds the following differential equations

$$\dot{\delta}_m = -ikv_m \tag{70}$$

$$\dot{v}_m = -\frac{2}{\eta}v_m + \frac{6i}{k\eta^2}\delta_m \tag{71}$$

From these equations is possible to obtain a second order differential equation for δ_m(we have take this from Dodelson 6.72)

$$\frac{d^2\delta_m}{da^2} + \frac{3}{2a}\frac{d\delta_m}{da} - \frac{3}{2a^2}\delta_m = 0 \tag{72}$$

The solution is

$$\delta_m = \left[\frac{3}{5}\frac{\delta_m}{a} + \frac{2}{5}\frac{d\delta_m}{da}\right]a + \left[\frac{2}{5}a^{3/2}\delta_m - \frac{2}{5}\frac{a^{5/2}d\delta_m}{da}\right]a^{-3/2} \tag{73}$$

These quantities are all evaluated at recombination. The second term is negligible at late times so we calculate the transfer function considering only the first term. From the Dodelson we know that the transfer function for the DM is

$$\delta_{DM} = \frac{3k^2}{5\Omega_{m0}H_0^2}\Phi(0)T_{DM}(k)a_{rec} \tag{74}$$

where $T_{DM}(k)$ is the transfer function. So the first term of the solution for the DM becomes

$$\frac{3}{5}\frac{\delta_{DM}}{a} + \frac{2}{5}\frac{d\delta_{DM}}{da} = \frac{3k^2}{5\Omega_{m0}H_0^2}\Phi(0)T_{DM}(k)a_{rec} \tag{75}$$

and taking the solution (65) for the baryon we have

$$\frac{3}{5}\frac{\delta_b}{a}+\frac{2}{5}\frac{d\delta_b}{da}=\frac{3\sqrt{3}}{10a}k\eta\Phi(0)\sin\frac{k\eta}{\sqrt{3}} \tag{76}$$

where we have considered only the second term since for $k\eta \gg 1$ dominates ($\frac{d\delta_b}{da}=-\frac{ik\eta}{2a}v_b=\frac{3\sqrt{3}}{4a}k\eta\Phi(0)\sin\frac{k\eta}{\sqrt{3}}$).
And so the total transfer function is given by

$$\delta_m=\Omega_b(a)\frac{3\sqrt{3}}{10a}k\eta\Phi(0)\sin\frac{k\eta}{\sqrt{3}}+\Omega_{DM}(a)\frac{3k^2}{5\Omega_{m0}H_0^2}\Phi(0)T_{DM}(k)a_{rec} \tag{77}$$

or better

$$T(k)=\Omega_{DM}(a)T_{DM}(k)+\frac{\sqrt{3}}{2}\Omega_b\frac{\Omega_{m0}H_0^2\eta}{ka_{rec}}\sin\frac{k\eta}{\sqrt{3}} \tag{78}$$

So the effects of the baryons are twice: first they suppress the transfer function at small scales since $\Omega_{DM}/\Omega_b < 1$ and second they imprint an oscillation on the transfer function and so on the power spectrum. It is simple to see that the period of these oscillation is

$$k=\frac{2\pi\sqrt{3}}{\eta_{rec}}\simeq 0.04Mpc^{-1} \tag{79}$$

These oscillations are the so famous Baryon Acoustic Oscillation that can be seen in the power spectrum even if with a very small contribution.

Arkani-Hamed 4

Starting from the action

$$S'=\int d^4x\sqrt{-g}[F'(A)(R-A)+F(A)] \tag{80}$$

and using the Eulero-Lagrange equation for the auxiliary field we have:

$$F''(A)(R-A)-F'(A)+F'(A)=0 \tag{81}$$

From which, for $F''(A)\neq 0$, we find $R=A$ and so

$$S'=S=\int d^4x\sqrt{-g}[F(R)] \tag{82}$$

For $F''(A) = 0$ we find $F = cA + b$ with a and b constants and

$$S'' = \int d^4x\sqrt{-g}[cR + b] \tag{83}$$

So we recover the general relativity plus a "cosmological constant".
Now, to demix A from the metric, is necessary to make a conformal transformation:

$$\tilde{g}_{\alpha\beta} = \Omega^2(x)g_{\alpha\beta} \tag{84}$$

Starting from the Ricci tensor $R_{\mu\nu}$

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu} = \partial_\lambda\Gamma^\lambda_{\mu\nu} - \partial_\mu\Gamma^\lambda_{\lambda\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\rho\lambda} + \Gamma^\lambda_{\nu\rho}\Gamma^\rho_{\mu\lambda} \tag{85}$$

constructed by the Christoffel symbol, we have the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$.
A conformal transformation first on the Christoffel symbols, then on the Ricci tensor and finally on the Ricci scalar gives

$$\tilde{R} = \Omega^{-2}R - 6\Omega^{-3}\Box\Omega - 6\Omega^{-4}\partial_\alpha\Omega\partial^\alpha\Omega \tag{86}$$

Now I define $\omega = \ln\Omega$ and then

$$\tilde{R} = \Omega^{-2}\Big[R - 6\Box\omega - 6\partial_\alpha\omega\partial^\alpha\omega\Big] \tag{87}$$

Substituting in the the action

$$S = \int d^4x\sqrt{-g}\Omega^4\Big\{F'(A)\Big[\Omega^{-2}(R - 6\Box\omega - 6\partial_\alpha\omega\partial^\alpha\omega) - A\Big] + F(A)\Big\} \tag{88}$$

where Ω^4 derives from the conformal transformation of the determinant of the metric.
Now I fix the conformal parameter to decouple the field from the metric

$$\Omega = \frac{1}{F'(A)^{1/2}} \tag{89}$$

and so the action becomes

$$S = \int d^4x\sqrt{-g}\Big[R - 6\Box\omega - 6\partial_\alpha\omega\partial^\alpha\omega - \frac{A}{F'(A)} + \frac{F(A)}{F'(A)^2}\Big] \tag{90}$$

I call $\sigma = -\ln F'(A)$ and so

$$S = \int d^4x\sqrt{-g}\Big[R - \frac{3}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma)\Big] \tag{91}$$

with $V(\sigma) = \frac{A}{F'(A)} - \frac{F(A)}{F'(A)^2}$.