\[ds^2 = dg^2 + \sinh^2 g \, dR_3^2\]

\[\int_{\Omega g} = 2\pi^2 \left( \sinh^2 g \, dg = 2\pi^2 \left( \frac{1}{3} \cosh^3 g_c - \cosh g_c + 2/3 \right) \right)\]

**Metric on bdy:**

\[ds^2 = \sinh^2 g_c \, dR_3^2\]

\[\partial g \eta_{ab} = -\sinh 2g \, dR_3^2\]

\[\frac{1}{2} h^{ab} \partial g h_{ab} = \frac{3}{\tanh g}\]

\[\Rightarrow \int_{\partial \Sigma_4} K \sqrt{-g} = 2\pi^2 \times \frac{3}{\tanh g_c} \cdot \sinh^3 g_c = 6\pi^2 \sinh^2 g_c \cosh g_c\]

\[\Rightarrow S_E = \frac{R \pi^2}{16\pi G} \left( -\frac{4\pi^2}{3} \left( \frac{1}{3} \cosh^3 g_c - \cosh g_c + 2/3 \right) - \frac{12\pi^2 \sinh^3 g_c \cosh g_c}{\tanh g_c} \right)\]

Keeping finite part give

\[\psi = e^{R \pi^2 / 16G}\]
Group \( \mathbb{U} \)

\[ S = \frac{1}{2} \rho \int \frac{d^{4}x}{\sqrt{g}} \left[ g_{11} \Phi_{t^2} \delta_{t^2} \Phi - 2\nabla \Phi - 2V \Phi \right] \]

De Sitter an massless:

\[ \Phi + 3H \Phi - 2 \frac{\partial \Phi}{\partial t} = 0 \]

For perturbations \( \delta \Phi(\mathbf{x}, t) \) where \( \Phi(\mathbf{x}, t) = \Phi(t) + \delta \Phi(\mathbf{x}, t) \):

\[ \delta \Phi + 3H \delta \Phi - 2 \frac{\partial \delta \Phi}{\partial t} = 0 \]

In Fourier space:

\[ \delta \Phi_k + 3H \delta \Phi_k - \frac{k^2}{a^2} \delta \Phi_k = 0 \]

To get in form of S.I.I. O.:

Define conformal time:

\[ dt = \frac{dt}{H^2} \rightarrow t = -\frac{1}{H} \]

\[ \delta \Phi_k'' + 2H \delta \Phi_k' + \frac{k^2}{a^2} \delta \Phi_k = 0 \quad \text{and} \quad \delta = 2 \]

Define new variable:

\[ \chi = a \delta \Phi_k \]

\[ \chi'' + \left( \frac{k^2 - \frac{2}{a^2}}{a^2} \right) \chi = 0 \]

\[ \delta = -\frac{1}{H^2} \rightarrow \frac{2}{a^2} = \frac{2}{H^2} \]

\[ \chi'' + \left( \frac{k^2 - \frac{2}{H^2}}{t^2} \right) \chi = 0 \]
Exact Solution is given by:

\[ V_2 = \frac{Ae^{-ikz}}{\sqrt{2k}} (1-i) + \frac{Be^{ikz}}{\sqrt{2k}} (1+i) \]

We need this to reduce to the zero point fluctuation of a free field in flat space-time when on scales corresponding to \( |kz| \gg 1 \):

\[ V_2 = \frac{e^{-ikz}}{\sqrt{2k}} \]

Hence:

\[ V_2 = \frac{e^{-ikz}}{\sqrt{2k}} (1-i) \]

And

\[ S \Psi_2 = \frac{1}{2} \frac{e^{-ikz}}{\sqrt{2k}} (1-i) \]

Two Point Function:

\[ \langle S \Psi_2, S \Psi_{2}^\prime \rangle = (2\pi)^3 S^3 (k+k') \frac{1}{2} \Psi_2^2 \]

\[ = (2\pi)^3 S^3 (k+k') \frac{1}{2} \frac{1}{(kz)^2} 2^2 \]

\[ = (2\pi)^3 S^3 (k+k') \frac{1}{2} \frac{1}{(1+k^2 z^2)} \]

\[ = (2\pi)^3 S^3 (k+k') \frac{1}{2} \frac{1}{(1+k^2 z^2)} \]

Superhorizon scales: \( |kz| \ll 1 \)
\[ \langle \delta \phi \delta \phi' \rangle = \left( \frac{\pi}{2} \right)^3 \, \delta^3(k+k') \frac{H^2}{2k^3} \]

Two point correlation function is the Fourier transform of the power spectrum, so:

\[ \delta(\tau) \sim H^2 \sqrt{\frac{1}{k^3}} \int e^{ik\cdot x} \, d^3k \]

which diverges for \( k = 0 \). (IR)

IR divergence is because we are in pure De-Sitter space. A more realistic calculation would be quasi-De-Sitter where IR divergence does not occur.
\[ P_a (n+1) = P_a (n) \bullet - \sum_b \gamma_{ba} P_a (n) + \sum_b \gamma_{ab} P_b (n) \]  

\[ = \sum_b \delta_{ab} P_b (n) + \sum_b \left( -\sum_c \gamma_{ca} \delta_{ab} + \gamma_{ab} \right) P_b (n) \]

\[ \Gamma_{ab} = -\delta_{ab} \sum_c \gamma_{ca} + \gamma_{ab} \]

From detailed balance,
\[ \gamma_{ab} n_b = \gamma_{ba} n_a \]

\[ \Rightarrow \frac{\gamma_{ab}}{\gamma_{ba}} = \frac{n_a}{n_b} = e^{S_a - S_b} \]

\[ \Rightarrow \gamma_{ab} e^{S_b} = \gamma_{ba} e^{S_a} \]

\[ M_{ab} = M_{ba} \]

Let \( P_a = e^{S_a/2} \phi_a \), transition equation becomes

\[ \phi_a (n+1) = \phi_a (n) - \sum_c \phi_a (n) + \sum_b M_{ab} e^{-S_b} e^{S_a} \frac{S_a}{2} \phi_b (n) \]

\[ = \phi_a (n) + \sum_b \left( -\sum_c \delta_{ab} M_{ca} e^{-S_a} + e^{-\frac{S_a}{2}} M_{ab} e^{-\frac{S_b}{2}} \right) \phi_b (n) \]

transition matrix manifestly symmetric.
zero mode \( \Phi_a \propto e^{\frac{S_a}{2}}: \)

\[
0 \neq \sum_b \left( -S_{ab} \sum_c \epsilon_{abc} M_{ac} e^{-S_a} + e^{-\frac{S_a}{2}} M_{ab} e^{-\frac{S_b}{2}} \right) e^{\frac{S_b}{2}}
\]

\[
= -\sum_c M_{ac} e^{-S_a} e^{\frac{S_a}{2}} + \sum_b e^{-\frac{S_a}{2}} M_{ab} = 0
\]

all other eigenvalues are non-positive:

let them be \( \lambda_i \), then in equation (\#)

the full transition matrix for \( \Phi_a \) is

\[
S_{ab} + \Gamma_{ab}
\]

The eigenvalues are \( 1 + \lambda_i \).

But since probability is conserved,

the full transition matrix must have

\[
1 + \lambda_i \leq 1, \quad i = 1, 2, \ldots
\]

otherwise \((S_{ab} + \Gamma_{ab})^n \sim (1 + \lambda_i)^n \) diverges.

Therefore \( \lambda_i \leq 0 \).
4.2 \quad V(\phi) = m^2 \phi \quad 0 < P < 2

\[ N = \int_H \phi \, d\phi' = \int_H \phi' \, d\phi = \int \frac{3H^2}{3H^2} \, d\phi' \]

Slow Loa: \quad 3H^2 \phi^2 - V, \psi

\[ 3H^2 \phi^2 - V \]

\[ N = \int \frac{V}{\mu^2 V} \, d\phi' \]

\[ V, \psi = \mu \phi^2 - \frac{1}{2} \phi^2 \]

\[ V = \frac{\phi}{\mu} \]

\[ N = \int \frac{\phi}{\mu^2} \, d\phi' = \frac{1}{2} \left( \phi^2 - \phi_{eq}^2 \right) \]

\[ \phi_{eq} = \frac{1}{\mu^2} \left( \phi^2 \right) = \frac{1}{2} \mu^2 \phi^2 \]

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\[ \phi_{eq} = \frac{1}{\mu^2} \left( \phi^2 \right) = \frac{1}{2} \mu^2 \phi^2 \]
\[ N = \frac{1}{2l} \left( \frac{\phi^2 - P}{4} \right) \]

\[ N \geq 60 \text{ To solve B.B problems.} \]

\[ \frac{1}{2l} \frac{\phi^2 - P}{4} > 60 \]

\[ \phi_m^2 > \frac{(60 + P)2P}{4} \]

\[ \phi_m > \sqrt{(60 + P)2P} \frac{l}{4} \]

\[ P \approx 1: \phi_m \sim P \ll 60 \]

\[ \phi_m \gtrsim \frac{\sqrt{120P}}{l} \text{ P.e.m. } \phi_m \gtrsim 10 \text{M which is no worry.} \]

\[ P = \text{max } \phi_m \sim \sqrt{120P} \text{M } \rightarrow \text{ no false e.g. case.} \]

\[ \text{Normalisation of Power Spectrum:} \]

\[ P_s^2 = H^4 \frac{(3H^2)^3}{\phi^2} = \frac{V^3}{3(3H\phi)^2} = \frac{3m^2V_1^2}{3m^2V_1^2} \]

\[ V_1 \phi = \mu \phi^{4-2p} \phi^p \]

\[ V^3 = \mu \phi^{4-2p} \phi^p \]

\[ V_1 \phi = \mu \phi^{4-2p} \phi^p \]

\[ P_s^2 = \frac{\mu^{4-p} \phi^{p+2}}{3m^2 \phi^2} \]
\[ y_{20A} \approx \sqrt{120 \, m^2} \]

\[ P_\phi^2 \approx \frac{m^4 - p^2}{(120 \, m^2)^2} \approx 10^{-9} \text{ (cos2)} \]

\[ m^4 - p^2 \approx 3 \, m_p^6 \, p^2 \cdot 10^{-9} \]

\[ P = 2 : \quad V(\psi) = \frac{m^2 \, \psi^2}{2} = \frac{1}{2} m^2 \psi^2 \]

\[ \frac{1}{2} m^2 \approx \frac{3 \, m_p^6 \cdot 4 \cdot 10}{(120 \cdot 2 \, m_p^2)^2} \]

\[ \approx \frac{10^{-8}}{m_p^2} \]

\[ \approx \frac{10^{-12}}{m_p^2} \]

Even though \( \phi_{\text{max}} \approx 10 \, m_p \)

\[ m \sim 10 \, m_p \quad \text{\textit{WEIER CORD THERMIONIC CATHODE}} \]

10 terms \( V \ll m_p \)

\[ \text{PARTIAL CORD THERMIONIC} \]
4. b) Radiative stability: \( \mu^- \bar{\nu}^\mu \)

\( p = 2 \implies \text{radiatively stable} \)

\( \mu^\mu \bar{\nu}^\mu \)

For \( p < 2 \), there is a term of \( \sim \phi^2 \) allowed of the form \( 1 \phi \pm \phi \sqrt{2} \phi \).

This term has the ability to drive the scale to \( M_{\text{eff}} \), as opposed to \( M \).

Thus, the \( p = 2 \) term is not radiatively stable, similar to the standard model lagrangian.

Field range is limited in order to make it such that things are still consistent.

For smaller \( p \), the range gets restricted even further.