1.1. Group 18

a) $\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(-\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi) = 0$

$\Rightarrow 0 = \partial_\mu\left(\frac{1}{\eta^2}\partial_\eta\phi - \frac{1}{\eta^2}\vec{\nabla}\phi\right) = \frac{1}{\eta^2}\partial_\eta^2\phi - \frac{2}{\eta^3}\partial_\eta\phi - \frac{1}{\eta^2}\vec{\nabla}^2\phi$

$\Rightarrow \partial_\eta^2\phi - \frac{2}{\eta}\partial_\eta\phi = -k^2\phi \circledast$ (Fourier Transformation)

Substitute $f \sim (1+i|k|\eta)e^{-i|k|\eta}$ into $\circledast$

$$\begin{aligned}\partial_\eta^2 f - \frac{2}{\eta}\partial_\eta f &= (|k|^2 - i|k|^3\eta)e^{-i|k|\eta} - \frac{2}{\eta}(k^2\eta)e^{-ik\eta} \\ &= -|k|^2(1+i|k|\eta)e^{-i|k|\eta} \\ &= -k^2 f \quad //\end{aligned}$$

b) $\mathcal{L} = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \frac{1}{\eta^2}\left((\partial_\eta\phi)^2 - (\vec{\nabla}\phi)^2\right)$

$$\begin{aligned}\pi(x) = \frac{\partial\mathcal{L}}{\partial(\partial_\eta\phi)} &= \frac{1}{\eta^2}(\partial_\eta\phi) \\ &= \frac{1}{\eta^2}\int d^3x\left[a^+\{i|k| - i|k|(1+i|k|\eta)\}e^{-i|k|\eta}\right. \\ &\qquad \left. + a\{-i|k| + i|k|(1-i|k|\eta)\}e^{i|k|\eta}\right]e^{-i\vec{k}\cdot\vec{x}} \\ &= \int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\frac{|k|^2}{\eta}(a^+e^{-i|k|\eta} + a e^{i|k|\eta})\end{aligned}$$

Let $f = A(1+i|k|\eta)e^{-i|k|\eta}$

Then,

$$\begin{aligned}i\delta^3(\vec{x}' - \vec{x}) &= [\phi(x'), \pi(x)] \\ &= \int d^3k'\int d^3k\, e^{i\vec{k}'\cdot\vec{x}'}e^{i\vec{k}\cdot\vec{x}}\left[a^+f + af^+, \frac{|k|^2}{\eta}(Aa^+e^{-ik\eta} + Aa\,e^{ik\eta})\right] \\ &= \int d^3k'\int d^3k\,\frac{A^2k^2}{\eta}\{(1+i|k|\eta)e^{i(k-k')\eta}[a^+, a] \\ &\qquad + (1-i|k|\eta)e^{i(k'-k)\eta}[a, a^+]\}e^{i\vec{k}'\cdot\vec{x}' + i\vec{k}\cdot\vec{x}} \\ &= +\frac{A^2k^2}{\eta}\{-(1+i|k|\eta) + (1-i|k|\eta)\} \\ &= -\frac{2iA^2k^2}{\eta}\end{aligned}$$

$\Rightarrow \underline{A = \frac{1}{\sqrt{2}}\frac{1}{k^{3/2}}}$ //

Group 18 .

Problem Sets for dS/CFT (Pr Maldacena).

1.1) Review of the 2-pt function computation in dS.
(sequel)

c) The Bunch-Davies vacuum $|BD\rangle$ is choosen such that $a|BD\rangle = 0$ for $\eta \to -\infty$ (very early times) in the dS space.
In fact, we have $-\eta^2 + \vec{x}^2 = R^2$ in dS space where R is the radius of curvature, thus two observers separated by a small distance ($\vec{x}^2 \ll \eta^2$) sees $|R| \sim |\eta|$ and so a huge curvature (asymptotically infinite). The dS space is there well approximated by the Minkowski space $\mathcal{M}_4$ and it is adapted to take the Bunch-Davies vacuum in this way.

The 2-pt function at instant time η in momentum's space is given by:

$$\langle BD| \phi_{\vec{k}}(\eta) \underbrace{\phi_{-\vec{k}}(\eta)}_{= \phi^*_{\vec{k}}(\eta) \text{ because the scalar field is real}} |BD\rangle$$

$$= \underbrace{\langle BD|\left(f_{\vec{k}} a^\dagger_{\vec{k}}\right.}_{=0} + f^*_{\vec{k}} a_{\vec{k}} \Big)\Big(f^*_{\vec{k}} a^\dagger_{\vec{k}} + \underbrace{\left. f_{\vec{k}} a_{\vec{k}} \right)|BD\rangle}_{=0}$$

for early times.

$= |f_{\vec{k}}|^2$ where $f_{\vec{k}}$ have been computed in b).

$$f_{\vec{k}} = \frac{1}{\sqrt{2}\, k^{3/2}} (1 + ik\eta) e^{-ik\eta}.$$

$$\Rightarrow \langle BD| \phi_{\vec{k}}(\eta) \phi_{-\vec{k}}(\eta) |BD\rangle = \frac{1}{2k^3} (1 + k^2\eta^2)$$

which is of order $\frac{1}{k}$ for early times.
This relation is true for all $\vec{k}$ inside the horizon i.e. $k < \eta^{-1} \Leftrightarrow k < H^{-1}$ where $H = \frac{1}{R}$ is the Hubble radius of the dS space.

d) The Fourier Transform^{-1} of $\langle BD|\phi_{\vec{k}}\phi^*_{\vec{k}}|BD\rangle$ is

$$\langle BD|\phi(\vec{x})\phi(\vec{x})|BD\rangle = \int \frac{d^3k}{(2\pi)^3}\langle BD|\phi_{\vec{k}}\phi^*_{\vec{k}}|BD\rangle e^{i\vec{k}\cdot\vec{x}}$$

$$= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2k^3}(1+k^2\eta^2)e^{i\vec{k}\cdot\vec{x}}.$$

$$= \frac{1}{2(2\pi)^2}\int_{+\infty}\frac{dk}{k}\int_{-1}^{+1} d\cos\theta \,(1+k^2\eta^2)\,e^{irk\cos\theta}$$

$$= \frac{1}{(2\pi)^2}\int_0 \frac{dk}{k}(1+k^2\eta^2)\frac{\sin(kr)}{kr}$$. which is not so useful...

The second term can be computed :

$$\frac{1}{(2\pi)^2}\int\frac{dk}{k}k^2\eta^2\frac{\sin kr}{kr} = \frac{\eta^2}{r(2\pi)^2}\,\mathrm{Im}\left\{\int_0^\infty dk\, e^{-\varepsilon k+ikr}\right\}$$

$$= \frac{\eta^2}{(2\pi)^2}\frac{1}{r^2}.$$

The first term is IR divergent.
This two terms are dS invariant because $1+k^2\eta^2$ is dS invariant (under rescaling, rotations and spatial translations). So this 2pt function is dS-invariant.

To come back on the divergence, we can say that the abscence of mass for the scalar field and the dS spacetime geometry are the physical origin of it.

Group 18

Creminelli #6:

The 3-point function is: (set $\eta=0$)

$$\langle \varphi_{k_1}(\eta=0)\,\varphi_{k_2}(\eta=0)\,\varphi_{k_3}(\eta=0)\rangle =$$

$$\langle 0|\bar{T} e^{i\int_{-\infty(1+i\epsilon)}^{0} H_I(\eta')d\eta'}\,\varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\, T e^{-i\int_{-\infty(1-i\epsilon)}^{0} H_I(\eta')d\eta'}|0\rangle$$

$$\simeq \langle 0|\bar{T}\left(1+i\int_{-\infty(1+i\epsilon)}^{0} H_I(\eta')d\eta'\right)\varphi_{k_1}\varphi_{k_2}\varphi_{k_3} T\left(1-i\int_{-\infty(1-i\epsilon)}^{0} H_I(\eta')d\eta'\right)|0\rangle$$

$$=\langle 0|-i\int_{-\infty(1-i\epsilon)}^{0} d\eta' \left[\varphi_{k_1}\varphi_{k_2}\varphi_{k_3}, H_I(\eta')\right]|0\rangle$$

Compute $H_I(\eta')$ in Fourier space: $H_I=\int \frac{M}{6}\varphi^3(x)\left(\frac{-1}{H\eta'}\right)d^3x$

$$H_I(\eta')=\frac{M}{6}\int d^3x \int\frac{d^3p_1}{(2\pi)^3}e^{i\vec{p}_1\cdot\vec{x}}\varphi_{p_1}(\eta')\int\frac{d^3p_2}{(2\pi)^3}e^{i\vec{p}_2\cdot\vec{x}}\varphi_{p_2}(\eta')$$
$$\int\frac{d^3p_3}{(2\pi)^3}e^{i\vec{p}_3\cdot\vec{x}}\varphi_{p_3}(\eta')\left(\frac{-1}{H\eta'}\right)$$

$$=\frac{M}{6}\int\prod_{i=1}^{3}\frac{d^3p_i}{(2\pi)^3}\varphi_{p_1}(\eta')\varphi_{p_2}(\eta')\varphi_{p_3}(\eta')(2\pi)^3\delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3)\left(\frac{-1}{H\eta'}\right)$$

$$\Rightarrow \langle \varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\rangle = -\frac{iM}{6}\int_{-\infty(1-i\epsilon)}^{0} d\eta'\left(\frac{-1}{H\eta'}\right)\int\prod_{i=1}^{3}\frac{d^3p_i}{(2\pi)^3}\langle 0|\left[\varphi_{k_1}\varphi_{k_2}\varphi_{k_3},\varphi_{p_1}(\eta')\right.$$
$$\left.(2\pi)^3\delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3)\,\varphi_{p_2}(\eta')\varphi_{p_3}(\eta')\right]|0\rangle$$

First term:

$$\langle 0|\varphi_{k_1}(0)\varphi_{k_2}(0)\varphi_{k_3}(0)\varphi_{p_1}(\eta')\varphi_{p_2}(\eta')\varphi_{p_3}(\eta')|0\rangle$$

To obtain fully connected diagrams, only contract k_i's with p_j's.

$$\Rightarrow \; = \langle \varphi_{k_1}(0)\varphi_{p_1}(\eta')\rangle\langle \varphi_{k_2}(0)\varphi_{p_2}(\eta')\rangle\langle \varphi_{k_3}(0)\varphi_{p_3}(\eta')\rangle$$
$$+ \text{ 5 permutations}$$

$$= \frac{H^2}{2k_1^3}\delta^3(\vec{k}_1-\vec{p}_1)(1-ip_1\eta')e^{ip_1\eta'}\frac{H^2}{2k_2^3}\delta^3(\vec{k}_2-\vec{p}_2)\cdot$$

$$\times(1-ip_2\eta')e^{ip_2\eta'}\frac{H^2}{2k_3^3}(1-ip_3\eta')e^{ip_3\eta'} + \text{permut.}$$

Plug back into above expression. The 5 permutations give similar contributions. The reverse terms in the commutator are complex conjugate of the term above.

$$\langle \varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\rangle = -iM\int_{-\infty(1-i\epsilon)}^{0} d\eta' \frac{(2\pi)^3}{-H\eta'}\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^6}{8k_1^3k_2^3k_3^3}$$

$$(1-ik_1\eta')(1-ik_2\eta')(1-ik_3\eta')e^{i|\vec{k}_1+\vec{k}_2+\vec{k}_3|\eta'} + \text{h.c.}$$

$$\langle \varphi_{k_1} \varphi_{k_2} \varphi_{k_3} \rangle = \frac{-iM\, H^5}{8 k_1^3 k_2^3 k_3^3} (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot$$

$$\times \int_{-\infty(1-i\epsilon)}^{0} \frac{d\eta'}{\eta'} (1 - i k_1 \eta')(1 - i k_2 \eta')(1 - i k_3 \eta') e^{i|\vec{k}_1 + \vec{k}_2 + \vec{k}_3|\eta'} + \text{h.c.}$$

$$= (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \frac{M H^5}{8 k_1^3 k_2^3 k_3^3} \frac{2}{(k_1 + k_2 + k_3)^3} \left(\sum_i k_i^3 + 4 \sum_{i \neq j} k_i k_j^2 + 11\, k_1 k_2 k_3 \right)$$

using numerical integration

Group 18

de Sitter hyperboloid:

3.1

$$-x_0^2 + \sum_{i=1}^{4} x_i^2 = \alpha^2 \qquad \text{(embedded in } \mathbb{R}^{4,1}\text{)}$$

For flat slicing, define

$$x_0 = \alpha \sinh(t/\alpha) + \frac{r^2}{2\alpha} e^{t/\alpha}$$

$$x_1 = \alpha \cosh(t/\alpha) - \frac{r^2 e^{t/\alpha}}{2\alpha}$$

$$x_i = e^{t/\alpha} y_i \quad , \qquad i = 2,3,4$$

$$r^2 = y_2^2 + y_3^2 + y_4^2$$

$\Rightarrow$ plug these in and we get

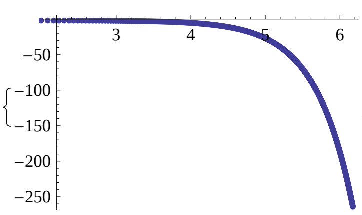
$$\boxed{ds^2 = -dt^2 + e^{2t/\alpha} \underbrace{\sum_i dy_i^2}_{\text{flat space metric}}}$$

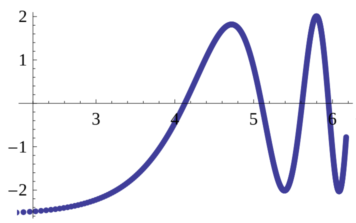
2.7 de Sitter in 4d has isometry group $O(4,1)$ with 10 generators: 3 rotations, 3 translations, 3 "boosts", 1 scaling symmetry
↳ involving time

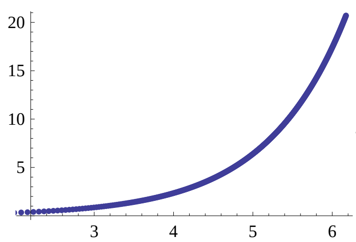
We evaluate correlation function $G(\eta_0)$ at a particular point in time, η_0. When we do this, we lose the isometries involving time because we pick out a specific point in time to evaluate the correlations. So we are only left with the 3 rotations, translations, + scale invariance - 7 symmetries.

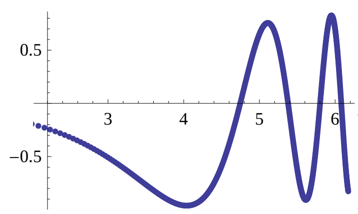
```
(* Group  18 *)

(* The system of coupled equations *)

Ωc = 0.95;
Ωm = 1;
Ωb = 0.05;
h = 0.5;
α = Sqrt[21.5 * Ωm h^2];
y[x_] := (α x)^2 + 2 α x;
yc[x_] := y[x] Ωc / Ωm;
yb[x_] := 1.68 y[x] Ωb / Ωm;
η [x_] := 2 α (α x + 1) (α ^2 x^2 + 2 α x);
x0 = 0.001;
κ0 = 0.001;
y0 := (α * x0)^2 + 2 α * x0;
η0 := 2 α (α x0 + 1) / (α ^2 x0^2 + 2 α x0);
Syst :=
  {
   D[δc[x ], x] == -κ vc[x] + 3 D[ϕ[x], x],
   D[δγ[x], x] == - 4/3 κ vγ[x] + 4 D[ϕ[x], x],
   D[vc[x], x] == - η[x] vc[x] + κ ϕ[x],
   D[vγ[x], x] == (- η[x] yb[x] vγ[x] + κ δγ[x] / 3) / (4 / 3 + yb[x]),
   D[ϕ[x], x] == - η[x] ϕ[x] + (3 η[x]^2 (vγ[x] (4 / 3 + y[x] + yc[x]) + yc[x] vc[x])) / (2 (1 + y[x]) κ),
   δc[x0] == 3/4 δγ[x0],
   δγ[x0] == -2 (1 + (3 y0)/16),
   vc[x0] == κ/η0 (δγ[x0]/4 + (2 κ^2 (1 + y0))/(9 η0^2 (4 / 3 + y0))),
   vγ[x0] == vc[x0],
   ϕ[x0] == 1
  };

(* A definition for the dots in the plots -- not very pretty but necessary *)

solution =
  {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, ... and so on ..., {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}};

(*At time x=0.1*)
```

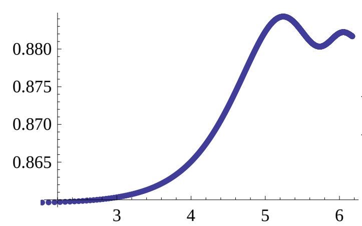
```
For[i = 1, i < 721, i++,
 {
  sol = NDSolve[Syst /. κ → i / 3, {δc, δγ, vc, vγ, ϕ}, {x, x0, 1}],
  solution⟦i, 1⟧ = Evaluate[δc[0.1] /. sol]⟦1⟧,
  solution⟦i, 2⟧ = Evaluate[δγ[0.1] /. sol]⟦1⟧,
  solution⟦i, 3⟧ = Evaluate[vc[0.1] /. sol]⟦1⟧,
  solution⟦i, 4⟧ = Evaluate[vγ[0.1] /. sol]⟦1⟧,
  solution⟦i, 5⟧ = Evaluate[ϕ[0.1] /. sol]⟦1⟧
 }
]
{ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦1⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦2⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦3⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦4⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦5⟧}, {n, 1, 720, 1}](*,Joined→True*)]}
```

{

, 

, 

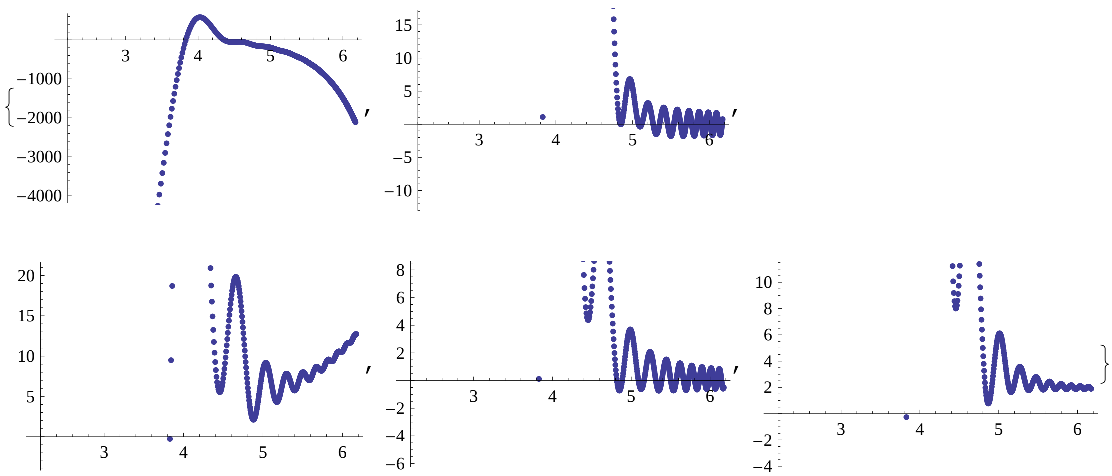
, 

, 

}

```
(*At time x=0.5*)
```

```
For[i = 1, i < 721, i++,
 {
  sol = NDSolve[Syst /. κ → i / 3, {δc, δγ, vc, vγ, ϕ}, {x, x0, 1}],
  solution⟦i, 1⟧ = Evaluate[δc[0.5] /. sol]⟦1⟧,
  solution⟦i, 2⟧ = Evaluate[δγ[0.5] /. sol]⟦1⟧,
  solution⟦i, 3⟧ = Evaluate[vc[0.5] /. sol]⟦1⟧,
  solution⟦i, 4⟧ = Evaluate[vγ[0.5] /. sol]⟦1⟧,
  solution⟦i, 5⟧ = Evaluate[ϕ[0.5] /. sol]⟦1⟧
 }
]
{ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦1⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦2⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦3⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦4⟧}, {n, 1, 720, 1}](*,Joined→True*)],
 ListPlot[Table[{Log[n / 1.5], solution⟦n⟧⟦5⟧}, {n, 1, 720, 1}](*,Joined→True*)]}
```



Group 18.

*) CMB and LSS (Professor Zaldarriaga).

Question 1).

From Ma & Bertschinger (95), we have the following dynamical equations for the density field and the divergence of the velocity field:

$$(I) \quad \delta' = -(1+w)(\theta - 3\phi') - 3\frac{a'}{a}\left(\frac{\delta P}{\delta \rho} - w\right)\delta$$

$$(II) \quad \theta' = -\frac{a'}{a}(1-3w)\theta - \frac{w'}{1+w}\theta + \frac{\delta P}{\delta \rho}\cdot\frac{k^2\delta}{1+w} - k^2\sigma + k^2\psi$$

These equations are valid for the net (massed-averaged) fields (δ_c, θ_c) and $(\delta_\gamma, \theta_c)$ of CDM and photons/baryons. In fact, because the two fluids interact by Thomson scattering, we need to add a term in the $\dot\theta_\gamma$ equation: $\frac{4}{3}\frac{\bar\rho_\gamma}{\bar\rho_b} a n_e \sigma_T (\theta_\gamma - \theta_b)$.

with $\sigma_T = 0{,}6652 \cdot 10^{-24}$ cm^2 and n_e the number density of electrons.

(I) and (II) are written in the conformal Newtonian gauge.

*) For CDM we obtain, with $w_c = 0$ and $\theta_c = k N_c$ in Fourier space:

- $$\delta_c' = -(k N_c - 3\phi') - 3H(w_c - w_c)\delta$$
$$= -k N_c + 3\phi'$$

with $'$ denoting $\partial/\partial\tau$ (conformal time)

Noting $\dot{(\ldots)} = \frac{\partial(\ldots)}{\partial x} = \tau_r \frac{\partial(\ldots)}{\partial \tau} = \tau_r (\ldots)'$ with $x = \frac{\tau}{\tau_r}$.

we thus have: $\dot\delta_c = -\kappa N_c + 3\dot\phi$

with $\kappa \equiv k \cdot \tau_r$.

- $$k N_c' = -\frac{a'}{a}\theta_c - \frac{w_\gamma'}{1+w_\gamma} + 0 - k^2\sigma + k^2\phi$$

$\uparrow$

$w_c = 0$; $c_s^2 = \frac{dP}{d\rho} = 0$; $\psi = \phi$

Group 18 | Robustness of GR (Arkani-Hamed)

Question 4)

Let us start with the action:

$$S = \int d^4x \sqrt{-g}\left[F'(A)(R-A) + F(A)\right] \quad \text{where } A \equiv A(x).$$

The equation of motion for A is given by:

$$\frac{\partial \mathcal{L}}{\partial A} = 0 = F''(A)(R-A) - \cancel{F'(A)} + \cancel{F'(A)}$$

so we must have: (i) either $A = R$

(ii) or $F''(A) = 0$.

Both these cases lead to the conclusion that S can be written as:

$$S = \int d^4x \sqrt{-g}\, F(R).$$

"Proof": (i) $A = R \Rightarrow S = \int d^4x \sqrt{-g}\left[0 + F(A=R)\right]$. ✓

(ii) $F''(A) = 0 \Rightarrow F(A) = C_1 A + C_2$. with C_1, C_2 two constants.

Then: $S = \int d^4x \sqrt{-g}\left[C_1(R-A) + C_1 A + C_2\right]$

$$= \int d^4x \sqrt{-g}\left[C_1 R + C_2\right] = \int d^4x \sqrt{-g}\, F(R) \quad ✓.$$

*) Let us now do a conformal transformation to demix $A(x)$ from the metric.

We take $g_{ab} \to \tilde{g}_{ab} = g_{ab} \cdot \Omega^2$ and so we have $\sqrt{-g} = \sqrt{-\tilde{g}}\, \Omega^{-4}$.

The scalar curvature transform as:

$$R \to \tilde{R} = \Omega^2\left[R + 6\frac{\Box\Omega}{\Omega}\right]$$

and thus $R = \Omega^2\left[\tilde{R} - 6\frac{\tilde{\Box}\Omega}{\Omega}\right]$.

Thus we have:

$$S = \int d^4x \sqrt{-\tilde{g}}\, \Omega^{-4}\left[F'(A)\cdot\Omega^2\left(\tilde{R} - 6\frac{\tilde{\Box}\Omega}{\Omega}\right) - F'(A)\cdot A + F(A)\right]$$

We choose Ω such that $F'(A) = \Omega^2$.

$$\Leftrightarrow S \overset{\downarrow}{=} \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} - \frac{6 \tilde{\Box} F'(A)^{1/2}}{F'(A)^{1/2}} - \frac{F'(A) \cdot A}{F'(A)^2} + \frac{F(A)}{F'(A)^2} \right]$$

defining $\sigma(x) = -\log F'(A)$ (i.e. $F'(A) = e^{-\sigma}$).

we have:

$$6 \frac{\Box e^{-\sigma/2}}{e^{-\sigma/2}} = 6 \frac{\tilde{g}^{\mu\nu} \nabla_\mu \nabla_\nu e^{-\sigma/2}}{e^{-\sigma/2}} = -3 e^{\sigma/2} \tilde{g}^{\mu\nu} \nabla_\mu (\partial_\nu \sigma \cdot e^{-\sigma/2})$$

$$= -3 e^{\sigma/2} \left[\tilde{g}^{\mu\nu} \nabla_\mu \partial_\nu \sigma \cdot e^{-\sigma/2} - \frac{\tilde{g}^{\mu\nu}}{2} (\partial_\mu \sigma)(\partial_\nu \sigma) e^{-\sigma/2} \right]$$

$$= -3 \tilde{g}^{\mu\nu} \nabla_\mu \partial_\nu \sigma \cdot + \frac{3}{2} (\partial \sigma)^2.$$

but the first term in the RHS is a total divergence and thus don't contribute to the action.

Defining $V(\sigma) = \frac{A}{F'(A)} - \frac{F(A)}{F'(A)^2}$, we have

for final result: $S = \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} - \frac{3}{2} \tilde{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right]$

Thus we have shown that the F(R)-gravity action was equivalent to a scalar field coupled to Einstein gravity.