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PiTP - 2010 - LECTURE 1

INTRODUCTION TO THE QUANTUM FIELD THEORY / QUANTUM GRAVITY DUALITY

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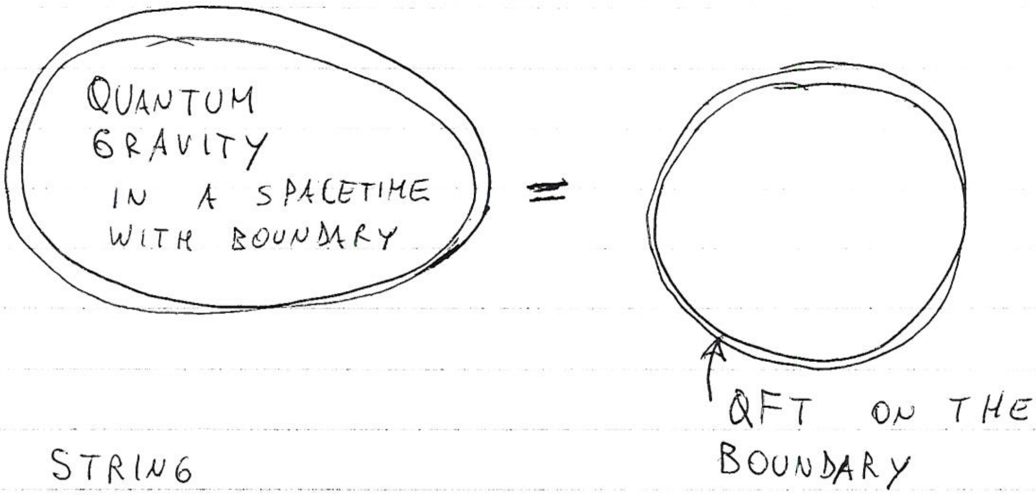
$\boxed{1}$

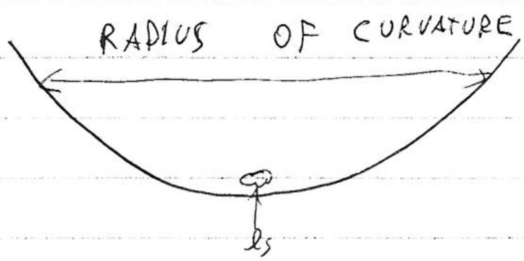
- WE GIVE A SHORT INTRODUCTION TO THE GAUGE GRAVITY DUALITY



STRING THEORIES. $\longleftrightarrow$ U(N) GAUGE THEORIES FOR LARGE N.

$$g^2_{string} \sim \frac{1}{G_N} \sim 1/N^2$$

$$\frac{SIZES}{l_{STRING}} \sim (g^2 N)^{\text{POSITIVE POWER}} \longleftrightarrow \lambda = g^2 N$$



DUALITY: $g^2 N \ll 1 \to$ PERT GAUGE THEORY

$g^2 N \gg \cdot \to$ GRAVITY IS SIMPLE.

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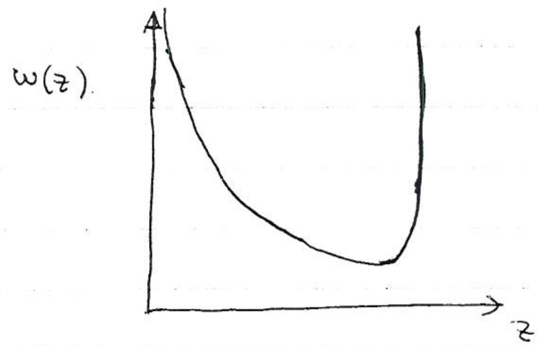
– WARP FACTORS:

$$ds^2 = \omega^2(z)\left(-dt^2 + dx^2 + dz^2\right).$$

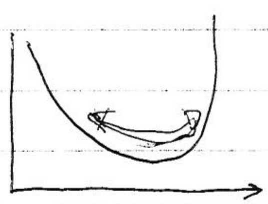
$D = d+1$ DIMENSIONS.

$d = 4$, $D = 5$



– GRAVITATIONAL POTENTIAL WELL.

- LOWEST MODES → MOVE IN 4 DIMENSIONS.
- TRUNCATE TO LOWEST MODES
  5 dim → 4 d theory – Nothing surprising
- AS WE INCLUDE HIGHER MODES
  → 4d WITH MORE FIELDS.



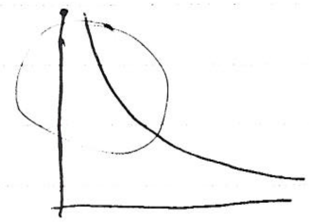
ALL RADIAL MODES

- NAIVELY → ∞ NUMBER OF FIELDS.
- IN A THEORY OF GRAVITY
  ⇝ A FINITE NUMBER OF FIELDS IS ENOUGH.
  (BLACK HOLES TRUNCATE THE SPECTRUM...)

$$N_{\text{FIELDS}} \approx \frac{R^3_{\text{CURV}}}{G_N^{(5)}}$$

. SPECIAL CASE: ANTI-DE-SITTER.

$$\omega = \frac{1}{z} \qquad ds^2 = R^2\frac{(-dt^2 + d\vec{x}^2)}{z^2}$$

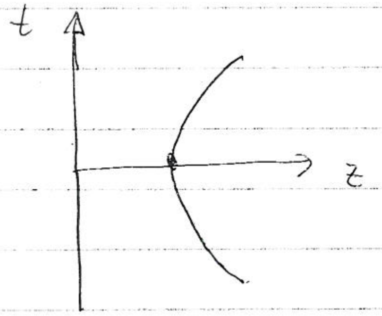


. MASSIVE PARTICLE OF MASS M.

$E \sim \frac{\partial}{\partial t}$ vs proper time $\frac{\partial}{\partial \tau}$ $\qquad d\tau^2 = \frac{R^2}{z^2}dt^2$.

$$E = \frac{R}{z_0}\cdot E_{PROPER} \sim \frac{Rm}{z_0}$$

PARTICLE AT REST AT z_0 (WILL START MOVING.



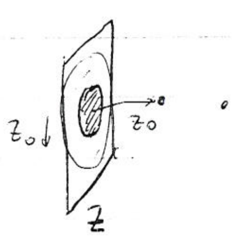
$z_0 \to \infty \qquad E \to 0$

$z_0 \to 0 \qquad E \to \infty$

. SCALING SYMMETRY $\qquad t, x \to \lambda(t, x)$

$\qquad z \to \lambda z$.

. IF PARTICLE AT z_0 → SIZE ~ z_0 in BDY.



. DO A SCALE TRANSF. ON BOTH SIDES.

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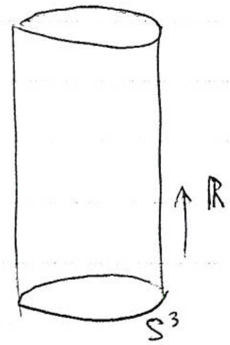
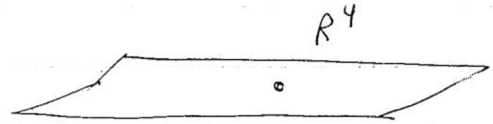
- ANTI-DE-SITTER.



$$ds^2 = R^2\left[-d\tau^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho \, d\Omega_3^2\right]$$

$\tau \to i\tau$.

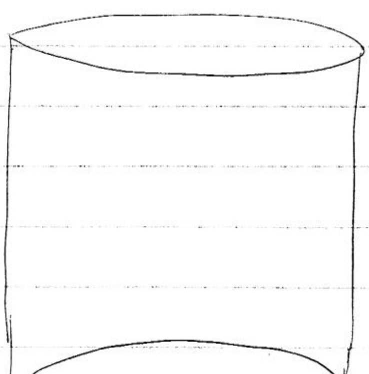


CYLINDER $\longleftarrow$

STATES $\longleftrightarrow$ OPERATORS IN $\mathbb{R}^4$

STATES IN CFT ON $\mathbb{R} \times S^3$

$=$ STATES FOR AdS IN GLOBAL COORDINATES.



$\Delta =$ DILATATION IN $\mathbb{R}^4$

$= \dfrac{\partial}{\partial \tau}$ IN CYLINDER.

$\Delta = mR$ MASSIVE PARTICLE $mR \gg 1$.

- FOR SMALLER MASSES $\rightarrow$ QUANTUM MECHANICS

(ACTION $\sim e^{-(mR)\tau}$

- POTENTIAL $\approx \cosh^2\rho$ $\sim$ FREQ

$\rightarrow$ WAVE EQN FOR A PARTICLE IN THIS BACKGROUND.

(WE WILL SEE AN EASIER WAY TO COMPUTE IT).

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SCALAR FIELD

$\Delta = 2 + \sqrt{(mR)^2 + 4}$; $d = 4$

MARGINAL COUPLING.

- $m = 0$ → also $S = 2$ gravitons → DIM OF STRESS TENSOR

- STRING THEORY

- STRING STATES: $m \sim \frac{1}{\sqrt{\alpha'}} \propto \sqrt{T}$ STRING TENSION.

$$\Delta_{\text{1-st massive string state}} = \frac{R}{\sqrt{\alpha'}} = \frac{R}{l_{STRING}}$$

- GRAVITY IS A GOOD APPROX IF

$$\frac{(\text{Energy spin} > 2)}{(\text{Energy spin } 2)} \sim \frac{R}{\sqrt{\alpha'}} \gg 1$$

- SPECTRUM IN GRAV or STRING THEORY

APPROX A FOCK SPACE

1 particle — 2 particle

E_1 — $E_1 + E_2 + \mathcal{O}(G_N) + \mathcal{O}(g^2) + \mathcal{O}(1/N^2)$.

↓

SINGLE TRACE

$\mathrm{Tr}[F^2]$ — $\mathrm{Tr}[F^2]\,\mathrm{Tr}[F^4]$ DOUBLE TRACE.

- IN WEAKLY COUPLED THEORIES.

MANY LIGHT OPERATORS WITH SPIN > 2.

$$Tr\left[\phi \left(\overleftrightarrow{\partial}_+\right)^s \phi \right] \sim \Delta - S = 2 + \mathcal{O}(g^2 N).$$

- ALL THESE STATES SHOULD GET A LARGE ANOMALOUS DIMENSION IF GRAVITY BECOMES A GOOD **APPROXIMATION** –

- WHAT ARE THE LIGHT STATES.
  - STRESS TENSOR, $\Delta = 4$, → MASSLESS GRAVITON.

LOCAL FIELD THEORIES $\longleftrightarrow$ GRAVITY IN THE INTERIOR

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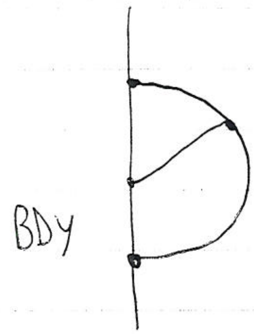
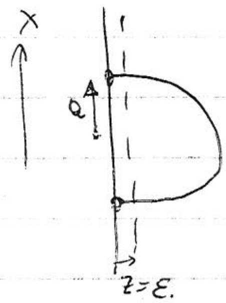
CORRELATION FUNCTIONS

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_N) \rangle \qquad \mathcal{O} \sim \mathrm{Tr}[\dots]$$





$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \rangle = e^{-\text{"}(\text{LENGTH})^{\text{"}\infty}}$$

GEODESICS → SEMI-CIRCLES $\quad z^2 + \cancel{x}^2 = a^2$.

LENGTH $\sim \quad \int ds = \int_{z=\varepsilon} \frac{d\theta}{z} \sim \int_{\theta \sim \varepsilon/a} \frac{d\theta}{\sin\theta} \approx 2 \log \varepsilon/x$

$$\langle \quad \rangle \sim e^{-mR\, 2 \log \varepsilon/x} \sim \left(\frac{\varepsilon}{a}\right)^{2(mR)} \sim \left(\frac{\varepsilon}{a}\right)^{2\Delta}$$

IF mR IS NOT $mR \gg 1$

→ FULL QM PATH INTEGRAL → SOLVE WAVE EQN.

→ RELATIVELY EASY.

$$\phi\Big|_{z=\varepsilon} = \phi_0(\vec{x}) \qquad \left\langle e^{\int \mathcal{O}\, \phi_0(\vec{x})} \right\rangle_{4D} \approx e^{-S_{CLASS}[\phi]}$$

↑ ON THE SOLUTIO

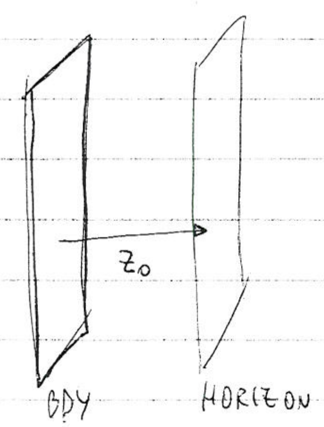
(EXCERCISE).

$$Z_{\substack{\text{FIELD}\\\text{THEORY}}}\left[\begin{array}{l}\text{- BOUNDARY METRIC.}\\\text{- COUPLINGS.}\end{array}\right] = Z_{\text{GRAVITY}}$$

WITH BOUNDARY CONDITIONS.
→ BOUNDARY METRIC.
BOUNDARY CONDITIONS FOR FIELDS, ET

SUM OVER <u>ALL</u> MANIFOLDS WITH THESE BOUNDARY CONDITIONS

- BLACK HOLES



$z_0 \propto \beta$ BY CONF SYMMETRY.

$$\beta F = -C V_3 \underbrace{\beta^{-3}}_{\text{CONF.}}$$

$$e^{\frac{1}{G_N}\int\sqrt{g}\,(R+\Lambda)} \rightsquigarrow \frac{(R^{ADS})^3}{G_N} \sim C.$$

FREE FIELDS.

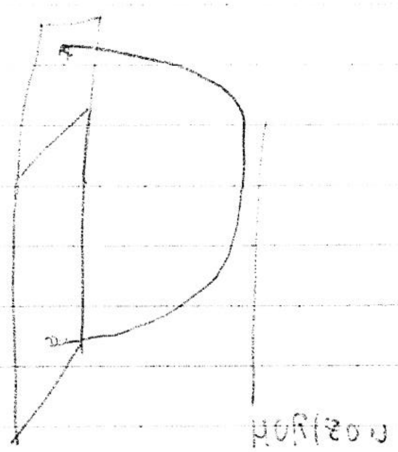
$$\beta F \sim -C V_3 \beta^{-3}.$$

$$S \propto C\, V_3 \beta^{-3} \sim C V T^3$$

Ex: CALCULATE ENTROPY FREE THEORY IN ADS_5 IN GLOBAL COORDINATES → CHECK CONSISTENCY

- WE CAN COMPUTE THERMODYNAMIC PROPERTIES OF STRONGLY COUPLED THEORIES VIA SIMPLE GRAVITY SOLUTIONS.

- CORRELATION FUNCTIONS IN THE PRESENCE OF A BLAK HOLE



$$\langle \mathcal{O}\mathcal{O} \rangle \sim e^{-\omega_I t}$$

DECAY BECAUSE PARTICLE IS FALLING INTO BLACK HOLE

QUASINORMAL MODES

$$\omega = \omega_{REAL} + i\,\omega_I$$

ABSORPTION BY BLACK HOLE

- $\langle T\,T \rangle \rightarrow$ INFORMATION ABOUT FLOW OF THE FLUID

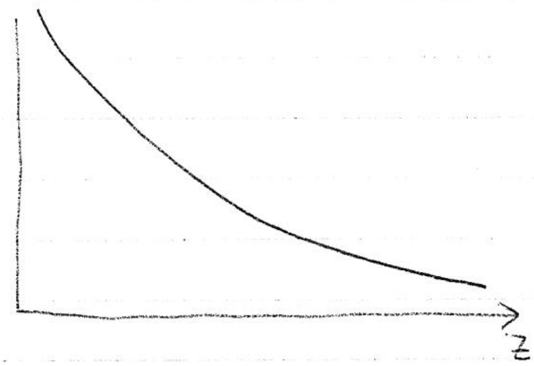
η = SHEAR VISCOSITY $\quad \frac{d(\rho \vec{v})}{dt} \simeq \eta \nabla^2 v$

$\rightarrow$ ABSORPTION $\sim$ AREA

$\frac{\eta}{s} = \frac{1}{4\pi}$ "SMALL VALUE" "PERFECT FLUID"

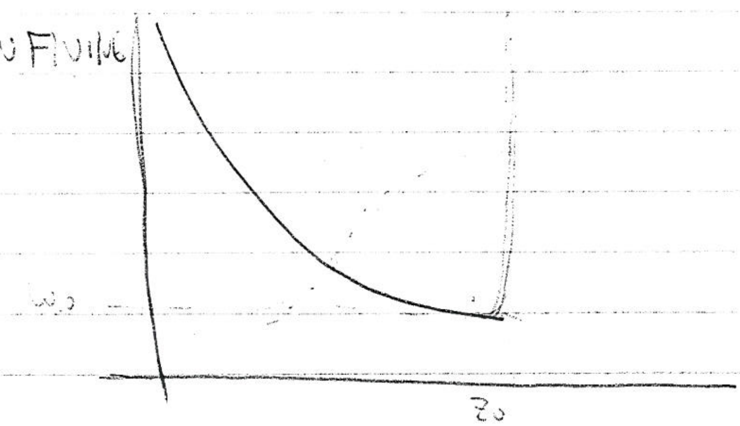
- CONFINING THEORIES

AdS.



NO MASS GAP

CONFINING



MASS GAP - $m_4 \approx w_0 m_{proper}$

GRAVITON → LOCALIZED IN z → $m_4 \neq 0$. MASSIVE SPIN 2 PARTICLE

Eq: EXTRA POLARIZATIONS

. STRING TENSION ←

$T \sim w_0^2 T_F$

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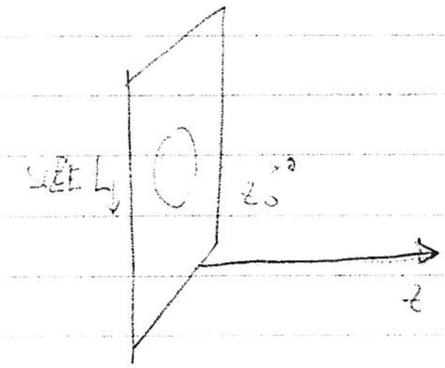
RADIAL DIRECTION



$$ds^2 = \frac{dx^2 + dz^2}{z^2}$$

$L \sim z_0$

- SMALL Z → SMALL L → UV OF BDY THEORY

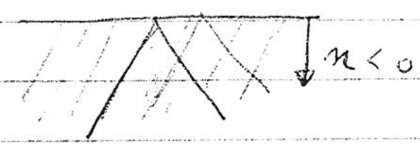
- ΔX FIXED AS $z \to 0$ ⇒ PROPER DISTANCE GOES TO ∞.

- POINTS BECOME DISCONNECTED
- LOCAL FIELD THEORY

- SIMILAR TO dS

$$ds_{dS} = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2}$$

CONF TIME.



- HORIZON.
- SEPARATE UNIVERSES @ LATE TIMES

- DIFFERENCES: - AdS CAN BE COMPLETELY STABLE (IN SUSY CASES)
- dS ALWAYS DECAYS

$\Psi[g^d, \phi^d] = Z_{FT}$

EUCLIDEAN AdS/CFT

$\mathbb{H}$ / CFT.

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GLOBAL SYMMETRIES_

- GLOBAL SYMMETRY IN THE BDY THEORY

$\rightarrow$ GAUGE SYMMETRY IN THE BULK.

(BLACK HOLES SHOULD PRESERVE IT)

$A_\mu \longleftrightarrow J_\mu$

A_μ ↑ BULK ↑ massless

J_μ ↑ CURRENT DIMENSION $d-1$

- SUPER SYMMETRY

$\rightarrow$ SUPERGRAVITY.

- CONFORMAL SYMMETRY:

- SCALINGS - DILATATIONS - SPECIAL CONFORMAL TRANSFORMATIONS.

$SO(2,4)$ → POINCARE + $x^\mu \rightarrow \frac{x^\mu}{x^2}$ (INVERSI... in $O(2,4$...

$[P_\mu, K_\nu] \sim \delta_{\mu\nu} D + \underbrace{M_{\mu\nu}}_{\text{LORENTZ}}$

OPERATORS $\mathcal{O}(0)$, $K^\mu \mathcal{O} = 0$. P, K LIKE CREATION & ANIHILATION OPERATORS

(IN 4-d) $\Delta > 1$ IF $S=0$, $\Delta = 1$ → FREE FIELD.

$\Delta - S \geq 2$ $S \neq 0$ $\Delta = S + 2$ → CONSERVED CURRENT_