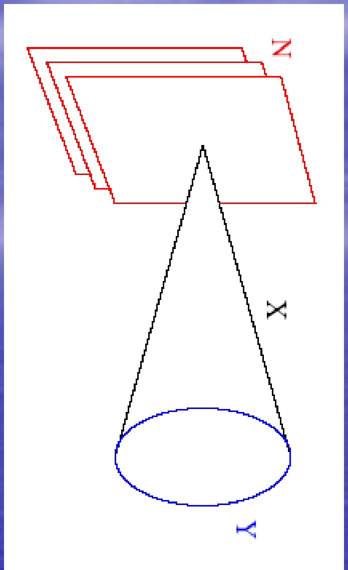
Conebranes



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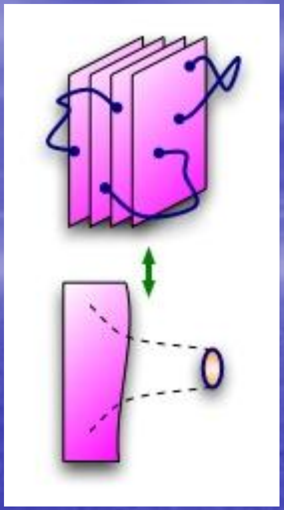
D-Branes vs. Geometry



- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric U(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of type IIB closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

whose radius is related to the coupling by

$$L^4 = g_{\rm YM}^2 N \alpha'^2$$

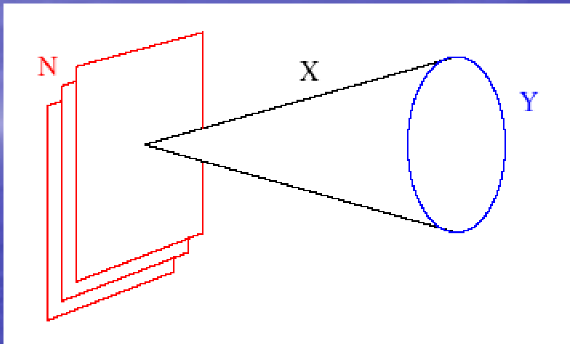
Super-Conformal Invariance

- In the $\mathcal{N}$=4 SYM theory there are 6 scalar fields ϕ^I (it is useful to combine them into 3 complex scalars: Z^j) and 4 gluinos interacting with the gluons. All the fields are in the adjoint representation of the U(N) gauge group.
- The Asymptotic Freedom is canceled by the extra fields; the beta function is exactly zero for any complex coupling. The theory is invariant under scale transformations $x_\mu \to \lambda x_\mu$. It is also invariant under space-time inversions $x_\mu \to x_\mu / x^2$. The full super-conformal group is SU(2,2|4).

Conebrane Dualities

- To reduce the number of supersymmetries in AdS/CFT, we may place the stack of N D3-branes at the tip of a 6-d Ricci-flat cone X whose base is a 5-d Einstein space Y:

$$ds_X^2 = dr^2 + r^2 ds_Y^2$$



- Taking the near-horizon limit of the background created by the N D3-branes, we find the space AdS_5 x Y, with N units of RR 5-form flux, whose radius is given by

$$L^4 = \frac{\sqrt{\pi}\kappa N}{2\,\mathrm{Vol}(Y)} = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\mathrm{Vol}(Y)}$$

- This type IIB background is conjectured to be dual to the IR limit of the gauge theory on N D3-branes at the tip of the cone X.

Orbifold Cones

Kachru, Silverstein; Lawrence, Nekrasov, Vafa

- The simplest set of examples is provided by cones that are orbifolds R^6/Γ , where Γ is a subgroup of the rotation group SO(6)~SU(4).
- For abelian orbifolds, all group elements can be brought to the form
- For Z_k orbifolds, the n-th group element is specified by three integers m_i defined mod k: $x_i = nm_i/k$.
- If none of the eigenvalues of the generator = 1, then all SUSY is broken; if one of the eigenvalues = 1, then $\mathcal{N}=1$ SUSY is preserved; if two of the eigenvalues = 1, then $\mathcal{N}=2$ SUSY is preserved.

$$\begin{pmatrix} e^{2\pi i x_1} & 0 & 0 & 0 \\ 0 & e^{2\pi i x_2} & 0 & 0 \\ 0 & 0 & e^{2\pi i x_3} & 0 \\ 0 & 0 & 0 & e^{-2\pi i(x_1+x_2+x_3)} \end{pmatrix}$$

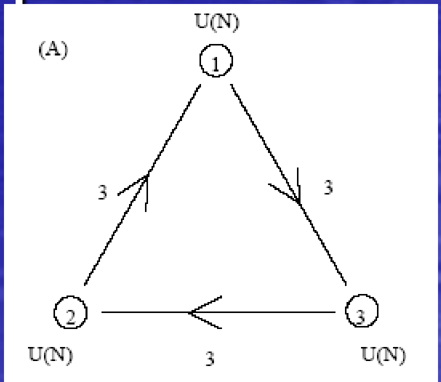
- The action in the n-th twisted sector on 3 complex coordinates of C^3, $Z^1 = X^1 + iX^2\ , \quad Z^2 = X^3 + iX^4\ , \quad Z^3 = X^5 + iX^6$

  and their complex conjugates, is

$$R(g_n) = \mathrm{diag}(\omega_k^{n(m_1+m_2)}, \omega_k^{n(m_1+m_3)}, \omega_k^{n(m_2+m_3)}, \omega_k^{-n(m_1+m_2)}, \omega_k^{-n(m_1+m_3)}, \omega_k^{-n(m_2+m_3)})$$

  where $\omega_k = e^{2\pi i/k}$

- If none of these phases = 1, then the orbifold acts freely on S^5/Γ. (The tip of the cone is a fixed point that is removed in the basic near-horizon limit.)
- A well-known example of a freely-acting orbifold is Z_3 with m_i=1. Since one of the eigenvalues of the generator = 1, i.e. $\Gamma \subset SU(3)$, this orbifold preserves $\mathcal{N}$=1 SUSY. The arrows represent bi-fundamental chiral superfields.



Construction of the quiver gauge theories Douglas, Moore

- Gauge theory on N D3-branes at the tip of R^6/Γ is found by applying projections to the U(Nk) gauge theory on the covering space. Retain only the fields invariant under the orbifold action combined with conjugation by a U(Nk) matrix γ acting on the gauge indices: $\gamma = \mathrm{diag}(I_N, e^{2\pi i/k} I_N, e^{4\pi i/k} I_N, \ldots, e^{-2\pi i/k} I_N)$

$$\psi^1 \to e^{2\pi i m_1/k} \gamma \psi^1 \gamma^{-1} , \qquad \psi^2 \to e^{2\pi i m_2/k} \gamma \psi^2 \gamma^{-1} , \ \ldots$$

$$Z^1 \to e^{2\pi i (m_1+m_2)/k} \gamma Z^1 \gamma^{-1} , \qquad Z^2 \to e^{2\pi i (m_1+m_3)/k} \gamma Z^2 \gamma^{-1} , \ \ldots$$

- Consider renormalization of quiver gauge theories on a stack of D3-branes at the tip of a cone R^6/Γ where the orbifold group Γ breaks all the supersymmetry.
- At first sight, the gauge theory seems conformal because the planar beta functions for all single-trace operators vanish. The candidate string dual is $AdS_5 \times S^5/\Gamma$. Kachru, Silverstein; Lawrence, Nekrasov, Vafa; Bershadsky, Johanson
- However, dimension 4 double-trace operators made out of twisted single-trace ones, $f\, O_n\, O_{-n}$, are induced. Their one-loop planar beta-functions have the Coleman-Weinberg form Dymarsky, IK, Roiban

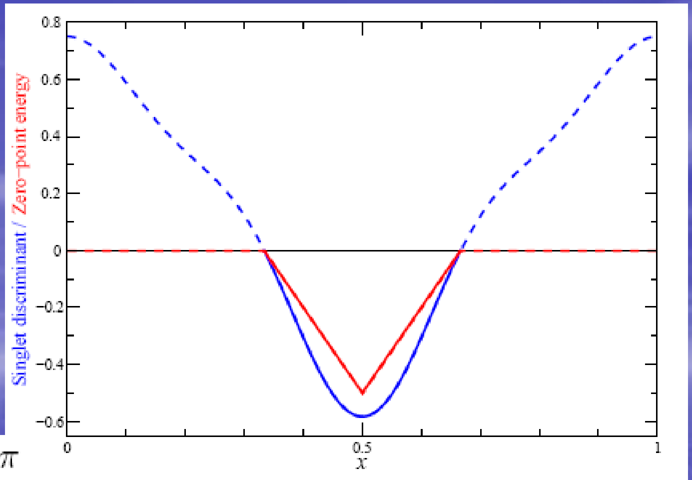
$$\beta_f = a\, \lambda^2 + 2\, \gamma\, f\, \lambda + f^2$$

$$\beta_\lambda = 0$$

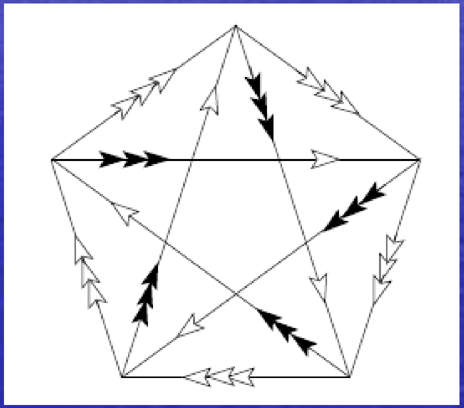
A Note on Normalizations

- The VEV of a single trace operator is of order N.
- The standard Yang-Mills action $S = -\int d^4x \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu}^2$ is of order N^2 in the `t Hooft limit.
- The double-trace operators f O_n O_{-n} make contributions of the same order (for the coupling constant f of order 1). They cannot be ignored in the leading large N limit.
- In fact, the tree-level potential of the $SU(N)^k$ quiver orbifold theories (with the interacting U(1)'s decoupled) contains such double-trace terms.

- If $D=\gamma^2 - a < 0$, then there is no real fixed point for f.
- A class of Z_k orbifolds with global SU(3) symmetry, that are freely acting on the 5-sphere, has the group action in the fundamental of SU(4) $r(g^n) = \mathrm{diag}(\omega_k^n, \omega_k^n, \omega_k^n, \omega_k^{-3n})$ $\omega_k = e^{i\alpha_k}\,, \qquad \alpha_k = \frac{2\pi}{k}$



- Here is a plot of a one-loop $SU(N)^k$ gauge theory discriminant, D. n=1, …, k-1 labels the twisted sector, and x=n/k.
- The simplest freely acting non-SUSY example is Z_5 where there are four induced double-trace couplings



$$\delta_{2\ \mathrm{trace}}\mathcal{L} = f_{8,1} O_1^{\langle i\bar{j}\rangle} O_{-1}^{\langle j\bar{i}\rangle} + f_{8,2} O_2^{\langle i\bar{j}\rangle} O_{-2}^{\langle j\bar{i}\rangle} + f_{1,1} O_1 O_{-1} + f_{1,2} O_2 O_{-2}$$

- For example, the SU(3) adjoints $O_n^{\langle i\bar{j}\rangle}$ are ($\alpha=2\pi/5$)

$$\sum_{k=1}^{5} \left(\Phi^i_{k,k+2}\Phi^{\bar{j}}_{k+2,k} - \frac{1}{3}\eta^{i\bar{j}}\Phi^l_{k,k+2}\Phi^{\bar{l}}_{k+2,k} \right) e^{in\alpha(k-1)}$$

- Any non-SUSY abelian orbifold contains unstable couplings. This appears to remove all such orbifold quivers from a list of large N perturbatively conformal gauge theories.
- The all-orders form of a double-trace beta-function is Pomoni, Rastelli

$$\beta_f = \frac{v(\lambda)}{1+\gamma(\lambda)} f^2 + 2\,\gamma(\lambda)\, f + a(\lambda)$$

- A real fixed point could appear for $\lambda > \lambda_c$

  This is suggested by dual string arguments: twisted operators are dual to strings stretched around the S^5/Γ .
- The $AdS_5 \times S^5/\Gamma$ background is tachyon-free at large radius. Yet, it has a non-perturbative tunneling instability. Horowitz, Polchinski, Orgera
- Upshot: Life is hard without SUSY!

D3-branes on the Conifold

- The conifold is a Calabi-Yau 3-fold cone X described by the constraint $\sum_{a=1}^{4} z_a^2 = 0$ on 4 complex variables.
- Its base Y is a coset $T^{1,1}$ which has symmetry $SO(4) \sim SU(2)_A \times SU(2)_B$ that rotates the z's, and also $U(1)_R$: $z_a \to e^{i\theta} z_a$
- The Sasaki-Einstein metric on $T^{1,1}$ is

$$\mathrm{d}s^2_{T^{1,1}} = \frac{1}{9}\left(\mathrm{d}\psi + \sum_{i=1}^{2} \cos\theta_i \mathrm{d}\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\right)$$

  where $\theta_i \in [0,\pi], \phi_i \in [0,2\pi], \psi \in [0,4\pi]$
- The topology of $T^{1,1}$ is $S^2 \times S^3$.

- To `solve' the conifold constraint det Z = 0 we introduce another set of convenient coordinates:

$$Z = \begin{pmatrix} z^3 + iz^4 & z^1 - iz^2 \\ z^1 + iz^2 & -z^3 + iz^4 \end{pmatrix} = \begin{pmatrix} w_1 & w_3 \\ w_4 & w_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix}$$

- The action of global symmetries is

$$SU(2) \times SU(2) \text{ symmetry} \quad : \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \to L \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \qquad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \to R \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\text{R-symmetry} \quad : \quad (a_i, b_j) \to e^{i\frac{\alpha}{2}} (a_i, b_j) \ ,$$

- There is a redundancy under

$$a_i \to \lambda\, a_i \quad , \quad b_j \to \frac{1}{\lambda}\, b_j \quad (\lambda \in \mathbf{C})$$

It may be fixed by identifying $a \sim e^{i\alpha} a, b \sim e^{-i\alpha} b$ and imposing $|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2 = 0$

- In the field theory this is implemented by a U(1) gauge symmetry and its D-flatness condition.
- In the IR gauge theory on D3-branes at the apex of the conifold, the coordinates a_1, a_2, b_1, b_2 are replaced by chiral superfields. For a single D3-brane it is necessary to introduce gauge group U(1) x U(1). The A's have charges (1,-1); the B's (-1,1). The `sum' U(1) is free.
- The moduli space of this gauge theory is the conifold.

- The $\mathcal{N}$=1 SCFT on N D3-branes at the apex of the conifold has gauge group SU(N)xSU(N) coupled to bifundamental chiral superfields A_1, A_2, in $(\overline{\mathrm{N}}, \mathrm{N})$, and B_1, B_2 in $(\mathrm{N}, \overline{\mathrm{N}})$. IK, Witten
- The R-charge of each field is ½. This insures $U(1)_R$ anomaly cancellation.
- The unique $SU(2)_A$x$SU(2)_B$ invariant, exactly marginal quartic superpotential is added:

$$W = \epsilon^{ij}\epsilon^{kl}\,\mathrm{tr}\,A_i B_k A_j B_l$$

- This theory also has a `baryonic' U(1) symmetry under which A_k -> e^{ia} A_k; B_l -> e^{-ia} B_l . It starts out as a U(1) gauge symmetry on D3-branes, but its gauge coupling flows to zero in the IR.

Comparison with a Z_2 Orbifold Quiver

- The simplest $\mathcal{N}$=2 SUSY quiver has k=2; $m_1=m_2=1$, $m_3=0$. The gauge group is again SU(N)xSU(N), but in addition to the bifundamentals A_i, B_j, there is one adjoint chiral superfield for each gauge group, with superpotential $g\mathrm{Tr}\Phi(A_1B_1 - A_2B_2) + g\mathrm{Tr}\tilde{\Phi}(B_1A_1 - B_2A_2)$
- Adding a Z_2 odd mass term $\frac{m}{2}(\mathrm{Tr}\Phi^2 - \mathrm{Tr}\tilde{\Phi}^2)$

and integrating out the adjoints, we obtain the superpotential of the conifold theory,

$$-\frac{g^2}{m}\left[\mathrm{Tr}(A_1B_1A_2B_2) - \mathrm{Tr}(B_1A_1B_2A_2)\right]$$

Problem 2

In a supersymmetric field theory, the trace anomaly coefficients a and c are given by the formulae

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - 3\mathrm{Tr}R\right) \ , \qquad c = \frac{1}{32}\left(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R\right) \ ,$$

where R refers to the $U(1)_R$ charges, and the trace is over all the chiral fermion fields.

a) Calculate a and c in the following two gauge theories: the $\mathcal{N} = 2$ supersymmetric Z_2 orbifold quiver, and in the $\mathcal{N} = 1$ SCFT on N D3-branes at the conifold.

b) For $AdS_5 \times Y$ with N units of RR 5-form flux, it was found at leading order in N that

$$a = c = \frac{N^2\pi^3}{4\mathrm{vol}(Y)} \ ,$$

where the radius of Y is normalized so that $R_{ij} = 4g_{ij}$ on Y. Compare this formula with the gauge theory results of part a).

Resolution and Deformation

- There are two well-known Calabi-Yau blow-ups of the conifold singularity.
- The `deformation' replaces the constraint on the z-coordinates by

$$z_1^2+z_2^2+z_3^2+z_4^2=\epsilon^2$$

- This replaces the singularity by a finite 3-sphere.
- In the `small resolution' the singularity is replaced by a finite 2-sphere. This is implemented by modifying the constraint on the a and b variables

$$|b_1|^2 + |b_2|^2 - |a_1|^2 - |a_2|^2 = u^2$$

- This suggests that in the gauge theory the resolution is achieved by giving VEV's to the chiral superfields. IK, Witten
- For example, we may give a VEV to only one of the four superfields: $B_2 = u 1_{N\times N}$
- The dual of such a gauge theory is a resolved conifold, which is warped by a stack of N D3-branes placed at the north pole of the blown up 2-sphere.

$$ds_{10}^2 = \sqrt{H^{-1}(y)}dx^\mu dx_\mu + \sqrt{H(y)}ds_6^2$$

- The explicit CY metric on the resolved conifold is

Pando Zayas, Tseytlin

$$\kappa(r) = \frac{r^2 + 9u^2}{r^2 + 6u^2}$$

$$\begin{aligned} ds_6^2 &= \kappa^{-1}(r)dr^2 + \frac{1}{9}\kappa(r)r^2\left(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\right)^2 \\ &\quad + \frac{1}{6}r^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{6}(r^2 + 6u^2)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) \end{aligned}$$

- The warp factor is the Green's function on this space with a source located at the D3-brane stack

IK, Murugan

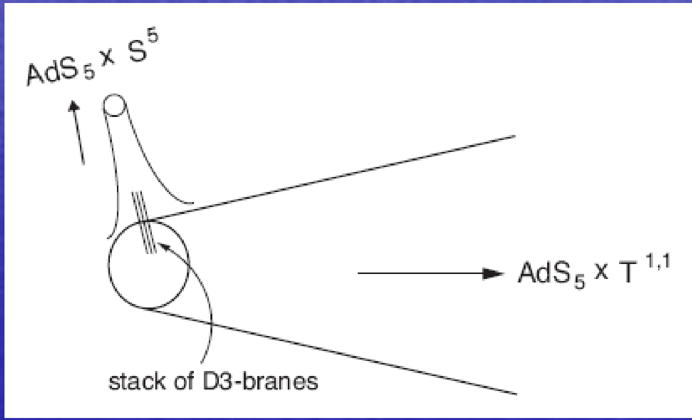
$$H(r,\theta_2) = L^4 \sum_{l=0}^{\infty} (2l+1) H_l^A(r) P_l(\cos\theta_2)$$

- The radial functions are hyper-geometric:

$$H_l^A(r) = \frac{2}{9u^2}\frac{C_\beta}{r^{2+2\beta}}\ {}_2F_1\left(\beta, 1+\beta; 1+2\beta; -\frac{9u^2}{r^2}\right)$$

$$C_\beta = \frac{(3u)^{2\beta}\Gamma(1+\beta)^2}{\Gamma(1+2\beta)}\,, \qquad \beta = \sqrt{1 + (3/2)l(l+1)}$$

- We get an explicit `localized' solution which describes SU(2)xU(1)xU(1) symmetric holographic RG flow from the $\mathcal{N}$=1 SU(N)xSU(N) SCFT in the UV to the $\mathcal{N}$=4 SU(N) SYM in the IR.



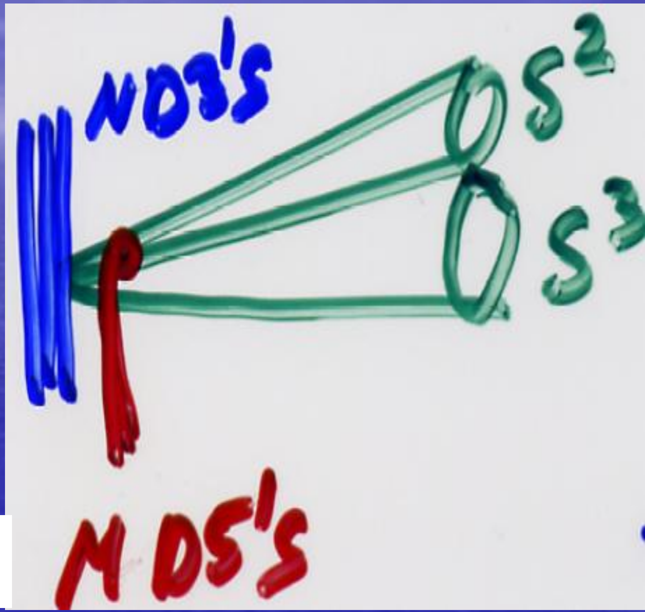
- A previously known `smeared' solution corresponds to taking just the l=0 harmonic. This solution is singular Pando Zayas, Tseytlin

$$\frac{2}{9u^2r^2} + \frac{4\beta^2}{81u^4}\ln r + \mathcal{O}(1) \quad \overset{0 \leftarrow r}{\longleftarrow} \quad H_l^A(r) \quad \overset{r \to \infty}{\longrightarrow} \quad \frac{2C_\beta}{9u^2r^{2+2\beta}}$$

Warped Deformed Conifold

- Let us add to the N D3-branes M D5-branes wrapped over the S^2 at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

$$ds_{10}^2 = h^{-1/2}(t)\left(-(dx^0)^2 + (dx^i)^2\right) + h^{1/2}(t)ds_6^2$$



- ds_6^2 is the metric of the deformed conifold, a simple non-compact Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The warp factor is finite at the `end of space' t=0, as required for the confinement: $h(t) = 2^{-8/3} \gamma I(t)$

$$I(t) = \int_t^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh 2x - 2x)^{1/3} , \qquad \gamma = 2^{10/3} (g_s M \alpha')^2 \varepsilon^{-8/3}$$

- The string tension $\sim h(t)^{-1/2}$ is minimized at t=0. It blows up at large t (near the boundary).
- The dilaton is exactly constant due to the self-duality of the 3-form background

$$\star_6 G_3 = i G_3 , \qquad G_3 = F_3 - \frac{i}{g_s} H_3$$

- The radius-squared of the S^3 at t=0 is g_sM in string units.
- When g_sM is large, the curvatures are small everywhere, and the SUGRA solution is reliable in `solving' this confining gauge theory.
- The details of the solution are reviewed in C. Herzog, IK, P. Ouyang, hep-th/0205100.

Log running in UV

- The large radius asymptotic solution is characterized by logarithmic deviations from $AdS_5 \times T^{1,1}$ IK, Tseytlin
- The near-AdS radial coordinate is $r \sim \varepsilon^{2/3} e^{t/3}$
- The NS-NS and R-R 2-form potentials:

$$F_3 = \frac{M\alpha'}{2}\omega_3 \ , \qquad B_2 = \frac{3g_s M\alpha'}{2}\omega_2 \ln(r/r_0)$$

$$\omega_2 = \frac{1}{2}(g^1 \wedge g^2 + g^3 \wedge g^4) = \frac{1}{2}(\sin\theta_1 d\theta_1 \wedge d\phi_1 - \sin\theta_2 d\theta_2 \wedge d\phi_2)$$

$$\omega_3 = \frac{1}{2} g^5 \wedge (g^1 \wedge g^2 + g^3 \wedge g^4)$$

- This translates into log running of the gauge couplings through

$$\frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} = \frac{\pi}{g_s e^{\Phi}} ,$$

$$\left[\frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2}\right] g_s e^{\Phi} = \frac{1}{2\pi\alpha'}\left(\int_{S^2} B_2\right) - \pi$$

$$\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2} = 6M\ln(r/r_s)$$

- This agrees with the β-functions in the gauge theory

$$\frac{d}{d\log(\Lambda/\mu)}\frac{8\pi^2}{g_1^2} = 3(N+M) - 2N(1-\gamma)$$

$$\frac{d}{d\log(\Lambda/\mu)}\frac{8\pi^2}{g_2^2} = 3N - 2(N+M)(1-\gamma)$$

$$\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2} = M\ln(\Lambda/\mu)[3 + 2(1-\gamma)]$$

- In the UV the anomalous dimension of operators $\mathrm{Tr}\, A_i B_j$ is γ ~ -1/2

- The warp factor deviates from the M=0 solution logarithmically.

$$h(r) = \frac{27\pi(\alpha')^2[g_sN + a(g_sM)^2\ln(r/r_0) + a(g_sM)^2/4]}{4r^4}$$

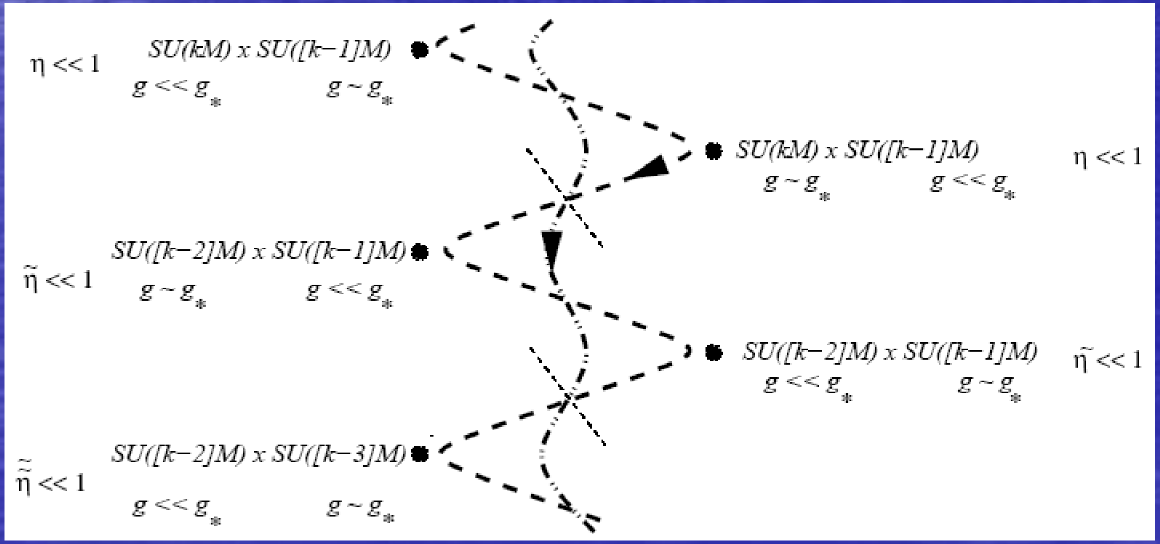
- Remarkably, the 5-form flux, dual to the number of colors, also changes logarithmically with the RG scale.

$$\tilde{F}_5 = \mathcal{F}_5 + \star\mathcal{F}_5 \ , \qquad \mathcal{F}_5 = 27\pi\alpha'^2 N_{eff}(r)\mathrm{vol}(\mathrm{T}^{1,1})$$

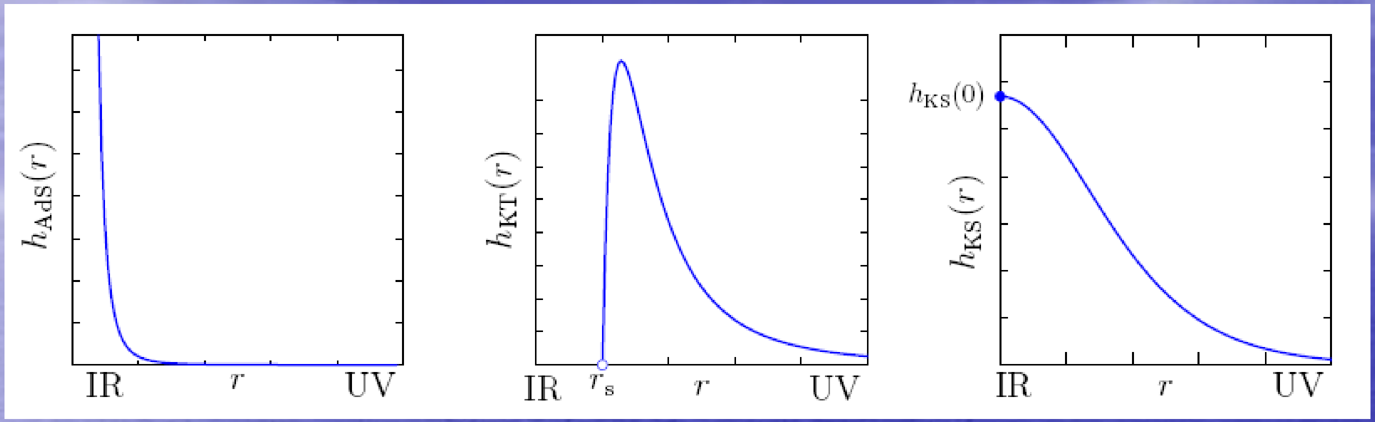
$$N_{eff}(r) = N + \frac{3}{2\pi} g_s M^2 \ln(r/r_0)$$

- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a duality cascade, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler

(diagram of RG flows from a review by M. Strassler)



- There is a scale where the SU(kM) coupling becomes very strong. This gauge group has N_f=2(k-1)M flavors.
- To understand further RG flow, perform Seiberg duality to SU(N_f-N_c)=SU((k-2)M).
- The resulting SU((k-1)M)xSU((k-2)M) theory has the same structure as the original one, but k is reduced by 1.
- The flow proceeds in a quasi-periodic fashion until k becomes O(1) in the IR.



- Comparison of warp factors in the AdS, warped conifold, and warped deformed conifold cases. The warped conifold (KT) solution, which has a naked singularity, should be interpreted as asymptotic (UV) approximation to the correct solution.

- The graph of quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD. The upper graph, from the Princeton Senior Thesis of V. Cvicek shows the string theory result for the warped deformed conifold.
- The lower graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm.

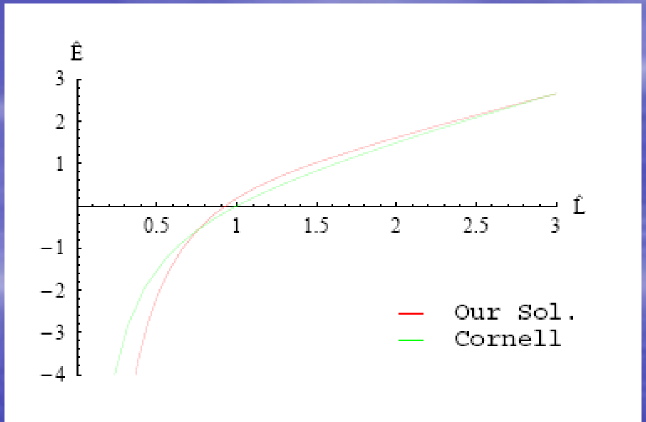
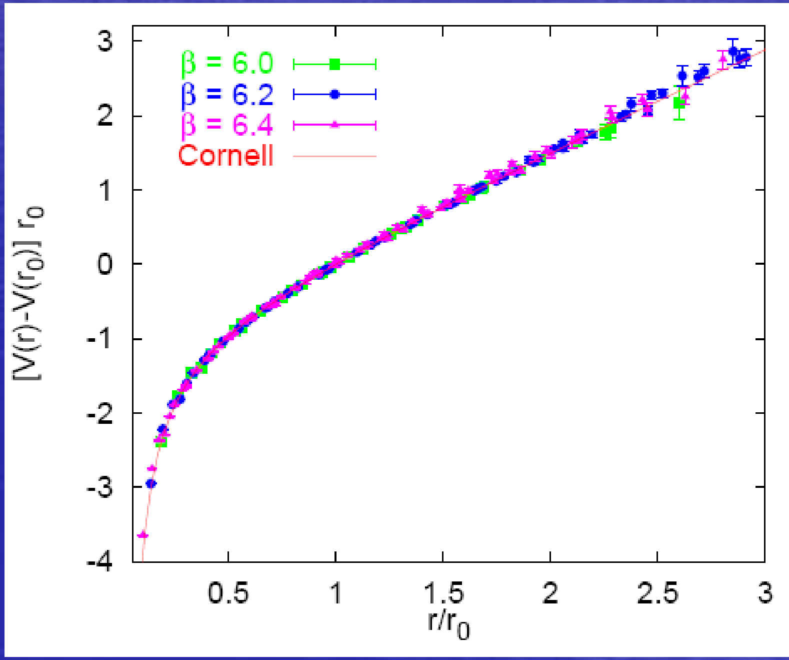


Figure 11: Comparison to the Cornell model



IR Behavior of the Duality Cascade

- Here the dynamical deformation of the conifold renders the solution smooth, and explains the IR dynamics of the gauge theory.
- Dimensional transmutation in the IR. The dynamically generated confinement scale is

$$\sim \varepsilon^{2/3}$$

- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: $Z_{2M} \to Z_2$
- Yet, for large $g_s M$ the IR gauge theory is somewhat more complicated.

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- In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f=N_c$.
- The baryon and anti-baryon operators Seiberg

$$\mathcal{A} = \epsilon^{i_1 \ldots i_{N_c}} A^{a_1}_{\alpha_1 i_1} \ldots A^{a_{N_c}}_{\alpha_{N_c} i_{N_c}}$$

$$\mathcal{B} = \epsilon_{i_1 \ldots i_{N_c}} B^{i_1}_{\dot{\alpha}_1 a_1} \ldots B^{i_{N_c}}_{\dot{\alpha}_{N_c} a_{N_c}}$$

acquire expectation values and break the U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$. Hence, we observe confinement without a mass gap: due to $U(1)_{baryon}$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner. There exists a baryonic branch of the moduli space

$$\mathcal{A} = i\Lambda_1^{2M} \zeta \ , \qquad \mathcal{B} = i\Lambda_1^{2M} / \zeta$$

- The KS solution is part of a moduli space of confining SUGRA backgrounds, resolved warped deformed conifolds. Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni
- To look for them we need to use the PT ansatz:

$$ds_{10}^2 = H^{-1/2} dx_m dx_m + e^x ds_6^2,$$

$$ds_6^2 = (e^g + a^2 e^{-g})(e_1^2 + e_2^2) + e^{-g} \sum_{i=1}^{2} (\epsilon_i^2 - 2a e_i \epsilon_i) + v^{-1}(\tilde{\epsilon}_3^2 + dt^2)$$

- H, x, g, a, v, and the dilaton are functions of the radial variable t.
- Additional radial functions enter into the ansatz for the 3-form field strengths. The PT ansatz preserves the SO(4) but breaks a Z_2 charge conjugation symetry, except at the KS point.

- All of this provides us with an exact solution of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for testing various ideas about strongly coupled gauge theory.
- The dimensional transmutation present in the model has been used to generate small scales in warped string compactifications (KKLT).
- Also provides an approach to metastable SUSY breaking by placing anti-D3 branes at t=0. Kachru, Pearson, Verlinde.