Where are there principled excuses to talk about strings & phenomenology? After all, in most scenarios, strings are relevant at $M_s \sim 10^{17}$ GeV $\gg$ any Experiment.

**Excuse 1:** The relevant phenomenology is dominated by $M_s$ or $M_p$ suppressed operators. This is true in many cases of central interest:

a) SUSY mediation mechanisms where

$$X = X + \Theta \Psi + \Theta^2 F_X$$

$$\langle F_X \rangle \gtrsim 10^{11} \text{ GeV}$$

- $\mathcal{O}$ terms

$$k \int d^4 \Theta \frac{X X \phi^+ \phi^-}{M_p^2}$$

**dominate** (or $O(1)$ contribute to) soft masses.
and need to be:

- suppressed, in anomaly mediation
- flavor blind, in gravity mediation

These are dim 6 $M_p$ suppressed $\rightarrow$ sensitive to UV physics.

b) Proton decay

c) Inflation

\[ V(y) = \frac{1}{2} M_p^2 \left( \frac{V'}{V} \right)^2 \]
\[ \eta = M_p^2 \frac{V''}{V} \]

So dim 6 operators can $\rightarrow O(1) \eta$;

but inflation requires $\eta, M \ll 1$ for slow roll (suitable generalization for non slow-roll)
also $\rightarrow$ sensitivity to $M_p$ suppressed ops.

Some ("large field", $\Delta \phi > M_p$) models are
sensitive to operators of \( \text{dim} \gg 6 \)

\[
V = V_{\text{ren}} + \sum_{n=1}^{\infty} \phi^n(n) \left( \frac{\phi}{M_p} \right)^n \quad \text{any \( \phi^n \)}
\]

In can ruin large field infl.

We may talk about a) & c) a bit more later. But first...

**Excuse 2:** The field theory dynamics involved is strongly coupled, in a way that is amenable to a weakly coupled dual gravity description.

a) DSB, needed to explain why \( F_x \ll M_{pl} \)

in SUGY models, is often a strong coupling phenomenon. Some DSB models have weakly coupled gravity descriptions.
b) Composite models (SUSY or not) often have a useful gravity dual description. Then phenomena like Yukawa hierarchies, soft mass hierarchies are geometrically explained.

I'll start by lecturing about 2 a) & b) in an EFT & string context, then move to 1 a) & (c) as time permits.

I. Hierarchies from a slice of AdSs.

(c.f. Randall/Sundrum papers)

A. Trapping gravity in AdSs

We can obviously live in a higher-D (say 5D) world if the extra dims are compact.
E.g.,

\[ dS^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dX^2 \]

5D Einstein action

\[ S_5 = \int d^5x \sqrt{-g} \, M_s^3 \, R_5 \]

Integrating out "extra" Xs dim \( \Rightarrow \)

\[ M_4^2 \sim M_s^3 \, R \]

\( \Rightarrow \) 4D gravity

\( \text{H} \text{G} \text{N} \text{N} \) fixed by this

For small \( R \), clearly compatible w/ experiment.

[At this level -- could worry about modulus, etc.]

More general: Metric can be warped.

Consider:

\[ S = \int d^5x \sqrt{-g} \left( R - \Lambda \right) + \int d^4x \sqrt{-g} \left( -V \text{brane} \right) \]

\[ g_{\mu\nu} = \delta_\mu^M \delta_\nu^N \, \delta^{MN} \, (x^5 = 0) \]

\( \mu = 1 \sim 4 \)

\( M = 1 \sim 5 \)
Following Randall & Sundrum, we take the most general $SO(3,1)$ symmetric ansatz:

$$ds^2 = e^{2A(x_5)} g_{\mu\nu} dx^\mu dx^\nu + dx_5^2$$

Then, Einstein's equations $\Rightarrow$

1. $6(A')^2 + \frac{1}{2} \Lambda = 0$
2. $3 A'' + \frac{1}{2} V \delta(x_5) = 0$

(choosing $\Lambda < 0$, can solve $(\star) \Rightarrow$

$$A = \pm k x_5 \quad k = \sqrt{-\frac{\Lambda}{12}}$$

Then integrating $(\star)$ from $x_5 = -\ell \rightarrow \ell$

to pick up $\delta$-function contribution $\Rightarrow$

$$3 \Delta (A') = -\frac{1}{2} V$$

$\xi$ discontinuity across $x_5 = 0$

So, to solve the EOM with our ansatz for the metric, we need to take:
\[ A = \begin{cases} 
-1 & x_s > 0 \\
1 & x_s < 0 
\end{cases} \]

Then we must tune brane tension \( V \) in terms of \( \Lambda \):

\[
V = 12k = 12 \frac{-\Lambda}{\sqrt{12}} \]

This yields a solution where

\[
ds^2 = e^{-2k|x_s|} g_{\mu\nu} dx^\mu dx^\nu + dx_s^2\]

- The warp factor is sharply peaked at \( x_s = 0 \), where the "Planck brane" is located.
- \( x_s \) is noncompact, but 3 4D gravity!

\[
\text{M}_{4D}^2 = M_s^3 \int dx_s \ e^{-2k|x_s|} < \infty
\]

This is finite! Such a solution has 4D gravity. (A Planck brane observer would see our Newton's law).
This is just a slice of the AdS$_5$ metric.

b. Relation to D3 metrics

The solution for a D3-brane stack (N of them) in IIB supergravity, is

$$ds^2 = h^{-1/2} dx_{11}^2 + h^{1/2} (dr^2 + r^2 dS^2)$$

$$h(r) = 1 + \frac{4\pi g N (\alpha')^2}{r_4}$$

Defining $U = \frac{r}{\alpha'}$, and taking $d^4 \to 0$ with fixed $U = 0$

$$ds^2 = \alpha' \left[ \frac{U^2}{\sqrt{4\pi g N}} dx_{11}^2 + \sqrt{4\pi g N} \frac{dU^2}{U^2} + \sqrt{4\pi g N} dS^2 \right]$$

This is AdS$_5 \times$S$^5$ with

$$R_{AdS} = R_{S^5} = \sqrt{4\pi g N} \alpha'$$.
Claim: The RS metric is the same as the 5D truncation of this, with some cutoff at $U = U_{\text{max}}$, + a $\mathbb{Z}_2$ copy of the sol'n inserted at $U_{\text{max}}$.

This is HW problem 1.

C. Hierarchies from IR branes

Consider now a case w/ 2 branes, located at $X_5 = 0$ + $X_5 = \pi$. $X_5 \in [0, \pi]$ now.

Then:

$$S = \int d^5x \sqrt{-g} \left( R^5 \right) + S_{\text{IR}} + S_{\text{UV}}$$

Here:
\[ S_{IR} = \int d^m x \sqrt{-g_{IR}} (L_{IR} - V_{IR}) \]
\[ S_{uv} = \int d^m x \sqrt{-g_{uv}} (L_{uv} - V_{uv}) \]

This $S \rightarrow$ hierarchies of scales in a natural way.

Again, consider

\[ ds^2 = e^{-2H(x_5)} g_{uv} dx^u dx^v + r^2 dx_5^2 \]

(\rightarrow \text{size of } x_5 \text{ interval is } rr).

Einstein eqns:

(1) \[ 6 \left( \frac{\Lambda}{r^2} \right)^2 + \frac{1}{2} \Lambda = 0 \]

(\triangle) \[ 3 \frac{A_{11}}{r^2} + \frac{1}{2} \frac{V_{uv} \delta(x_5)}{r} + \frac{1}{2} \frac{V_{IR} s(x_5 - \Pi)}{r} = 0 \]

Defining \[ h = \sqrt{-\Lambda} \Rightarrow A(x_5) = \frac{|k|}{h} \mid x_5 \mid \]

from (1).

* consistent w/ a $\exists$ to find a $\exists$ inv't sol'n
with \(-\pi < x_5 < \pi\), then quotient by \(\mathbb{Z}_2\).

Now \(A = \text{ker} |x_5| \to\)

\[ A'' = 2i\pi \left[ s(x_5) - s(x_5 - \pi) \right] \]

Then to solve (\(\Delta\)) need

\[ V_{uv} = -V_{ir} = 12k \]

\[ \text{tune of CC} \]

\[ \text{This is to get Poincaré invt metric}. \]

Now

\[ ds^2 = e^{-2i\pi x_5} \left( g_{uv} dx^u dx^v + r^2 dx_5^2 \right) \]

\[ 0 \leq x_5 \leq \pi \]

Computing \(M_4\):

\[ M_4^2 = M_5^3 \left( 1 - e^{-2i\pi \tau} \right) \]

\(\Rightarrow M_4\) depends weakly on \(r\); the 4D graviton must be localized @ UV-brane.
Now also notice

\[ g_{\mu\nu} = \eta_{\mu\nu} \]

\[ g_{12} = M_{12} e^{-2\pi r \mp} \]

In particular, a scalar on 12 brane w/ cutoff scale mass \( M_* \) has

\[ L \sim \int d^n x \left( g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M_*^2 \phi^2 - \int g_{12} \right. \]

\[ \sim \int d^n x \left( e^{-2\pi r \mp} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} e^{-4\pi r \mp} \phi^2 \cdot M_*^2 \right) \]

And letting \( \tilde{\phi} = e^{-\pi r \mp} \phi \rightarrow \)

\[ L \sim \int d^n x \left( \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \frac{1}{2} e^{-2\pi r \mp} \tilde{\phi}^2 \cdot M_*^2 \right) \]

\[ = D \quad \tilde{M} = e^{-\pi r \mp} M_* \]

So the "natural" energy scale @ 12 brane is \( \tilde{M} \ll M_* \rightarrow D \) light scalars are natural for \( 12 r \sim \text{few} \).
This is just the gravity dual of dimensional transmutation...

d. $\text{AdS/CFT duality}$

$IIB$ string theory $\overset{\sim}{\rightarrow} N = 4 \text{ } SU(N)$ on $\text{AdS}_5 \times S^5$ gauge theory

with \[ \frac{R_{\text{AdS}}}{\ell_s} \sim \frac{4 \pi \alpha'}{\ell_s^3} \]

Precise map $(G/P, W)$:

- Bulk field $\Phi \in \text{AdS}$ and operator $\Theta \in \text{CFT}$

\[ ds^2 = e^{-2ky} dx^m dx^n + dy^2 \]

\[ \kappa = 1/R_{\text{AdS}} \]

- Given a boundary value

\[ \Phi(x^m, y = -\infty) = \Phi_0 (X^m) \]

...
then
\[ - \langle d^4x \Phi_0, 0 \rangle_{\text{CFT}} = e^{-\Pi(\Phi_0)} \]
\[ \Pi(\Phi_0) = \text{SUGRA (strong) action of sol'n} \]
\[ \text{w/ boundary value } \Phi_0. \]

What do the cutoffs at UV \& IR brane correspond to?

We saw in \( \square \) that energies redshift like \( e^{-ty} \).

- Cutoff at \( y_{\text{max}} \) (IR) \( \Rightarrow \) minimal energy scale \( \sim e^{-ty_{\text{max}}} \) \( \Rightarrow \) CFT develops a mass gap at \( y_{\text{max}} \)!

- Cutoff at \( y_{\text{min}} = 0 \) \( \Rightarrow \) maximal energy scale; CFT is cutoff in UV here \& coupled to strings/quantum gravity.
How can we use this?

Ambitious: Forget SUSY.

\[ y = y_{\text{max}} \quad y = y_{0} \]

\{ \text{Planck brane} \}

\{ \text{Higgs} \}

\{ \text{SM gauge fields, matter} \}

We'll see that varying matter field properties moderately can explain Yukawa hierarchies.

[Will explain why matter on IR brane is bad next time].

Still interesting:

\[ y = y_{\text{max}} \quad y = y_{0} \]

\{ \text{Use gauge/gravity} \}

\{ \text{to geometrize DSB!} \}

\[ \text{Asusy} \ll M_{p} \text{ via warping. We'll see that:} \]
- Can make simple string realizations
- Can still explain Yukawas if you wish

The Yukawa explanations in the 2 cases are opposite:

<table>
<thead>
<tr>
<th>Non-SUSY</th>
<th>SUSY</th>
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<tbody>
<tr>
<td>top</td>
<td>point-like</td>
</tr>
<tr>
<td>1st 2</td>
<td>composite</td>
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<tr>
<td>gens</td>
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We'll focus on the SUSY models:
- $\rightarrow$ models of single-sector SUSY
- $\rightarrow$ (in a limit) general gauge mediation