# **D-BRANE MODEL BUILDING, PART III**

## Luis E. Ibáñez

Instituto de Física Teórica (IFT-UAM/CSIC)

Universidad Autónoma de Madrid

PITP School, IAS Princeton, July 2008

# **VI- NEW COUPLINGS FROM STRINGY INSTANTONS**

## U(1) global symmetries are not forever

- We saw how in generic IIA and IIB D-brane models some U(1)'s (anomalous or not) can get a mass combining with some RR fields.
- The corresponding  $U(1)_X$  symmetry remains perturbatively to all orders as an effective global symmetry. E.g. any superpotential operator

$$\frac{1}{M_s^{n-3}}\Phi_{q_1}...\Phi_{q_n} = 0 \ ; \ \sum_i q_i \neq 0$$
 (1)

- That is the case e.g. baryon number and lepton number in MSSM-like models of the type we have described. That could be a problem for baryo- or lepto-genesis and/or the existence of Majorana neutrino masses.
- It has been recently realized however that non-perturbative stringy instanton effects may give rise to superpotential operators

$$\frac{1}{M_s^{n-3}} e^{-M} \Phi_{q_1} \dots \Phi_{q_n} \neq 0 \ ; \ \sum_i q_i \neq 0$$
 (2)

with M a IIA complex structure (IIB Kahler modulus) field whose imaginary part shifts under a  $U(1)_X$  gauge transformation of parameter  $\Lambda_x$  like

$$M \longrightarrow M + \Lambda_x \left(\sum_i^n q_i\right)$$
 (3)

• This shift is such that the the operator is fully gauge invariant.

#### Majorana neutrino masses

- Let us recall some well known facts. The simplest explanation for the smallness of neutrino masses is the celebrated see-saw mechanism.
- If there are right-handed neutrinos  $\nu_R^a$  with large Majorana masses  $M_M$  and standard Dirac masses  $M_D$ , the lightest eigenvalues have masses of order

$$M_{\nu} \simeq \frac{M_D^2}{M_M} \,, \tag{4}$$

of order experimental results for  $M_D$  of order of standard charged lepton masses and for  $M_M \propto 10^{10}-10^{13}$  GeV.

- Why  $\nu_R$  exists? They are natural in left-right symmetric extensions of the SM like SO(10),  $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ .
- The  $\nu_R$  may get Majorana masses through Yukawa couplings

$$\nu_R \nu_R \phi_M$$

(5)

In SO(10) one has  $\phi_M = 126$  , in  $3 \times 2 \times 2 \times 1$  it is (1, 1, 3, 2) .

• Or else through some non-renormalizable couplings

$$\frac{1}{M_{B-L}}\nu_R\nu_R < \bar{N}_R\bar{N}_R > \tag{6}$$

• In the first case in the MSSM R-parity is automatic but the Higgs sector is ugly.... In the second case the Higgs sector is simple but R-parity is not automatic.....

#### Some general features for $\nu_R$ masses in string models

- Note that unlike field theory models, string vacua are quite rigid. Given a compactification the spectrum and couplings are fixed (as a function of the moduli). We cannot 'complete our model' with the fields and couplings we would like to have!!
- The presence of  $\nu_R$ 's is generic in SM-like vacua.
- Dirac neutrino masses are generically present but couplings generating Majorana v<sub>R</sub>-masses are absent!!. The required Higgs fields have 'charge too big' to be in the massless spectrum.
- Practically all MSSM-like models constructed to date have a  $U(1)_{B-L}$  gauge boson beyond the SM group.
- Efforts up to now in order to obtain ν<sub>R</sub> masses in string models typically resort to higher-dimensional ν<sub>R</sub>ν<sub>R</sub> φ<sup>n</sup> operators. But then typically one gets
   1) Too small ν<sub>R</sub> masses 2) R-parity violation and 3) Often fast proton decay.

- In D-brane models B and L symmetries are U(1) gauged symmetries. Then right-handed neutrinos  $\nu_R$  are perturbatively massless.
- Even if  $U(1)_{B-L}$  gets massive by combining with a RR-field, it will remain perturbatively massless.
- However String Theory instantons may generate  $\nu_R$ -masses through operators <sup>a</sup>

$$e^{-U} \nu_R \nu_R M_{string} \tag{7}$$

with U axion-like fields which shift under  $U(1)_{B-L}$ .

<sup>a</sup>L.E.I. and A. Uranga, hep-th/0609213; R. Blumenhagen, M. Cvetic, T. Weigand hep-th/0609191.



## **String Theory Instantons**

- We saw in Type IIA orientifolds gauge groups live on D6-branes wrapping 3-cycles in the CY.
- We now consider the effect of D2 euclidean branes also wrapping a 3-cycle. They have Drichlet b.c. in Minkowski, localized in space and time, instantons
   a
- They may contribute in semiclassical tunneling processes, as standard gauge theory instantons do.
- Standard gauge theory instantons would correspond to D2-branes wrapping precissely the same 3-cycle as the D6-branes where the gauge group lives.
- However, in string theory there are other instanton varieties corresponding to D2-branes wrapping other cycles and intersecting the D6-branes present.
- At the D2-D6 brane intersections live fermionic zero modes which are <sup>a</sup>Becker<sup>2</sup>,Strominger (95);Witten (96,99);Harvey,Moore (99)

#### charged under the D6 gauge group.

• They may give rise to superpotentials involving 4-D matter fields <sup>b</sup>. The kind of operator which is interesting for as is:

$$e^{-S_{D2}}\nu_R\nu_R \tag{8}$$

- The operator  $\nu_R \nu_R$  has charge = 2 under both  $U(1)_{B-L}$  and  $U(1)_R$ . Thus the operator  $e^{-S_{D2}}$  has to transform with charges = -2.
- Transition amplitude induced by D2-instanton M is proportional to ( $S_{D^2}$ = Born-Infeld action)

$$e^{-S_{D2}} = \exp\left(-\frac{V_{\Pi_M}}{\lambda} + i\sum_r q_{M,r}a_r\right)$$
(9)

(e.g. in toroidal models  $q_M = n_M m_M m_M, n_M n_M n_M$  in terms of wrapping numbers of D2)

<sup>b</sup>Ganor (96);Florea,Kachru,Mc Greevy,Saulina (06)

• We already saw that the  $U(1)_A$  gauge bosons have non-trivial couplings to a set of basic 2-forms  $B_r$  in the 4d theory

$$S_{BF} = \sum_{A,r} N_A p_{Ar} \int_{4d} B_r \wedge F_A \tag{10}$$

(e.g. in toroidal models  $p_A = m_A n_A n_A$  or  $m_A m_A m_A$ )

• This implies that under a  $U(1)_A$  gauge transformation :  $A_A \rightarrow A_A + d\Lambda_A$  the  $a_r$  scalar dual to  $B_r$  transforms:

$$\boldsymbol{a_r} \to \boldsymbol{a_r} + N_A \left( p_{Ar} - p_{A^*r} \right) \Lambda_A \tag{11}$$

$$\sum_{r} q_{M,r} \sum_{A} N_A \left( p_{Ar} - p_{A^*r} \right) \Lambda_A = \sum_{A} N_A \left( I_{MA} - I_{MA^*} \right) \Lambda_A$$
(12)

where  $I_{MA} = \Pi_M \cdot \Pi_A$  is the intersection number of M and A,



<sup>a</sup>L.E.I. and Uranga (06); Blumenhagen, Cvetic and Weigand (06)



can give masses to unwanted extra zero modes.

<sup>a</sup>Witten (96)

## The Microscopic Mechanism

- How such a term is generated? There are open strings stretching between D2 and the background D6.
- Quantization of these open strings shows there are fermionic zero modes  $\alpha_i, \gamma_i = 1,2$  at the Mc and Md\* intersections respectively.
- There are Mc-cd\*-d\*M cubic couplings involving the scalar  $\nu_R$ :

$$L_{cubic} \propto d_a^{ij} \left( \alpha_i \, \nu^a \gamma_j \right) \,, a = 1, 2, 3$$
 (16)



• We have to integrate over zero modes  $\alpha_i, \gamma_i$ : (recall e.g.  $\int d\alpha \alpha = 1, \int d\alpha = 0$ )

$$\int d^2 \alpha \, d^2 \gamma \, e^{-d_a^{ij} \, (\alpha_i \nu^a \gamma_j)} \propto -\nu_a \nu_b \int d^2 \alpha \, d^2 \gamma \, \alpha_i \alpha_j \gamma_k \gamma_l \, d_a^{ik} d_b^{jl}$$
$$= \nu_a \nu_b \left( \epsilon_{ij} \epsilon_{kl} d_a^{ik} d_b^{jl} \right)$$
(17)

yielding a right-handed neutrino mass term:

$$\nu_{a}\nu_{b}M_{s}\left(\epsilon_{ij}\epsilon_{kl}d_{a}^{ik}d_{b}^{jl}\right)\exp\left(-\frac{V_{\Pi_{M}}}{\lambda}+i\sum_{r}q_{M,r}a_{r}\right)$$
(18)

- The gauge  $U(1)_c, U(1)_d$  transformation of the bilinear piece and the  $e^{-S_{D2}}$  factor nicely cancel.
- Note that the flavor structure is controlled by the  $d_a^{ik}$  coefficients which are given by the triangle correlators.

## Size of $\nu_R$ Majorana masses

• The  $\nu_R$  Majorana masses are of order:

$$M_{\nu_R} \simeq M_s \, d^2 \exp\left(-\frac{V_{\Pi_M}}{\lambda}\right) = M_s \, d^2 \exp\left(-\sum_r q_{M,r} Re U_r\right)$$
(19)

• In the N = 1 SUSY case  $U_r$  are the complex structure moduli.

- For usual gauge instantons this would be strongly supressed since  $\sum_{r} q_{M,r} ReU_{r}$  is nothing but the inverse gauge coupling constant.
- For the non-gauge instantons here considered that quantity is unrelated to SM gauge couplings, may be of order one.
- One expects  $\nu_R$  masses to be e.g. a few orders of magnitude below the string scale  $M_s$ .

## **R-parity and instanton induced Majorana masses**

- The composite field  $\exp(-U)$  has  $U(1)_{B-L}$  and  $U(1)_R$  charges = -2. It is a sort of effective Majoron-like field.
- This means that there is an unbroken gauge  $Z_2$  soubgroup of  $U(1)_{B-L}$ .
- This is nothing but R-parity which is an automatic symmetry if  $\nu_R$  masses are generated this way.
- Of course this is so in the absence of other possible instantons which might violate R-parity. One has to check in each compactification for the absence of these other instantons.

## **Further remarks**

- Note that it is possible that in the preseence of RR and/or NS fluxes many or all extra zero modes may be lifted. (E.g. that happens in IIB orientifolds with 3-form fluxes in wich certain D3-instanton zero modes are lifted). In that case superpotentials may be generated event if apparently there are too many zero modes.
- Other class of extra zero modes may arise if the instanton M intersects 'hidden sector' D6-branes. In such a case operators of the general form

$$\nu_a \nu_b \Phi_H \dots \Phi_H \tag{20}$$

may appear, with  $\Phi_H$  hidden sector D=4 fields . These may give rise tu  $\nu_R$  masses upon  $\Phi_H$  vevs.

• Note that although our discussion uses the language of N = 1 SUSY the mechanism is general and should exist also in non-SUSY models.

## **Direct generation of Weinberg operator**

- In addition to the see-saw contribution there might be a direct generation of the Weinberg operator  $L\bar{H}L\bar{H}$ .
- Instantons W with intersections

$$I_{Wc} - I_{Wc^*} = I_{Wd} - I_{Wd^*} = -2$$
<sup>(21)</sup>



• A Weinberg operator is generated:

$$L_{a}\bar{H}L_{b}\bar{H}\frac{1}{M_{s}}\left(\epsilon_{ij}\epsilon_{kl}\sum_{s}c_{a}^{ik}(s)c_{b}^{jl}(s)\right)\exp\left(-U_{s}\right)$$
(22)

• This contribution may be comparable to the see-saw one.

## **Neutrino Mass Matrices : the Weinberg Operator**

• The flavour structure is particularly simple for Sp(2) instantons <sup>a</sup> In that case the fermionic zero modes are Sp(2) doublets and

$$c_a^{ij} = \epsilon^{ij} c_a ; d_a^{ij} = \epsilon^{ij} d_a , a = 1, 2, 3$$
 (23)

• Then the left-handed neutrino masses from Weinberg operators are:

$$M^{\nu_{L}}{}_{ab} = \frac{2 < \overline{H} >^{2}}{M_{s}} \sum_{r} c_{a}^{(r)} c_{b}^{(r)} e^{-U_{r}}$$
(24)

• The sum is over different instanton contributions. One thus has a structure

$$M^{\nu_{L}} = \frac{2V^{2}}{M_{s}} \sum_{r} e^{-U_{r}} \operatorname{diag}\left(c_{1}^{(r)}, c_{2}^{(r)}, c_{3}^{(r)}\right) \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \cdot \operatorname{diag}\left(c_{1}^{(r)}, c_{2}^{(r)}, c_{3}^{(r)}\right)$$
(25)

• Each instanton makes one particular combination of  $\nu_L$  massive. A hierachy among 3 eigenvalues naturally appears for 3 instantons with different  $exp(-U_r)$ 

<sup>a</sup>L.I.,B.Schellekens, A. Uranga, hep-th/0704.1079

• One can get a structure consistent with experiment <sup>b</sup>. For example, if two instantons  $D2^{(2)}$  and  $D2^{(3)}$  dominate with  $exp(-ReU_3)/exp(-ReU_2)\sim 5$  and

$$(c_1^{(2)}, c_2^{(2)}, c_3^{(2)}) = \frac{1}{\sqrt{3}} (1, 1, 1), (c_1^{(3)}, c_2^{(3)}, c_3^{(3)}) = \frac{1}{\sqrt{2}} (0, -1, 1).$$
 (26)

• Asuming small mixing in the charged lepton mass matrix one obtains hierarchy of masses and (aproximately) Tri-bimaximal mixing for PMNS matrix

$$\boldsymbol{U}_{\rm tri} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0\\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2}\\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}.$$
 (27)

• This is just an example. More generaly, there are ranges of  $c_a^{(i)}$  consistent with the data.

<sup>b</sup>Antusch,L.I.,Macri Arxiv:0760.2132 [hep-ph].

#### **Neutrino Mass Matrices: See-Saw**

• Again, for Sp(2) instantons  $\nu_R$ -masses flavor dependence simplified:

$$M^{\nu_{R}}{}_{ab} = 2M_{s} \sum_{r} d^{(r)}_{a} d^{(r)}_{b} e^{-U_{r}}$$
(28)

• Again, each instanton makes one particular combination of  $\nu_R$  massive. A hierarchy among 3 eigenvalues naturally appears for 3 instantons with different  $exp(-U_r)$ .

$$M^{\nu_{L}}(\text{see-saw}) = \frac{\langle \overline{H} \rangle^{2}}{2M_{s}} h_{D}^{T} (\sum_{r} d_{a}^{(r)} d_{b}^{(r)} e^{-U_{r}})^{-1} h_{D}$$
(29)

- This case is more model dependent, structure depends on ordinary Yukawa coupling contant matrices  $h_D$ . Again, agreement with experimant may be obtained e.g.for  $d_a$  aligned with diagonal  $h_D$
- Both Weinberg and see-saw mechanisms may be simultaneously present



- i) The SM group should be extended by a  $U(1)_{B-L}$  gauge symmetry. There are  $\nu_R$  's in the massless spectrum to begin with.
- ii) The gauge boson  $U(1)_{B-L}$  gets a Stuckelberg mass combining with an axion-like closed string field.
- iii) String Dp-instantons exist with the appropriate fermionic zero modes.
- Local examples exist in which indeed the neutrino mass is generated <sup>a</sup> It requires D2-branes wrapping rigid cycles.
- This is stringy mechanism, because these are string instanton effects with no obvious field theory counterpart.
- The same mechanism exists in other known compactifications like Type IIB orientifolds and heterotic (with U(1) bundles).

<sup>&</sup>lt;sup>a</sup>Cvetic,Richter,Weigand hep-th/0703028.

## **Other Interesting Instanton-induced Operators**

• It is clear that in general OTHER instantons may exist generating operators violating some massive U(1) symmetry:

$$e^{-S_{Ins}}\phi...\phi\tag{30}$$

- Examples are
  - The  $\mu$ -term in the MSSM

$$e^{-S_{Ins}} H\overline{H}$$
(31)

- Some Yukawa couplings which may be forbidden perturbatively (e.g., for the 1-st generation). For example Lepton Yukawas in the LR-symmetric D3-brane model.
- R-parity violating couplings in MSSM :  $L\bar{H}$  , UDD .
- Superpotential couplings involving hidden sector fields, possibly usefull in fixing moduli and/or breaking SUSY<sup>a</sup>

<sup>a</sup>Florea et al.; Akerblom et al. 2006

# **VII- FLUXES AND SUSY BREAKING**

## **SUSY breaking and fluxes**

 Imagine we succed in building a string vacuum with the structure of the MSSM. Does string theory give us information about the structure of SUSY breaking soft terms?.

$$L_{g} = \frac{1}{2} \sum_{a} M_{a} \lambda_{a} \lambda_{a} + h.c.$$

$$L_{m^{2}} = -m_{H_{d}}^{2} |H_{d}|^{2} - m_{H_{u}}^{2} |H_{u}|^{2} - m_{Q_{ij}}^{2} Q_{i} Q_{j}^{*} - m_{U_{ij}}^{2} U_{i} U_{j}^{*}$$

$$- m_{D_{ij}}^{2} D_{i} D_{j}^{*} - m_{L_{ij}}^{2} L_{i} L_{j}^{*} - m_{E_{ij}}^{2} E_{i} E_{j}^{*}$$

$$L_{A,B} = -A_{ij}^{U} Q_{i} U_{j} H_{u} - A_{ij}^{D} Q_{i} D_{j} H_{d} - A_{ij}^{L} L_{i} E_{j} H_{d} - B H_{d} H_{u} + h.c.$$
(32)

- There are plenty of possibilities for SUSY breaking and its mediation. In essentially all approaches there is a hidden sector of SM singlets in which SUSY breaking resides.
- In string theory the Kahler moduli  $T_i$ , dilaton S and complex structure  $U_i$  fields are natural candidates to form part of the hidden sector.
- One simple possibility is to asume that the source of SUSY breaking resides in the auxiliary fields of the dilaton/moduli fields , e.g. ,  $F_S$ ,  $F_{T_i}$ .
- If we have a knowledge of the Kahler potential and gauge kinetic function one can then derive predictions for the soft terms. In the Heterotic it was found:

## – $F_T \neq 0$ : MODULUS DOMINANCE

This gives rise to a vanishing c.c. to leading order, which is a nice point. However no-soft terms appear (again to leading order).

–  $F_S \neq 0$  : DILATON DOMINANCE

This gives rise to interesting universal soft terms:

$$m^2 = \frac{1}{3} |M|^2; A^{ijk} = -h^{ijk} M_a$$
 (33)

- However no obvious microscopic source for such  $F_S \neq 0$  was found in the context of the heterotic string.
- As we have seen, Type II orientifolds, e.g. Type IIB orientifolds compactified on a CY offer new possibilities for the embedding of the SM in string theory.
- In a different development it has been realized the important role played by antisymmetric field fluxes in Type IIB orientifold compactifications.
- Fluxes in Type IIB orientifold theories may fix both the dilaton and the complex-structure moduli  $M_i$ . Including non-perturbative effects depending on the volume moduli  $T_i$  all the moduli in these compactifications could possibly be determined.
- Here we are going to discuss another consequence of the presence of fluxes
   they may GENERATE SUSY BREAKING SOFT TERMS.



## **Closed string fluxes in IIB**

- Type II string theory contains antisymmetric fields from the RR and NS sectors. Their field strengths may be non-vanishing and the corresponding fluxes through closed surfaces are quantized.
- These provide for new (discrete) degrees of freedom in each compactification.
- The best understood case is that of 3-form fluxes in Type IIB orientifolds. There are NS  $H_3$  and RR  $F_3$ . They would verify for any 3-cycle  $\Sigma$  in the CY

$$\frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma} \boldsymbol{F_3} \in \boldsymbol{Z} \; ; \; \frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma} \boldsymbol{H_3} \in \boldsymbol{Z}$$
(34)

 These fluxes contribute positively to the vacuum energy. That lead to certain no-go theorems stating that no IIB compactifications with fluxes are consistent with equations of motion. • Fluxes also contribute to tadpoles of RR  $C_4$  form due to the CS coupling

$$\int_{M_4 \times CY} \frac{H_3 \wedge F_3 \wedge C_4}{(35)}$$

One has in fact

$$N_{flux} = \frac{1}{(4\pi^2 \alpha')^2} \int_{CY} H_3 \wedge F_3 = \frac{1}{(4\pi^2 \alpha')^2} \frac{1}{(S+S^*)} \int_{CY} \frac{G_3 \wedge \overline{G_3}}{(36)}$$

where S is the complex dilaton and

$$G_3 = F_3 - iSH_3$$
 (37)

Both things (vacuum energy and RR tadpoles) may be cancelled in the orientifold case, the orientifolds have negative tension and negative charge.
 Solutions of equations of motion are obtained (Giddings, Kachru Polchinski) if G<sub>3</sub> is imaginary self-dual (ISD):

$$*_6G_3 = i G_3$$
 (38)

• The presence of fluxes generates a superpotential for the complex structure  $U_i$  and dilaton S fields

$$W_{GVW} = \int_{CY} G_3 \wedge \Omega^{CY}$$
(39)

where  $\Omega^{CY}$  is the holomorphic 3-form of the CY, which contains dependence on complex structure fields, (e.g. for tori  $\Omega = (dx_1 + \tau_1 dy_1) \wedge (dx_2 + \tau_2 dy_2) \wedge (dx_3 + \tau_3 dy_3)$ ).

• Minimization of the effective potential generically fixes the complex structure and dilaton fields  $U_i$ , S. (But not the Kahler moduli  $T_i$ ).



## **ISD and IASD 3-form fluxes**

• The flux  $G_{mnp}$  transforms as a 20-dimensional reducible SO(6)representation, decomposing as  $\mathbf{20} = \overline{\mathbf{10}} + \mathbf{10}$  (imaginary self-dual (ISD)  $G^+_{(3)}$  and imaginary anti self-dual (IASD)  $G^-_{(3)}$  parts), respectively

$$G_{(3)}^{\pm} = \frac{1}{2} (G_{(3)} \mp i *_6 G_{(3)}) ; *_6 G_{(3)}^{\pm} = \pm i G_{(3)}^{\pm}$$
 (40)

• It is useful to classify the components of the ISD and IASD parts of  $G_3$ according to their behavior under SU(3) (Graña, Polchinski), 10 = 6 + 3 + 1

	ISD			IASD	
SU(3) rep.	Form	Tensor	SU(3) rep.	Form	Tensor
1	(0,3)	$G_{ar{1}ar{2}ar{3}}$	1	(3,0)	$G_{123}$
$\overline{6}$	$(2,1)_P$	$S_{\overline{i}\overline{j}}$	6	$(1,2)_P$	$S_{ij}$
3	$(1,2)_{NP}$	$A_{ij}$	3	$(2,1)_{NP}$	$A_{\overline{i}\overline{j}}$

• Here one defines (and similarly for  $S_{\overline{ij}}, A_{ij}$ )

$$S_{ij} = \frac{1}{2} (\epsilon_{ikl} G_{j\bar{k}\bar{l}} + \epsilon_{jkl} G_{i\bar{k}\bar{l}}) ; \quad A_{\bar{i}\bar{j}} = \frac{1}{2} (\epsilon_{\bar{i}\bar{k}\bar{l}} G_{kl\bar{j}} - \epsilon_{\bar{j}\bar{k}\bar{l}} G_{kl\bar{i}})$$
(41)

- Subindex P=primitive  $\leftrightarrow G_3 \wedge J = 0$ . (In general Non-primitive fluxes not present in CY, since if  $G_3 \wedge J \neq 0$  5-cycles should exist, and they do not in CY).
- (0,3) ISD ( (3,0) IASD) flux contributes to RR-charge with same sign as D3 ( $\overline{D3}$ )-branes
- (2,1) fluxes preserve SUSY whereas (0,3) do not.

## **Effect of fluxes on D3-brane fields**

- Let us consider a stack of D3 branes at a (smooth) point in the CY (the results for an orbifold singularity may be obtained by projection). In terms of N = 1 SUSY we have super-Yang-Mills and 3 chiral adjoint multiplets  $\Phi_i$ , i=1,2,3.
- The vev of the 3 worldvolume scalars  $\phi_i$  correspond to the three complex transverse coordinates  $x^m=2\pi \alpha' \phi^m$
- The effective action in the presence of fluxes may be obtained by expanding the Dirac-Born-Infled action (as extended by Myers to non-Abelian case):

$$S_{BI} = -\mu_3 \int d^4 x \operatorname{Tr} \left( e^{-\phi} \sqrt{-\det\left(P\left[E_{\mu\nu} + E_{\mu m}(Q^{-1} - \delta)^{mn} E_{n\nu}\right] + \sigma F_{\mu\nu}} \right) \det(Q)} \\ E_{MN} = G_{MN} - B_{MN} \\ Q^m{}_n = \delta^m{}_n + i\sigma \left[\phi^m, \phi^p\right] E_{pn}$$

$$\sigma = 2\pi \alpha'$$

$$(43)$$

as well as the Chern-Simons action  $S_{CS}$  which includes RR couplings (plus fermionic terms).

• Plugging the closed string background one obtains <sup>a</sup> the SUSY breaking soft terms (we take  $A_{mn} = 0$ ).

$$m_{ij}^{2} = \frac{g_{s}}{6} (|G_{123}|^{2} + \sum_{ij} |S_{ij}|^{2} - Re(G_{123}G_{\overline{1}\overline{2}\overline{3}} + \frac{1}{4}S_{lk}S_{\overline{l}\overline{k}}))$$

$$A^{ijk} = -h^{ijk}\frac{g_{s}^{1/2}}{\sqrt{2}}G_{123}$$

$$M^{a} = \frac{g_{s}^{1/2}}{\sqrt{2}}G_{123}$$

$$\mu_{ij} = -\frac{g_{s}^{1/2}}{2\sqrt{2}}S_{ij}$$
(44)

• Note that if only ISD fluxes (  $G_{\bar{1}\bar{2}\bar{3}}$ ,  $S_{\bar{l}\bar{k}}$ ) are present, no soft terms are

<sup>a</sup>M. Graña hep-th/0209200, P. Camara et al.hep-th/0311241; Graña et al. hep-th/0312232.

generated!. This is like in the heterotic modulus dominance situation.

• On the other hand in the presence of IASD (3,0) fluxes  $G_{123}$  one gets

$$m^{2} = \frac{g_{s}}{6} |G_{123}|^{2}; M^{a} = \frac{g_{s}^{1/2}}{\sqrt{2}} G_{123}; A^{ijk} = -h^{ijk} \frac{g_{s}^{1/2}}{\sqrt{2}} G_{123}$$
(45)

• Note the relationships

$$A^{ijk} = -h^{ijk}M_a \; ; \; m^2 \; = \; \frac{1}{3}|M|^2$$
 (46)

- They correspond to dilaton dominated SUSY-breaking previously discussed in heterotic context.
- Note that anti-D3branes in (0,3) backgrounds have these soft terms.
- On the other hand only ISD fluxes are known to solve the equations of motion.
- One may conclude that, if one wants to have SUSY breaking induced by ISD fluxes, locating the MSSM at at D3 branes is not a good idea. Let us locate the SM at D7-branes.



 $K3 \times C$  and assume constant backgrounds over  $\Sigma_4$ . Similar features expected in more complicated geometries.

- We will asume our D7 brane transverse coordinate is  $x_3$ . There are now two types of chiral matter fields when reducing to 4 dimensions:
  - $\phi_3$  which parametrizes the location of the D7-branes in transverse space
  - $-\phi_{1,2}$  which come from the zero modes of the gauge fields in 8 dimensions.
- Doing a similar expansion of the  $S_{DBI} + S_{CS}$  as in the D3 case one finds soft terms depending on the fluxes as follows <sup>a</sup>:

$$\begin{split} m_{1\bar{1}}^2 &= m_{2\bar{2}}^2 = 0 \; ; \; B_{ij} = 0 \; , \; i, j \neq 3 \\ m_{3\bar{3}}^2 &= \frac{g_s}{18} \left( |G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4} |S_{\bar{3}\bar{3}}|^2 + \frac{1}{4} \sum_{i,j=1,2} |S_{ij}|^2 \right) \\ B_{33} &= \frac{g_s}{9} \left( \frac{1}{4} (S_{12})^{*2} - \frac{1}{2} (G_{\bar{1}\bar{2}\bar{3}})^* (S_{\bar{3}\bar{3}})^* - \frac{1}{4} (S_{22})^* (S_{11})^* \right) \end{split}$$

<sup>a</sup>P. Camara et al hep-th/0408036; Lust et al. hep-th/0406092.

$$A^{ijk} = -h^{ijk} \frac{g_s^{1/2}}{3\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^*$$

$$M^a = \frac{g_s^{1/2}}{3\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^*$$

$$\mu_{33} = -\frac{g_s^{1/2}}{6\sqrt{2}} (S_{\bar{3}\bar{3}})^*$$

$$\mu_{ij} = -\frac{g_s^{1/2}}{6\sqrt{2}} S_{ij}, i, j = 1, 2$$

## The case of ISD fluxes

• These include fluxes (0,3)  $(G_{ar{1}ar{2}ar{3}})$  and (2,1)  $(S_{ar{i}ar{j}})$  . One finds

$$m_{\Phi_{77}}^{2} = \frac{g_{s}}{18} |G_{\bar{1}\bar{2}\bar{3}}|^{2}; M^{(77)} = \frac{g_{s}^{1/2}}{3\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^{*}; A^{ijk} {}^{(77)} = -h^{ijk} \frac{g_{s}^{1/2}}{3\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^{*}$$
(48)  
$$\mu_{(77)} = -\frac{g_{s}^{1/2}}{6\sqrt{2}} (S_{\bar{3}\bar{3}})^{*}; B_{33} = -\frac{g_{s}}{18} (G_{\bar{1}\bar{2}\bar{3}})^{*} (S_{\bar{3}\bar{3}})^{*} = 2M\mu_{(77)}$$
(49)

• Interestingly enough, for ISD backgrounds the scalar potential is positive definite

$$V_{ISD} = |-M_{77}^* \Phi_{77}^3^* - \mu_{(77)} \Phi_{77}^3 + \Phi_{77}^1 \Phi_{77}^2 + \Phi_{73} \Phi_{37}|^2 = |-M_{77}^* \Phi_{77}^3^* + \partial_{\phi}|_{3W}$$
(50)

• Note that the scheme is very constrained yielding relationships

$$A^{ijk\ (77)} = -h^{ijk}M^{(77)}; \ m^2_{\Phi^3_{77}} = |M^{(77)}|^2; \ B_{33} = 2M\mu_{(77)}$$
(51)

2



<sup>a</sup>Marchesano, Shiu hep-th/0409132.



• Here W is the superpotential generated by the flux. The supergravity auxiliary field for a chiral field with metric  $K_{i\bar{j}}$  is given by

$$\overline{F}^{i} = \exp(K/2M_p^2) \ K^{\overline{i}j} \ D_j W/M_p^2 \tag{53}$$

Applying to the above Kahler potential and superpotential one obtains

$$F^{S} = (S + S^{*})^{1/2} (T + T^{*})^{-3/2} \int G_{(3,0)} \wedge \Omega$$

$$F^{T} = -(S+S^{*})^{-1/2} (T+T^{*})^{-1/2} \int G_{(0,3)} \wedge \Omega$$
 (54)

• This shows that a (3,0) flux corresponds to a non-vanishing auxiliary field for the complex dilaton S, whereas a (0,3) corresponds to the overall Kähler field T. One then has

$$m_{3/2}^2 = e^K |W|^2 = (S+S^*)^{-1} (T+T^*)^{-3} |\int G_{(0,3)} \wedge \Omega|^2$$
. (55)

• One can now consider the presence of D7 branes in the CY . Consider the case of a toroidal/orbifold orientifold compactification with  $D7^i$ -branes wrapping the  $T^4$  transverse to the i - th complex plane. One has gauge kinetic functions

$$f_{7_i} = T_i \tag{56}$$

• We saw that the two types of matter fields in the worldvolume of D7-branes

turn out to have Kahler metrics (ignoring here c.s. fields):

$$K_{ii}^{i} = \frac{1}{S+S^{*}}; K_{ii}^{j} = \frac{1}{T_{k}+T_{k}^{*}}, i \neq j \neq k$$
 (57)

• On the other hand for states for intersecting  $D7_i - D7_j$ -branes we already mentioned (for the T-dual of intersecting D6-branes):

$$K_{ij} = rac{1}{(S+S^*)^{1/2}(T+T^*)^{1/2}}$$
 (58)

• In terms of the dependence on the overall Kahler modulus  $T = T_i$  one can summarize for the gauge kinetic function and the matter metrics the result

$$f = T ; K_{\xi} = \frac{1}{s^{1-\xi}t^{\xi}}$$
 (59)

where  $t = T + T^*$ ,  $s = S + S^*$  and the 'modular weights'  $\xi = 0, 1/2, 1$ depending the origin of the matter field.

- One can argue that for large volume these 3 types of matter fields coming from D7-branes are general, not particular to the toroidal case.
- The D7 branes generically may have magnetic flux  $F_i$  in their worldvolume in order to obtain chirality. As we saw this flux modifies the Kahler moduli in a toroidal setting. One finds for  $D7^i$  branes within the same stack the result

$$K_{(7^{i}7^{i})_{j}} = \frac{1}{t^{k}} \left| \frac{1 + iF^{k}}{1 + iF^{j}} \right| \; ; \; K_{(7^{i}7^{i})_{i}} = \frac{1}{s} \left( 1 + |F^{j}F^{k}| \right), \quad (60)$$

where  $i \neq j \neq k$  label the 3 2-tori and  $F^i$  is the magnetic flux going through the i-th 2-torus which may be written as

$$F^{i} = n^{i} \left(\frac{st_{i}}{t_{j}t_{k}}\right)^{1/2}, \tag{61}$$

with  $n^i$  quantized integer fluxes.

 For states coming from open strings in between (magnetized) branes D7<sup>a</sup>, D7<sup>b</sup> wrapping different 4-tori we already mentioned (for the T-dual case) one has

$$K_{7^{a}7^{b}} = \frac{1}{(st_{1}t_{2}t_{3})^{1/4}} \prod_{j=1}^{3} u_{j}^{-\theta_{ab}^{j}} \sqrt{\frac{\Gamma(\theta_{ab}^{j})}{\Gamma(1-\theta_{ab}^{j})}}, \quad (62)$$

where  $u_j$  are the real parts of the complex structure moduli,  $\Gamma$  is the Euler Gamma function and

$$\theta_{ab}^{j} = \arctan(F_{b}^{j}) - \arctan(F_{a}^{j}).$$
(63)

• The gauge kinetic functions are also modified in the presence of magnetic fluxes as

$$Ref_{i}^{a} = T_{a} \left( 1 + |F_{a}^{j}F_{a}^{k}| \right).$$
 (64)

• These results apply for a toroidal and/or orbifold setting. However we can try to model out what could be the effect of fluxes in a more general setting following the above structure. To model out the possible effect of fluxes we consider the limit with  $t_i = t$  and diluted fluxes  $|F_i| = F$ , i.e. large t and

ignore the dependence on the complex structure  $u_i$  fields. Then one gets from the above formulae:

$$K^{i}_{(7^{i}7^{i})_{j}} = \frac{1}{t} ; K_{(7^{i}7^{i})_{i}} = \frac{1}{s} \left(1 + a_{i} \frac{s}{t}\right), \qquad (65)$$

$$K_{7^{a}7^{b}} = \frac{1}{(s^{1/2}t^{1/2})} (1 + c_{ab} \frac{s^{1/2}}{t^{1/2}}), \qquad (66)$$

$$\operatorname{Re} f_i = t + a_i s , \qquad (67)$$

where  $a_i, c_{ab}$  are constants (including the flux quanta) of order one.

• These three formulae may again be summarised by:

$$K_{matter} = \frac{1}{s^{(1-\xi)}t^{\xi}} \times (1 + c_{\xi}(s/t)^{1-\xi}) = \frac{1}{s^{(1-\xi)}t^{\xi}} + \frac{c_{\xi}}{t},$$
(68)

with  $c_{\xi}$  some flux-dependent constant coefficient whose value will depend on the modular weight  $\xi$  and the magnetic quanta.

• In the dilute flux  $t \to \infty$  one recovers the fluxless case.



- Insist that the D-brane configuration is such that there is at least one Yukawa coupling (top quark) of order g (the gauge coupling constant).

- Gauge coupling unification may be natural if the MSSM resides at F-theory
   7-branes (rather than D7-branes) (Beasley, Heckman, Vafa; Donagie, Wijnholt).
- A further simplification is that we will consider a single local Kahler modulus *t* coupling to the MSSM brane system. t is not in general an overall volume modulus but a local modulus.
- Then, using the Kahler metrics and gauge kinetic function discussed above one can compute the MSSM SUSY-breaking soft terms in the standard way.

## Soft term computation from the effective action

- Using the effective  ${\cal N}=1$  supergravity Lagrangian one can compute soft terms as

$$M_{i} = \frac{1}{2 \operatorname{Re} f_{i}} F^{M} \partial_{M} f_{i} ,$$
  

$$m_{I}^{2} = m_{3/2}^{2} - \sum_{M,N} \bar{F}^{\bar{M}} F^{N} \partial_{\bar{M}} \partial_{N} \log(\tilde{K}_{I\bar{I}}) ,$$
  

$$A_{IJL} = F^{M} [\hat{K}_{M} + \partial_{M} \log(Y_{IJL}) - \partial_{M} \log(\tilde{K}_{I\bar{I}} \tilde{K}_{J\bar{J}} \tilde{K}_{L\bar{L}})]$$
  

$$B = \left( F^{m} \left[ \hat{K}_{m} + \partial_{m} \log \mu - \partial_{m} \log(K_{H_{u}} K_{H_{d}}) \right] - m_{3/2} \right)$$

Here  $F^M$  are auxiliary fields of moduli,  $\tilde{K}_{I\bar{I}}$ ,  $\hat{K}_M$  the metric of matter and moduli and  $f_i$  gauge kinetic functions.

• In Type IIB orientifolds the holomorphic perturbative superpotential is independent of the Kahler moduli so that the derivatives of  $\widehat{Y}^{(0)}$  in the expression for A vanish.

• Using these formulae one then obtains <sup>a</sup> general soft terms as follows (the gaugino mass M is a free parameter)::

$$m_{\alpha}^{2} = (1 - \xi_{\alpha})|M|^{2} , \ \alpha = Q, U, D, L, E, H_{u}, H_{d},$$
(69)  

$$A_{U} = -M(3 - \xi_{H_{u}} - \xi_{Q} - \xi_{U}),$$
  

$$A_{D} = -M(3 - \xi_{H_{d}} - \xi_{Q} - \xi_{D}),$$
  

$$A_{L} = -M(3 - \xi_{H_{d}} - \xi_{L} - \xi_{E}),$$
  

$$B = -M(2 - \xi_{H_{u}} - \xi_{H_{d}}).$$

where we have neglected for the moment the corrections from magnetic fluxes (i.e. dilute flux limit ).

• Within the philosophy of gauge coupling unification one can assume unified modular weights:

$$\xi_f = \xi_Q = \xi_U = \xi_D = \xi_L = \xi_E .$$
 (70)

<sup>a</sup>L. Aparicio, L.E.I., D. G.Cerdeño, hep-ph/0805.2943.

- On the other hand the Higgs fields could have different modular weight than fermion fields. So we will take  $\xi_H = \xi_{H_u} = \xi_{H_d} = 0, 1, 1/2.$
- We have three type of 7-brane matter fields  $\phi$ , A, I corresponding to modular weights 0,1,1/2 respectively.
- It turns out that there are only renormalizable couplings of three types.

$$(A - A - \phi)$$
;  $(I - I - A)$ ;  $(I, I, I)$  (71)



• They correspond to modular weights (1,1,0)), (1,1/2,1/2) and (1/2,1/2,1/2).

- The same types of Yukawa couplings exist in F-theory compactifications (Vafa et al.).
- For each of these three configurations the results for soft terms are shown in the table.

$(\xi_L,\xi_R,\xi_H)$	Coupling	М	$m_L^2$	$m_R^2$	$m_H^2$	A	В
(1, 1, 0)	(A-A- <i>φ</i> )	M	0	0	$ M ^2$	-M	-2M
(1/2, 1/2, 1)	(I-I-A)	M	$\frac{ M ^2}{2}$	$\frac{ M ^2}{2}$	0	-M	0
(1/2, 1/2, 1/2)	(1-1-1)	M	$\frac{ M ^2}{2}$	$\frac{ M ^2}{2}$	$\frac{ M ^2}{2}$	-3/2M	-M

Table 1: Modulus dominated soft terms for choices of modular weights  $\xi_{\alpha}$  which are consistent with the existence of trilinear Yukawa couplings in 7-brane systems.

• Note that in the scenarios with couplings (A-A- $\phi$ ) and (I-I-A) it is natural to assume that the Higgs field is identified with fields of type  $\phi$  and A respectively and these are the cases displayed in the table.

- Concerning the *B* parameter it is obtained assuming an explicit  $\mu$ -term.
- The modular weights in the first line reproduces the results obtained from (0, 3) ISD fluxes. (No prediction from fluxes exists for particles at intersecting D7-branes).
- The modular weights in the third case are universal, a particular case of CMSSM boundary conditions.
- One can also estimate the possible effect of magnetic fluxes in the dilute limit  $t \to \infty$ . Using the corrected formulae for gauge kinetic function and Kahler metrics discussed before one finds

Coupling	$m_f^2$	$m_{H}^{2}$	А	В			
(A-A- <i>φ</i> )	0	$ M ^2(1-2 ho)$	-M(1- ho)	$-2M(1 \cdot$	- <i>p</i> )		
(I-I-A)	$\frac{ \boldsymbol{M} ^2}{2}(1-\frac{3}{2}\rho_f)$	0	$-M(1-\rho_f)$	0			
(1-1-1)	$\frac{ \boldsymbol{M} ^2}{2}(1-\frac{3}{2}\rho_f)$	$rac{ \pmb{M} ^2}{2}(1-rac{3}{2}\pmb{ ho}_{\pmb{H}})$	$-\frac{1}{2}M(3-\rho_H-2\rho_f)$	-M(1 - M)	$ ho_H)$		
where							
$\rho = \frac{(c_H - as)}{t}; \ \sigma = \frac{as}{t}; \ \rho_f = \frac{c_f}{t^{1/2}}; \ \rho_H = \frac{c_H}{t^{1/2}}, \tag{72}$							



$$\mu B = \frac{1}{2} \sin 2\beta \left( m_{H_d}^2 + m_{H_u}^2 + 2\mu^2 \right), \tag{74}$$

where

$$tan\beta = \langle H_u/H_d \rangle \quad . \tag{75}$$

- In addition there are a number of experimental constraints
  - LEP limits on SUSY particles and lightest Higgs boson.
  - $2.85 \times 10^{-4} \leq BR(b \rightarrow s\gamma) \leq 4.25 \times 10^{-4}$  (Heavy Flavour Averaging Group).
  - BR $(B^0_s \to \mu^+ \mu^-) < 5.8 imes 10^{-8}$  at 95% c.l. (CDF)
  - Anomalous magnetic moment of the muon,  $11.6 \times 10^{-10} \le a_{\mu}^{\text{SUSY}} \le 43.6 \times 10^{-10}.$
  - WMAP limits on cold dark matter (applied to neutralino LSP),  $0.1037 \leq \Omega h^2 \leq 0.1161.$

## The intersecting (I-I-A) and (I-I-I) schemes

• These two cases may be analysed together by taking the Higgs modular weight  $\xi_H$  as a free parameter with  $\xi_H = 1, 1/2$  reproducing the two limits . Then soft erms are

$$m_{L,E,Q,U,D}^{2} = |M|^{2}/2, \qquad (76)$$

$$m_{H_{u},H_{d}}^{2} = (1-\xi_{H})|M|^{2},$$

$$A_{U,D,L} = -M(2-\xi_{H}),$$

$$B = -2M(1-\xi_{H}).$$

• A sample of the resulting SUSY spectrum is shown in the figure as a function of  $tan\beta$  for M=400 GeV and  $\mu < 0$  for  $\xi_H = 1/2, 1$ .



Figure 1: Low-energy supersymmetric spectrum as a function of  $\tan \beta$  for  $\xi_H = 1/2$ , (left) and  $\xi_H = 1$  (right) with M = 400 GeV and  $\mu < 0$ . The ruled area for large  $\tan \beta$  is excluded by the occurrence of tachyons in the slepton sector.

• Note for not too large  $tan\beta$  the lightest neutralino is the LSP. For large  $tan\beta$  the lightest stau becomes lighter (and even tachyonic).

- This is because the Yukawa coupling goes like  $1/\cos\beta$  and r.g.e. decreases the mass<sup>2</sup> for large Yukawa. One thus has  $\tan\beta \le 45$  GeV (for  $\xi_H = 1/2$ ) and  $\tan\beta \le 55$  (for  $\xi_H = 1$ ).
- The effect of various experimental constraints is shown in the next figure for  $\xi_H = 0.5, 0.6, 0.8, 1$  (condition for consistent radiative EW breaking given by a line).



- In order to get neutralino dark matter in agreement with WMAP results one should be in the coannihilation region with  $m_{\chi^0}\simeq m_{ ilde{ au}}$ .
- On the other hand in order to achieve correct EW symmetry breaking in this coannihilation region one needs  $\xi_H \simeq 0.6$  so that

$$m_f^2 = 1/2 |M|^2,$$
  

$$m_H^2 \approx (1/2 - 0.1) |M|^2,$$
  

$$A_{U,D,L} \approx (-3/2 + 0.1) M,$$
  

$$B \approx (-1 + 0.2) M.$$
(77)

• This in very close to the configuration with all particles residing at intersecting 7-branes. The small deviations may be atributed to subleading corrections .





Figure 4: Left) Low-energy supersymmetric spectrum as a function of  $\tan \beta$  for case (A-A- $\phi$ ). Right) Effect of the various experimental constraints on the  $(M, \tan \beta)$  plane for case (A-A- $\phi$ ).

• The LSP is the stau. One does not get consistent REW.

# LHC

• Making use of the missing energy signal for squarks and gluinos LHC will be able to test the intersecting brane scheme for

Int. Lumin.	M	$m_{ ilde q}$	$m_{ ilde{g}}$	$m_{\chi^0}\simeq m_{ ilde{ au}}$
1 $fb^{-1}$	$\leq 650$	$\leq 1.3$	$\leq 1.5$	$\leq 300$
10 $fb^{-1}$	$\leq 900$	$\leq 1.8$	$\leq 2.0$	$\leq 400$

Spectra:

