Macroscopic Supersymmetry

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Lecture 1

(Supplementary Material)

The Standard Model

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	
Q_i^α	$\mathbf{3}$	$\mathbf{2}$	$1/3$	$i = 1, 2, 3$
$\overline{u}_i^\alpha$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-4/3$	
$\overline{d}_i^\alpha$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$2/3$	
L_i^α	$\mathbf{1}$	$\mathbf{2}$	-1	
$\overline{e}_i^\alpha$	$\mathbf{1}$	$\mathbf{1}$	2	
H	$\mathbf{1}$	$\mathbf{2}$	1	
$G_{\mu\nu}$	$\mathbf{8}$	$\mathbf{1}$	0	
$W_{\mu\nu}$	$\mathbf{1}$	$\mathbf{3}$	0	
$B_{\mu\nu}$	$\mathbf{1}$	$\mathbf{1}$	0	

Lagrangian

$$-\frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4g^2}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4g_s^2}G^a_{\mu\nu}G^{a\mu\nu}$$

$$+ \psi_i^* i \not{D} \psi_i + \lambda_u^{ij} Q_i H \overline{u}_j + \lambda_d^{ij} Q_i H^c \overline{d}_j + \lambda_e^{ij} L_i H^c \overline{e}_j$$

$$+ (D_\mu H)^\dagger D^\mu H + m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$H^c = i\sigma^2 H^*$$

$$D_\mu = \partial_\mu + i\frac{1}{2}Y B_\mu + iT^a W^a_\mu + iT^a G^a_\mu$$

$$\mathrm{Tr}\,(T^a T^b) = \frac{1}{2}\delta^{ab}$$

$$Q = T_3 + \frac{1}{2}Y$$

The Minimal Supersymmetric Standard Model (MSSM)

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/3$	$i = 1, 2, 3$
$\overline{u}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-4/3$	
$\overline{d}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$2/3$	
L_i	$\mathbf{1}$	$\mathbf{2}$	-1	
$\overline{e}_i$	$\mathbf{1}$	$\mathbf{1}$	2	
H_u	$\mathbf{1}$	$\mathbf{2}$	1	
H_d	$\mathbf{1}$	$\mathbf{2}$	-1	
G_α	$\mathbf{8}$	$\mathbf{1}$	0	
W_α	$\mathbf{1}$	$\mathbf{3}$	0	
B_α	$\mathbf{1}$	$\mathbf{1}$	0	

Lagrangian

$$\int d^2\theta \left(-\frac{1}{4g'^2} B^\alpha B_\alpha - \frac{1}{4g^2} W^{a\alpha} W^a_\alpha - \frac{1}{4g_s^2} G^{a\alpha} G^a_\alpha \right) \;+\; \text{h.c.}$$

$$+ \int d^4\theta \left(\Phi_i^\dagger e^V \Phi_i + H_u^\dagger e^V H_u + H_d^\dagger e^V H_d \right)$$

$$V = \frac{1}{2} Y V_B + T^a V_W^a + T^a V_G^a$$

$$+ \int d^2\theta \left(\lambda_u^{ij} Q_i H_u \overline{u}_j + \lambda_d^{ij} Q_i H_d \overline{d}_j + \lambda_e^{ij} L_i H_d \overline{e}_j + \mu H_u H_d \right) \;+\; \text{h.c.}$$

Most General MSSM Renormalizable Lagrangian

$$\int d^2\theta \left(-\frac{1}{4g'^2} B^\alpha B_\alpha - \frac{1}{4g^2} W^{a\alpha} W^a_\alpha - \frac{1}{4g_s^2} G^{a\alpha} G^a_\alpha \right) \ + \ \text{h.c.}$$

$$+ \int d^4\theta \left(\ \Phi_i^\dagger e^V \Phi_i + H_u^\dagger e^V H_u + H_d^\dagger e^V H_d \ \right)$$

$$+ \int d^2\theta \left(\lambda_u^{ij} Q_i H_u \overline{u}_j + \lambda_d^{ij} Q_i H_d \overline{d}_j + \lambda_e^{ij} L_i H_d \overline{e}_j + \mu H_u H_d \right) \ + \ \text{h.c.}$$

$$+ \int d^2\theta \left(\lambda'^{\ [ij]k} L_i L_j \overline{e}_k + \lambda''^{\ ijk} Q_i L_j \overline{d}_k + \lambda'''^{\ i[jk]} \overline{u}_i \overline{d}_j \overline{d}_k + \mu^i L_i H_u \right) + \text{h.c.}$$

Flavor Symmetry of the (MS)SM

	$U(3)_Q$	$U(3)_{\bar{u}}$	$U(3)_{\bar{d}}$	$U(3)_L$	$U(3)_{\bar{e}}$
Q	$\mathbf{3}$				
$\bar{u}$		$\overline{\mathbf{3}}$			
$\bar{d}$			$\overline{\mathbf{3}}$		
L				$\mathbf{3}$	
$\bar{u}$					$\overline{\mathbf{3}}$
λ^u	$\overline{\mathbf{3}}$	$\mathbf{3}$			
λ^d	$\overline{\mathbf{3}}$		$\mathbf{3}$		
λ^e				$\overline{\mathbf{3}}$	$\mathbf{3}$

$$\lambda^u \neq 0, \lambda^d \neq 0, \lambda^e \neq 0 \quad : \quad U(3)^5 \to U(1)_B \times U(1)_{L_i}$$

$U(1)_R$ Symmetry Charges

	$U(1)_R$	
ϕ	R	Scalar
ψ	$R-1$	Fermion
F	$R-2$	Auxillary
θ	$+1$	Grassman Coordinate
$\int d\theta$	-1	Integral Quarter Superspace
$\int d^2\theta$	-2	Integral Half Superspace
$\int d^4\theta$	0	Integral All Superspace
W	2	Superpotential
λ^α	$+1$	Gaugino
A^μ	0	Gauge Field
D	0	Auxillary

Discrete Symmetry Charges

	Z_2 $R-$parity	Z_2 Matter parity	Z_2 Baryon parity	Z_2 Lepton parity	Z_2 $B-L$ parity	Z_5 X
Q	-1	-1	-1	0	$+1$	3
$\overline{u}$	$+1$	-1	-1	0	-1	3
$\overline{d}$	$+1$	-1	-1	0	-1	-1
L	-1	-1	0	-1	$+1$	-1
$\overline{e}$	$+1$	-1	0	-1	-1	3
H_u	0	0	0	0	0	2
H_d	0	0	0	0	0	-2
$LL\overline{e}$	-1	-1	0	-1	$+1$	1
$QL\overline{d}$	-1	-1	0	-1	$+1$	1
$\overline{u}\overline{d}\overline{d}$	$+1$	-1	-1	0	-1	1
$QH_u\overline{u}$	0	0	-1	0	0	0
$QH_d\overline{d}$	0	0	-1	0	0	0
$LH_d\overline{e}$	0	0	-1	0	0	0
$\mathrm{Tr}(X)$						0
$2\mathrm{Tr}(XSU(2)_L^2)$	0	-1	-1	-1	0	0
$2\mathrm{Tr}(XSU(3)_C^2)$	0	0	0	0	0	0

Discrete Generator Z_k : $e^{2\pi i X/k}$ (X only defined mod k)

↑ ↑ Accidental Z_2 $R-$parity

Exercises: Lecture 1

1. Work out the form of the component field interactions in the renormalizable MSSM with R-parity conservation in the supersymmetric limit. [Hint: Don't forget any cross terms from auxillary components of the superpotential].

2. Write the most general dimension five interactions in the MSSM in the supersymmetric limit. Do any of these interactions violate R-parity? Do any of these interactions cause proton decay? If so determine the proton decay modes which can arise from these interactions.

3. Suppose that one of the sneutrinos in the renormalizable MSSM with general soft supersymmetry breaking terms and R-parity conservation is tachyonic and gains an expectation value. Show that there is a massless Goldstone boson associated with this spontaneous breaking. Estimate the neutrino mass for the partner of the tachyonic sneutrino. What MSSM particle(s) play the role of right handed neutrino.

4. Dimensional regularization of a local quantum field theory has no divergences worse than logarithmic. Explain why this does not imply that all mass scales in a local quantum field theory are technically natural, and does not solve hierarchy problems.