SUSY Breaking

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1. Introduction

This is a shortened version of K. Intriligator and N. Seiberg, "Lectures on Supersymmetry Breaking," arXiv:hep-ph/0702069. Throughout these talks we will not give references. References can be found there.

Hierarchy of solutions to the hierarchy problem:

1. In MSSM: explicit (soft) SUSY breaking
2. Since SUSY is gauged (SUGRA), need spontaneous SUSY breaking
3. More elegant and aesthetically natural – dynamical SUSY breaking (DSB)

The purpose of these talks is to describe models which break supersymmetry. They should be used as modules with the various messenger schemes which were described here by others.

Since $m_{3/2} \ll M_P$, can use global SUSY – field theory.

2. Spontaneous SUSY Breaking

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger) + \int d^2\theta W(\Phi) + \int d^2\overline{\theta}\, \overline{W}(\Phi^\dagger)$$

SUSY is unbroken iff $E = 0 \Leftrightarrow \partial W = 0$.

2.1. Simplest example

$$K = X^\dagger X \qquad ; \qquad W = M^2 X$$

$$V = |M|^4.$$

Spectrum: massless fermion (Goldstone fermion) and a massless complex scalar X.

Flat direction: X. We refer to this space as pseudo-moduli space because unlike more common flat directions (see also below) they are not stable.

The spectrum looks SUSY even though SUSY is broken.

2.2. Second simplest example

$$K = K(X^\dagger, X) \qquad ; \qquad W = M^2 X.$$

K includes higher dimension operators. This is a non-renormalizable theory. It is an effective theory obtained by integrating out heavy fields.

$$V = \frac{|M|^4}{\partial_X \partial_{X^\dagger} K}.$$

This lifts the flat direction and the spectrum is in general not SUSY.

2.3. Renormalizable model - O'Raifeartaigh model

$$K = X^\dagger X + \Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_2 \qquad ; \qquad W = \frac{h}{2} X \Phi_1^2 + m \Phi_1 \Phi_2 + f X$$

$$V = |\frac{h}{2}\Phi_1^2 + f|^2 + |m\Phi_1|^2 + |hX\Phi_1 + m\Phi_2|^2.$$

We cannot set all the terms to zero and hence SUSY is broken.

Flat direction, $\Phi_2 = -\frac{hX\Phi_1}{m}$. This is a pseudo-moduli space of vacua.

For $y = |hf/m^2| < 1$ the global minimum as at $\Phi_1 = \Phi_2 = 0$ with arbitrary X and $V_{min} = |f|^2$. Classically, the effective theory along this flat direction is like our simplest example.

Radiative corrections lift the degeneracy. The one loop effective potential (Coleman-Weinberg potential) is

$$\frac{1}{64\pi^2} \mathrm{STr} \left(\mathcal{M}^4 \log \frac{\mathcal{M}^2}{M_{cutoff}^2} \right),$$

where STr is a supertrace (sum over bosons minus sum over fermions), $\mathcal{M}$ is the mass matrix.

Note:

1. The equal number of bosons and fermions leads to the absence of a quarticly divergent constant.
2. In any renormalizable SUSY Lagrangian $\mathrm{STr}\mathcal{M}^2 = 0$ (prove in the problem set). Hence there is no quadratically divergent term.
3. The logarithmically divergent term $\sim \mathrm{STr}\mathcal{M}^4$ can be absorbed in renormalization of the coupling constants.

4. In general, the expression above cannot be written in terms of a SUSY effective Lagrangian with a renormalized Kahler potential. An appropriate SUSY effective Lagrangian exists but it includes terms with arbitrarily many covariant derivatives.

In the case of the O'Raifeartaigh model this effective potential leads to a minimum at the origin $X = 0$. Work it out in the problem set.

For $y \ll 1$ or for $|X| \to \infty$ we can describe the potential with an effective Kahler potential as in our second example. However, for larger y and small X we need also terms with covariant derivatives.

2.4. Criteria for SUSY breaking

With a generic superpotential $W(\Phi^i)$, $i = 1, ..., n$, and smooth Kahler potential K we need to solve

$$\partial_i W = 0 \qquad ; \qquad i = 1, ..., n.$$

These are n equations with n unknowns so there are isolated solutions (recall W is generic). Therefore, SUSY is unbroken.

If there is a $U(1)_R$ symmetry under which Φ^i has charge r_i, then

$$W = (\Phi^1)^{\frac{2}{r_1}} f\left(z^a = \frac{\Phi^a}{(\Phi^1)^{\frac{r_a}{r_1}}}\right) \qquad ; \qquad a = 2, ..., n.$$

If the $U(1)_R$ symmetry is spontaneously broken, we can take $\Phi^1 \neq 0$, and then the conditions for unbroken SUSY are

$$f = \partial_a f = 0 \qquad ; \qquad a = 2, ..., n$$

i.e. n equations for the $n - 1$ unknowns z^a. Therefore, SUSY is broken.

More generally, with generic r_i the change of variables is singular at the point $\Phi^1 = 0$ and this triggers SUSY breaking. There are however notable exceptions; e.g. $W = X(\Phi^2 - 1)$.

We conclude: With generic Lagrangian SUSY is broken if and only if the theory has a $U(1)_R$ symmetry.

Consider the effect of small explicit breaking of $U(1)_R$:

$$W = W_0 + \epsilon W_1.$$

For $\epsilon = 0$ SUSY is broken with a stable minimum at some $\langle\Phi\rangle = \Phi_0$. For nonzero $\epsilon \ll 1$ the effect of the perturbation around Φ_0 is negligible and therefore it remains a local minimum. However, SUSY is restored at some $\langle\Phi\rangle$ such that

$$\lim_{\epsilon\to 0}\langle\Phi\rangle \to \infty.$$

We conclude: Approximate $U(1)_R$ symmetry leads to a metastable SUSY breaking minimum.

This is important for model building. We want to break SUSY. Hence we need $U(1)_R$. For gaugino masses we need to break it. If the breaking is only spontaneous, there is a massless Goldstone boson. Hence $U(1)_R$ must be explicitly broken and hence we must live in a metastable state!

Exceptions: nongeneric W, use gravitational effects.

2.5. Homework

1. Consider
$$K = X^\dagger X - \frac{1}{\mu^2}(X^\dagger X)^2 + \mathcal{O}(|X|^4) \qquad ; \qquad W = M^2 X$$
around $X = 0$. Find the minimum of the potential and the spectrum of bosons and fermions. What are the symmetries of this system? What are the spontaneously broken symmetries?
2. Prove that for any renormalizable Kahler potential and superpotential $\mathrm{STr}\mathcal{M}^2 = 0$.
3. Use the one loop Coleman-Weinberg potential to show that for $y < 1$ the only minimum of the effective potential of the O'Raifeartaigh model is at the origin.
4. Generalize the O'Raifeartaigh model to
$$W = \frac{h}{2}X\Phi_1^2 + m\Phi_1\Phi_2 + fX + \frac{\epsilon}{2}m\Phi_2^2$$
with $\epsilon \ll 1$. Find the classical minima of the potential. Note that for $\epsilon = 0$ (the O'Raifeartaigh model) the theory has a $U(1)_R$ symmetry. For nonzero $\epsilon \ll 1$ this symmetry is approximate.
5. Combine the two previous problems. Find metastable SUSY breaking minima in the quantum version of the generalized O'Raifeartaigh model of problem 4. It is enough to solve the problem to leading order in $\epsilon, h \ll 1$.

3. Dynamical SUSY breaking (DSB)

Models of SUSY breaking similar to those we discussed can solve the technical hierarchy problem, but not the aesthetic problem of why $M_W \ll M_P$.

Witten suggested to solve this problem with dynamical SUSY breaking. The tree level theory has only dimensionless parameters and does not break SUSY. Because of the nonrenormalization theorems SUSY remains unbroken to all orders in perturbation theory. Nonperturbatively, because of dimensional transmutation, a low scale

$$\Lambda = M_P e^{-c/g^2(M_P)} \ll M_P$$

is generated. SUSY breaking at that scale solves the hierarchy problem. This is similar in spirit to the idea behind technicolor.

In most known examples the low energy effective theory below Λ is similar to the previous examples in which SUSY is broken at tree level.

In order to understand DSB we need to understand the nonperturbative dynamics of gauge theories. This is a huge field, and we cannot do justice to it in one hour. So we'll be brief, trying to convey only the main ideas.

3.1. Use gaugino condensation

Consider an $SU(N_c)$ gauge theory with no matter fields. It has N_c vacua with gaugino condensation

$$\frac{1}{32\pi^2}\langle \mathrm{Tr}\lambda\lambda\rangle = \Lambda^3 e^{\frac{2\pi i n}{N_c}} \qquad ; \qquad n = 1, ..., N_c$$

where

$$\Lambda = M_{cutoff} e^{-\frac{8\pi^2}{3N_c g(cutoff)^2}} \ll M_{cutoff}$$

is the dynamically generated scale of the theory.

Now, take a theory which classically breaks SUSY. Add to it an almost decoupled $SU(N_c)$ gauge theory and replace the parameters of dimensions 1 and 2 by W_α^2/M_{cutoff}^2 and W_α^2/M_{cutoff} where W_α is the field strength, and M_{cutoff} is some heavy scale. Then, all the dimensionful coefficients in the classical theory are in front of higher dimension/non-renormalizable/irrelevant operators. They are suppressed by powers of $1/M_{cutoff}$.

Ignoring the nonperturbative dynamics SUSY is unbroken. This is true classically and to all orders in perturbation theory. Non-perturbatively, gaugino condensation in the gauged $SU(N_c)$ theory generates low scales and triggers spontaneous SUSY breaking.

For example,

$$\int d^2\theta \left(-\frac{8\pi^2}{g^2(cutoff)} + \frac{1}{M_{cutoff}} X\right) S \qquad ; \qquad S = -\frac{1}{32\pi^2} \mathrm{Tr}\, W^\alpha W_\alpha$$

leads to

$$W = N_c \Lambda^3 \exp\left(\frac{X}{N_c M_{cutoff}}\right) = N_c \Lambda^3 + \frac{\Lambda^3}{M_{cutoff}} X + \mathcal{O}(X^2).$$

This is our first example. The full expression typically leads to runaway but with appropriate K there can be a metastable SUSY breaking state at $X = 0$.

Similarly, the O'Raifeartaigh theory can be obtained by using

$$\int d^2\theta \, \left[\frac{h}{2} X \Phi_1^2 + \left(-\frac{8\pi^2}{g^2(cutoff)} + \frac{1}{M_{cutoff}} X + \frac{1}{M_{cutoff}^2} \Phi_1 \Phi_2\right) S\right].$$

It is important that in all the theories which are constructed this way the SUSY breaking minimum is metastable.

3.2. SQCD for $N_f < N_c$ and SUSY breaking

We would like to use SUSY QCD with matter to dynamically break SUSY. We first discuss the classical theory and then the nonperturbative dynamics.

Consider an $SU(N_c)$ gauge theory with N_f flavors of quarks Q in $\mathbf{N_c}$ and anti-quarks $\widetilde{Q}$ in $\overline{\mathbf{N_c}}$.

The classical potential is

$$V \sim \sum_a (D^a)^2 = \sum_a \left(\mathrm{Tr}(Q T^a Q^\dagger - \widetilde{Q}^* T^a \widetilde{Q}^T)\right)^2.$$

The equations determining the ground states – the zeros of this potential – have a large space of solutions. These are true moduli – because of unbroken SUSY the accidental vacuum degeneracy is not broken to all orders in perturbation theory. The specific form of these spaces depends on N_c and N_f.

For $N_f < N_c$ it is (up to global and gauge symmetries)

$$Q = \widetilde{Q} = \begin{pmatrix} v_1 & & & & & \\ & v_2 & & & & \\ & & v_3 & & & \\ & & & . & & \\ & & & & v_{N_f} & \end{pmatrix}.$$

This moduli space of vacua can be parameterized by the meson operator

$$M = \widetilde{Q} Q^T \qquad ; \qquad M_a^{\widetilde{a}} = \widetilde{Q}_i^{\widetilde{a}} Q_a^i$$

where $i = 1, ..., N_c$ is a color index and $a, \widetilde{a} = 1, ..., N_f$ are flavor indices.

Nonperturbative effects generate a superpotential along the moduli space of vacua

$$W_{dynamical} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}} .$$

Note that this term is proportional to a positive power of

$$\Lambda = M_{cutoff} \exp \left(- \frac{8\pi^2}{(3N_c - N_f) g(M_{cutoff})^2} \right)$$

and hence it is nonperturbative. It leads to a potential which slopes to zero at infinity and the system does not have a ground state.

Now, use this physics. Consider a theory based on the gauge group $SU(3) \times SU(2)$ with chiral superfields: Q in $(\mathbf{3}, \mathbf{2})$, $\widetilde{u}$ in $(\overline{\mathbf{3}}, \mathbf{1})$, $\widetilde{d}$ in $(\overline{\mathbf{3}}, \mathbf{1})$, L in $(\mathbf{1}, \mathbf{2})$.

The classical flat directions are characterized by

$$X = Q\widetilde{d}L \qquad , \qquad Y = Q\widetilde{u}L \qquad , \qquad Z = QQ\widetilde{u}\widetilde{d}.$$

As a first approximation we neglect the $SU(2)$ dynamics and then, as with $N_f = 2 < N_c = 3$ above the $SU(3)$ dynamics generates a superpotential

$$W_{dynamical} = \frac{\Lambda_3^7}{Z}$$

where Λ_3 is the dynamically generated scale of the $SU(3)$ gauge theory. With this superpotential the theory does not have a ground state.

We add to the model a tree level superpotential

$$W_{tree} = \lambda Q\widetilde{d}L = \lambda X.$$

The combined model with $W_{dynamical}$ and W_{tree} breaks SUSY. The vacuum energy, vevs and the spectrum can be calculated at leading order in $\lambda \ll 1$.

3.3. Nonperturbative dynamics for $N_f = N_c$ and SUSY breaking

The classical moduli space of vacua for $N_f \geq N_c$ is (up to global and gauge symmetries)

$$Q = \begin{pmatrix} v_1 & & & & \\ & v_2 & & & \\ & & v_3 & & \\ & & & \cdot & \\ & & & & v_{N_c} \\ & & & & \\ & & & & \end{pmatrix} \quad ; \quad \widetilde{Q} = \begin{pmatrix} \widetilde{v}_1 & & & & \\ & \widetilde{v}_2 & & & \\ & & \widetilde{v}_3 & & \\ & & & \cdot & \\ & & & & \widetilde{v}_{N_c} \\ & & & & \\ & & & & \end{pmatrix}$$

$$|v_i|^2 - |\widetilde{v}_i|^2 \text{ independent of } i.$$

The $SU(N_c)$ gauge symmetry is completely broken. The moduli space of vacua can be parameterized by $M_a^{\widetilde{a}} = \widetilde{Q}_i^{\widetilde{a}} Q_a^i$ as well as

$$B_{a_1,a_2,\ldots,a_{N_c}} = \epsilon_{i_1,i_2,\ldots i_{N_c}} Q_{a_1}^{i_1} Q_{a_2}^{i_2} \ldots Q_{a_{N_c}}^{i_{N_c}}$$

$$\widetilde{B}^{\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_{N_c}} = \epsilon^{i_1,i_2,\ldots i_{N_c}} \widetilde{Q}_{i_1}^{\widetilde{a}_1} Q_{i_2}^{\widetilde{a}_2} \ldots Q_{i_{N_c}}^{\widetilde{a}_{N_c}}$$

These fields are not independent. For example, for $N_f = N_c$ there is only one $B = \det Q$ and one $\widetilde{B} = \det \widetilde{Q}$ and they satisfy

$$\det M - B\widetilde{B} = 0.$$

This constraint is true at every point in the moduli space; i.e.

$$\mathcal{M} = \{M, B, \widetilde{B} | \det M - B\widetilde{B} = 0\}.$$

In the quantum theory this classical moduli space of vacua is deformed and the moduli space of quantum vacua is

$$\mathcal{M}_{quantum} = \{M, B, \widetilde{B} | \det M - B\widetilde{B} = \Lambda^{2N_c}\}.$$

We can describe it by adding a Lagrange multiplier field X and the superpotential

$$W = X(\det M - B\widetilde{B} - \Lambda^{2N_c}).$$

This description is analogous to the linear sigma model as opposed to the nonlinear model for pions.

Now use this physics to break SUSY. Consider the $SU(N_c)$ theory with $N_f = N_c$ and add fields $S^a_{\widetilde{a}}$, b and $\widetilde{b}$ and a superpotential

$$W_{tree} = S^a_{\widetilde{a}} \widetilde{Q}^{\widetilde{a}}_i Q^i_a + b \det \widetilde{Q} + \widetilde{b} \det Q.$$

Classically $Q = \widetilde{Q} = 0$ and SUSY is unbroken.

In the quantum theory we get the effective superpotential

$$W_{effective} = S^a_{\widetilde{a}} M^{\widetilde{a}}_a + b\widetilde{B} + \widetilde{b}B + X(\det M - B\widetilde{B} - \Lambda^{2N_c})$$

which breaks SUSY.

Note, we can describe the classical theory by this superpotential but with $\Lambda = 0$. The only effect of the quantum dynamics is to generate $-X\Lambda^{2N_c}$ in the superpotential and thus break SUSY.

3.4. Nonperturbative dynamics for $N_c < N_f < 3N_c/2$ and SUSY breaking

In this range of N_f and N_c the theory is dual to another gauge theory. It has gauge group $SU(N_f - N_c)$, N_f flavors of quarks q and $\widetilde{q}$, and N_f^2 gauge singlets M with superpotential

$$W = \frac{1}{\mu} \mathrm{Tr}\, \widetilde{q}^T M q + \mathrm{Tr}\, m\, M,$$

where m is an $N_f \times N_f$ matrix representing the masses of the fundamental quarks. This M is identified with the meson operator of the original theory. It has dimension 2 and hence the dimensionful factor $\frac{1}{\mu}$ which can be absorbed in the normalization of M. This theory has N_c SUSY vacua with nonzero $\langle M \rangle$.

Looking for supersymmetric vacua we need to solve

$$\frac{1}{\mu} q\widetilde{q}^T + m = 0.$$

These are $N_f \times N_f$ dimensional matrices. However, the first term has rank N_c, and the second has rank N_f. Therefore these equations cannot be solved and SUSY is broken. The actual minimum after taking the radiative corrections into account is, up to symmetry transformations, at

$$\langle M \rangle = 0 \qquad , \qquad \langle q \rangle = \langle \widetilde{q} \rangle = \begin{pmatrix} i\sqrt{m_0 \mu} \\ 0 \end{pmatrix},$$

where for simplicity we have taken $m = m_0 I_{N_f}$ proportional to the unit matrix and the upper block of $\langle q \rangle = \langle \widetilde{q} \rangle$ is $(N_f - N_c) \times (N_f - N_c)$ dimensional.

Comments:

1. Nonperturbatively in this dual gauge theory SUSY is restored with some nonzero $\langle M \rangle$.
2. For $N_f = N_c + 1$ there is another term $\sim \det M$ in the superpotential. It does not affect our conclusions. This term is crucial for finding the SUSY vacua.
3. The tunneling from the metastable minimum to the stable SUSY preserving minimum is suppressed.
4. All the approximations here can be justified for $m_0 \ll \Lambda$.

3.5. Conclusion

Dynamical SUSY breaking in four dimensions is possible and in fact generic.

3.6. Homework

Use DSB to construct a viable predictive SUSY model. Publish your results.