

# LHC for SUSY Theorists

## 2. Standard Model Backgrounds to SUSY

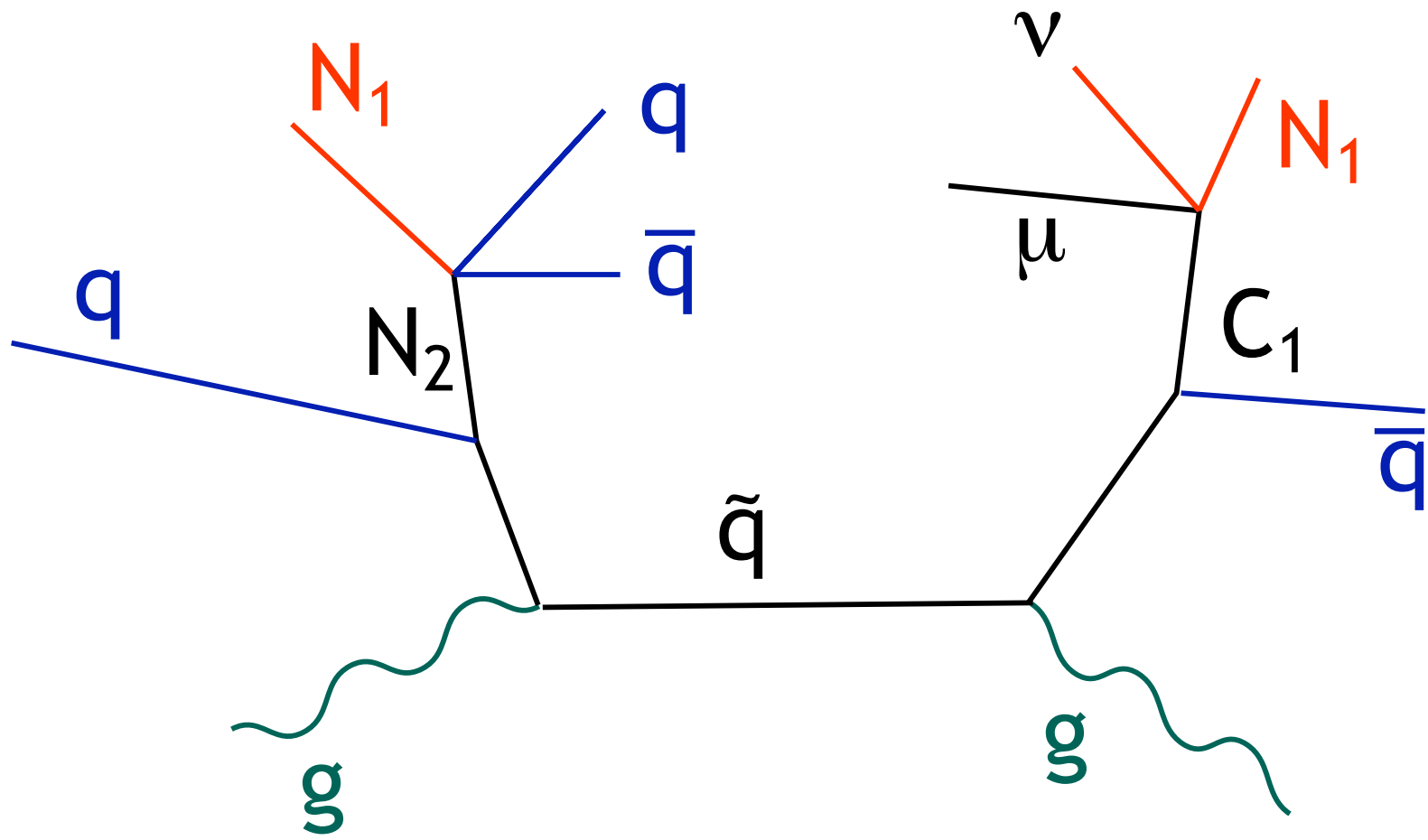
M. E. Peskin  
PiTP Summer School  
July 2007

2. Can we understand the Standard Model at a sufficient level to claim that data is inconsistent with its predictions? SUSY predicts complex events. How do we compute the Standard Model backgrounds to these complex event topologies?

As Scott Thomas has explained to you, there are many types of SUSY models, with many possible signatures. Some of these are dramatic and 'unmissable', e.g., the presence of two stable charged sleptons in each event.

In these lectures, I will concentrate on the more canonical SUSY signatures of pair production and decay of heavy particles. If there is a SUSY connection to dark matter, the final decay product will be invisible and so we will also see unbalanced  $p_T$ .

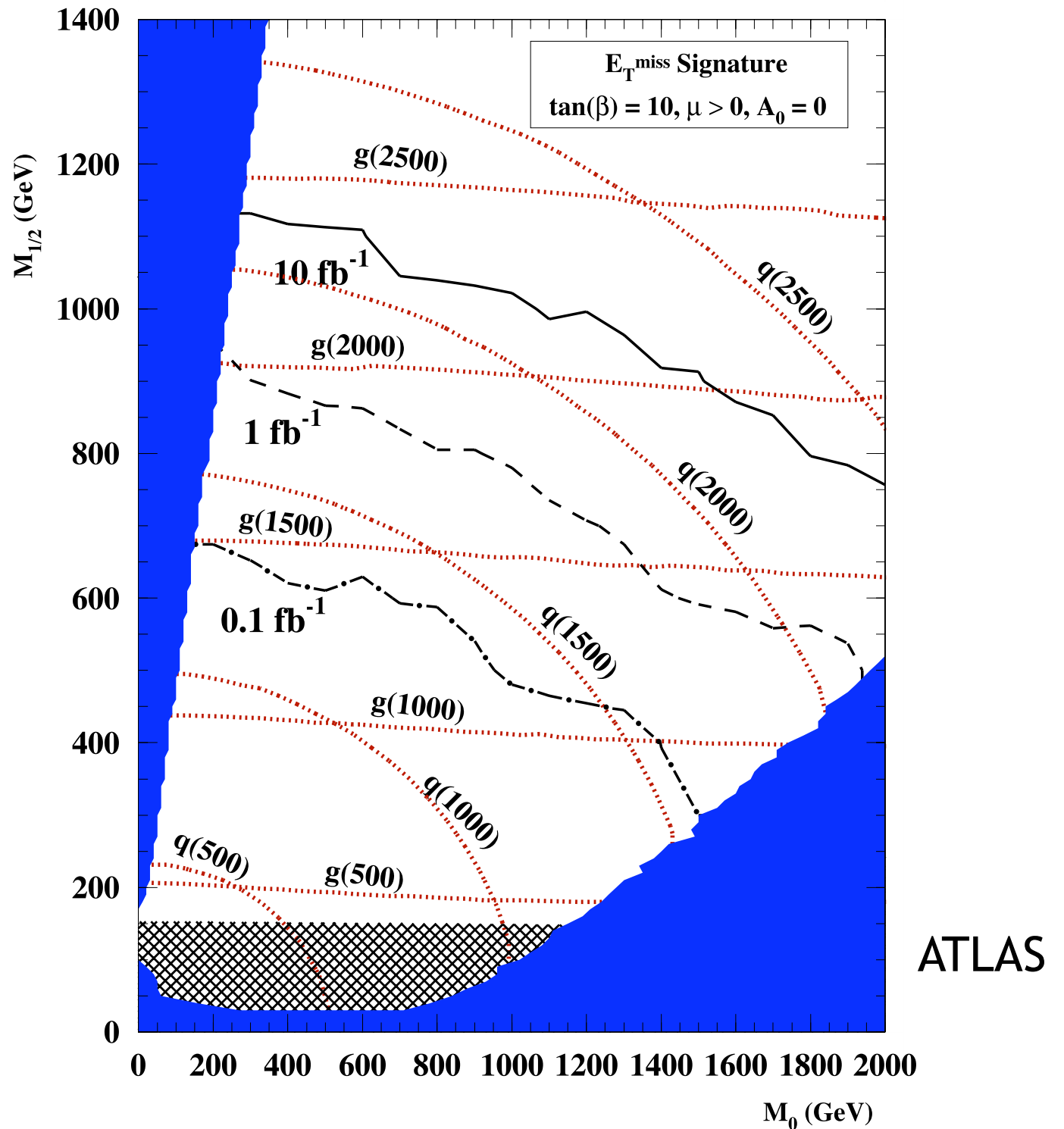
This same general set of signatures apply to most of the models that Nima Arkani-Hamed will discuss in the BSM lectures.



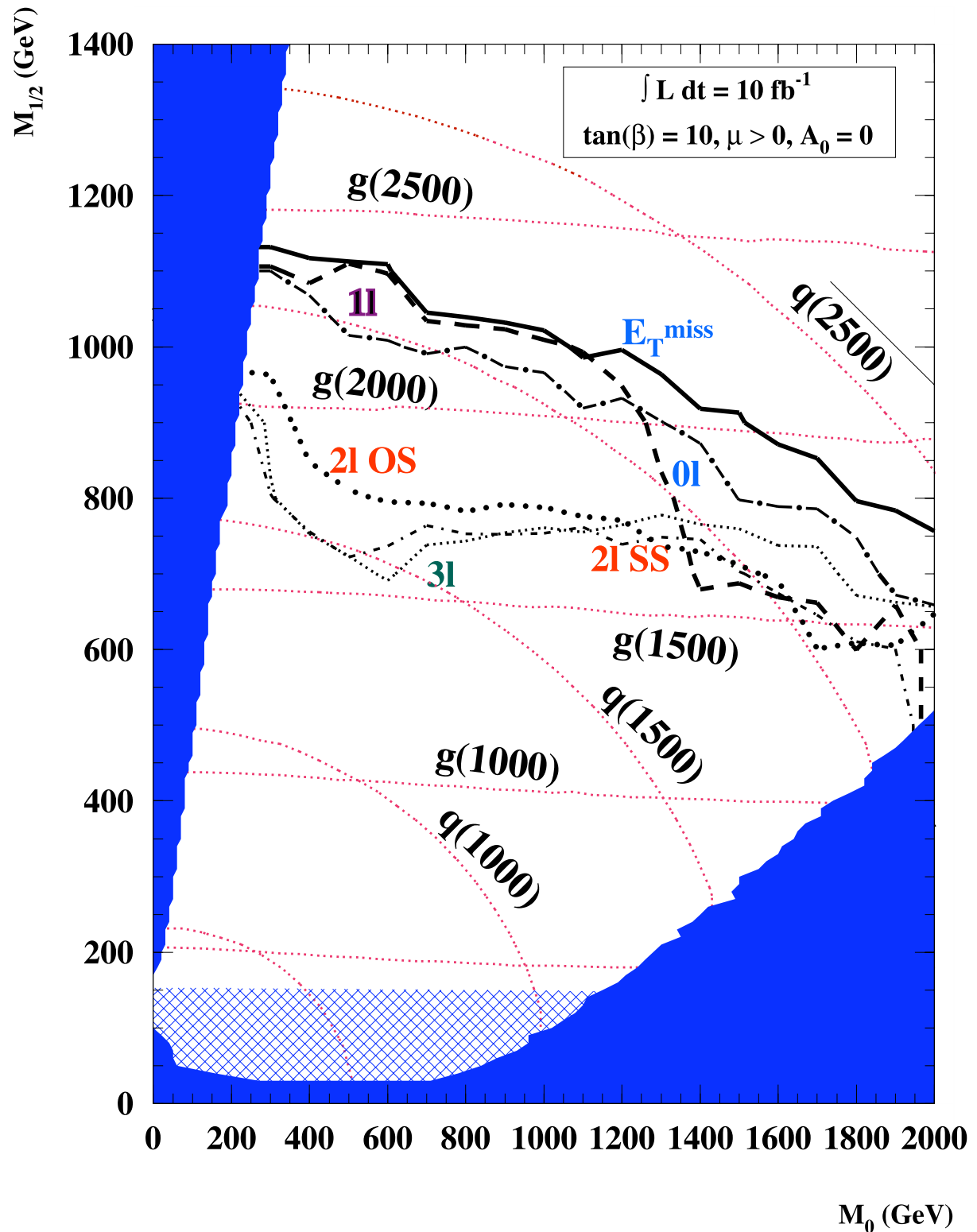
It is expected that events of the kinds that we are discussing will appear as a very significant signal above background.

Here are the estimates of [Tovey \(2003\)](#) for supersymmetry models with universal scalar and gaugino masses at the GUT scale.

For squark and gluino masses below 1 TeV, the missing energy signature should be significant with a very small amount of integrated luminosity.



At the same time, many different signatures of new physics should be seen above the Standard Model expectation.



However, the expectation of large signals above Standard Model background does not mean that we can be complacent.

The theoretical background levels must be understood very well both absolutely and in relation to the actual data.

In the previous lecture, we discussed the problems of overall detection and recognition of events, and the measurement of missing ET.

Assuming that all of these issues can be understood, we now come to the irreducible backgrounds - Standard Model events with heavy particle production, leptons, and missing energy.



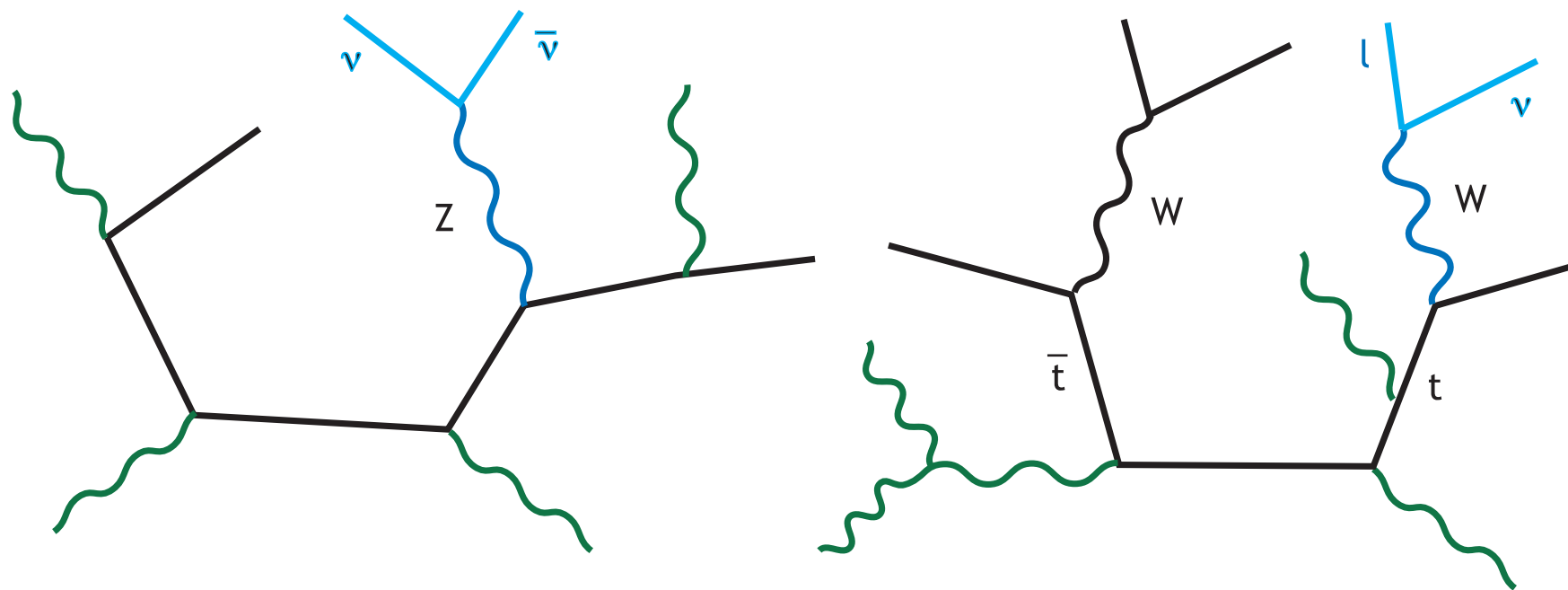
In the physics studies of ATLAS and CMS, the dominant backgrounds to new physics come from a different source, heavy particle production within the Standard Model, production of  $W, Z, t\bar{t}$  plus jets.

These reactions already offer missing energy, leptons, and hadronic activity. They populate the region of large transverse energy deposition associated with new physics to the extent the additional jets are radiated along with the heavy particles.

I remind you of the hierarchy of cross sections:

$\sigma_{tot}$	100 mb
jets w. $p_T > 100$	1 $\mu$ b
Drell-Yan	100 nb
$t\bar{t}$	800 pb
SUSY ( $M < 1$ TeV)	1-10 pb

This is genuinely scary. Processes such as



have cross sections comparable to the SUSY signal and might compete with it.

A similar problem arises in the search for top quark pair production at the Tevatron. The similarities to our problem are:

The cross section is  $10^{-10}$  of the hadron collider total cross section.

The irreducible background  $p\bar{p} \rightarrow W + ng + X$  produces events qualitatively similar to top quark production at a similar rate. These events can even contain b quarks (and c faking b) through gluon splitting  $g \rightarrow b\bar{b}, c\bar{c}$ .

# e + 4 jet event

40758\_44414

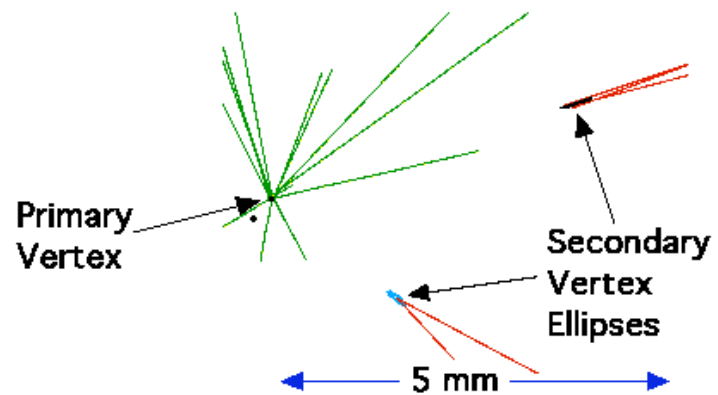
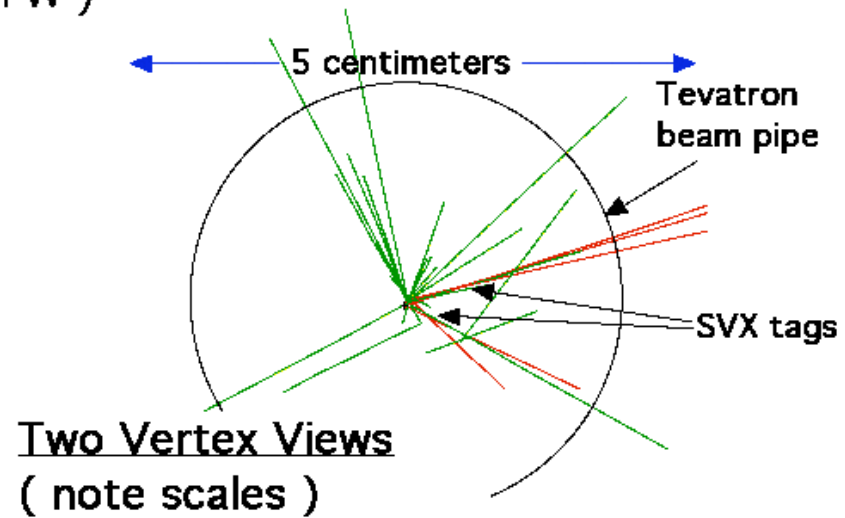
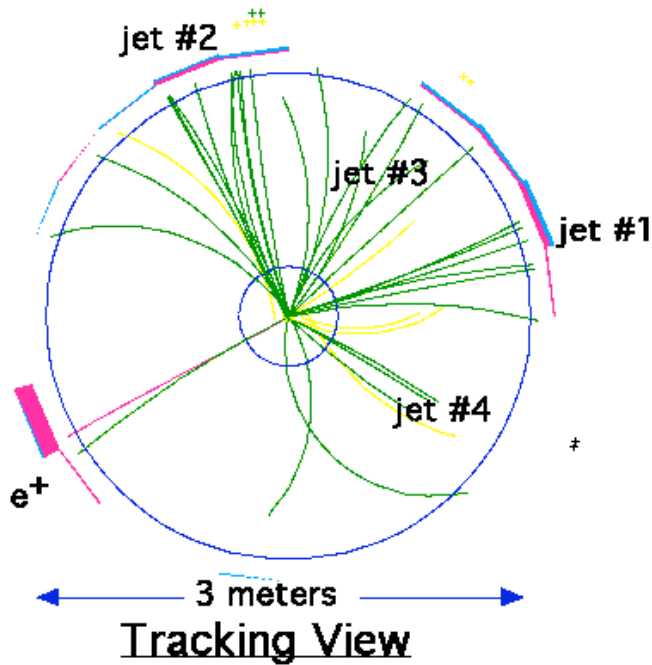
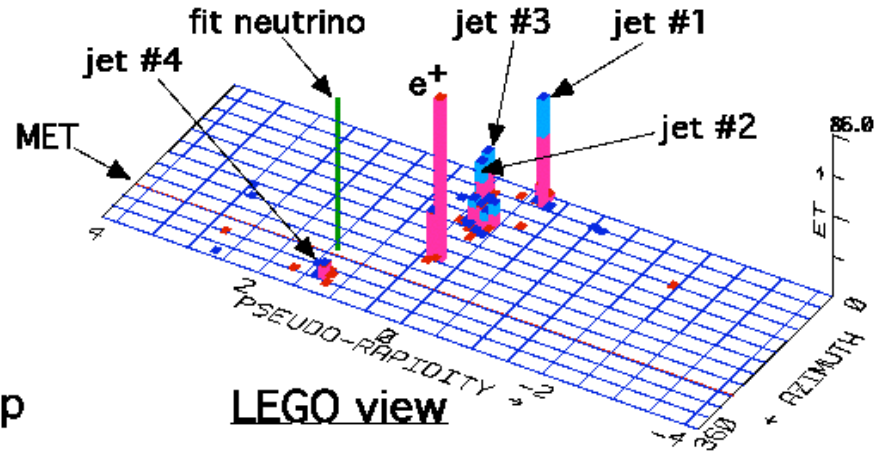
24-September, 1992

TWO jets tagged by SVX

fit top mass is  $170 \pm 10$  GeV

$e^+$ , Missing  $E_t$ , jet #4 from top

jets 1,2,3 from top ( 2&3 from W )



How can we decide whether events from our sample of this type come from  $p\bar{p} \rightarrow W^+ + \text{jets}$  or  $p\bar{p} \rightarrow t\bar{t}$  ?

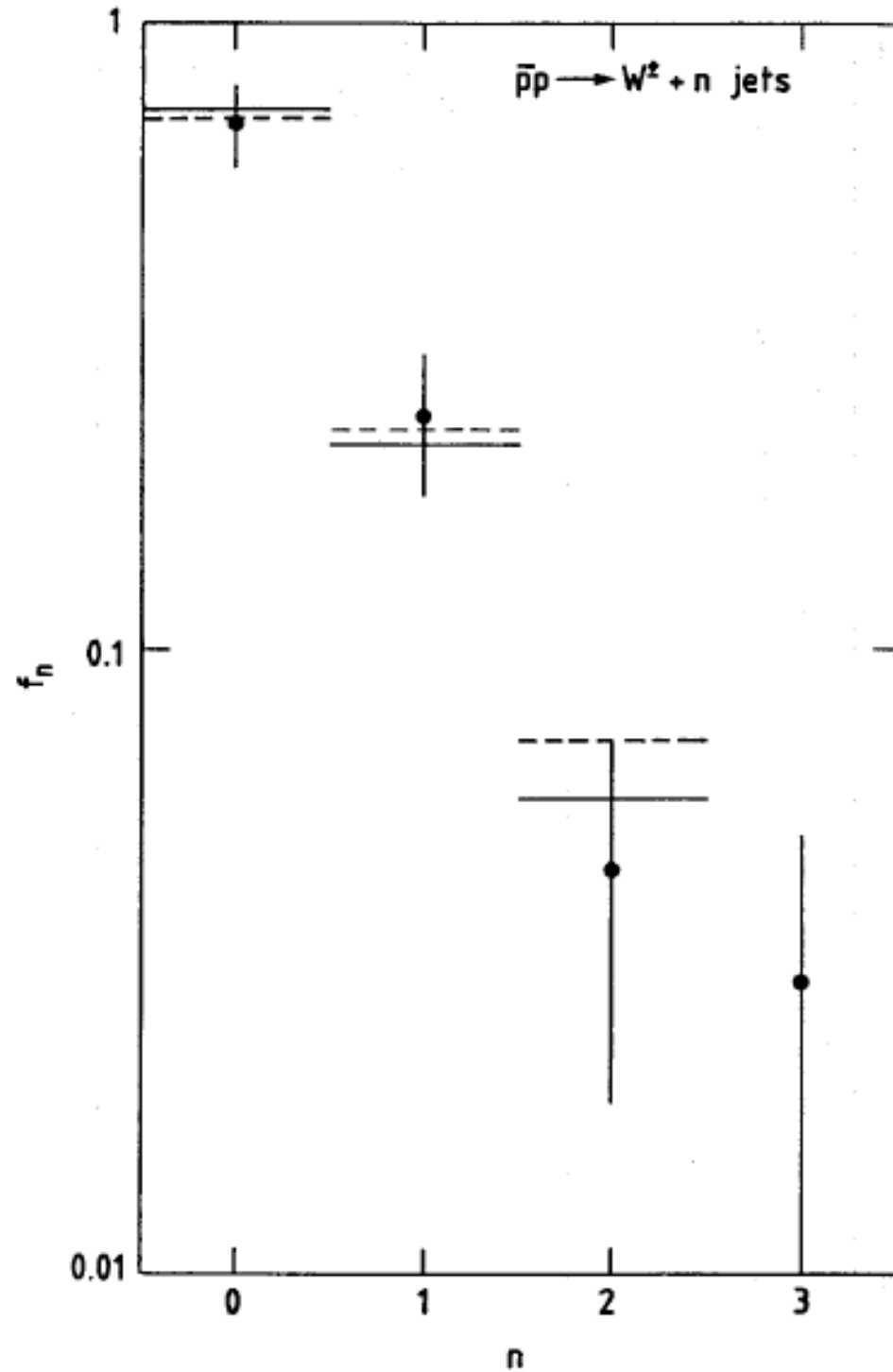
One way to compute the background cross section precisely. This is difficult but not impossible, and such calculations are underway for LHC. An easier task is to compute the **systematic dependence of the background cross section** on variables that characterize heavy particle production:

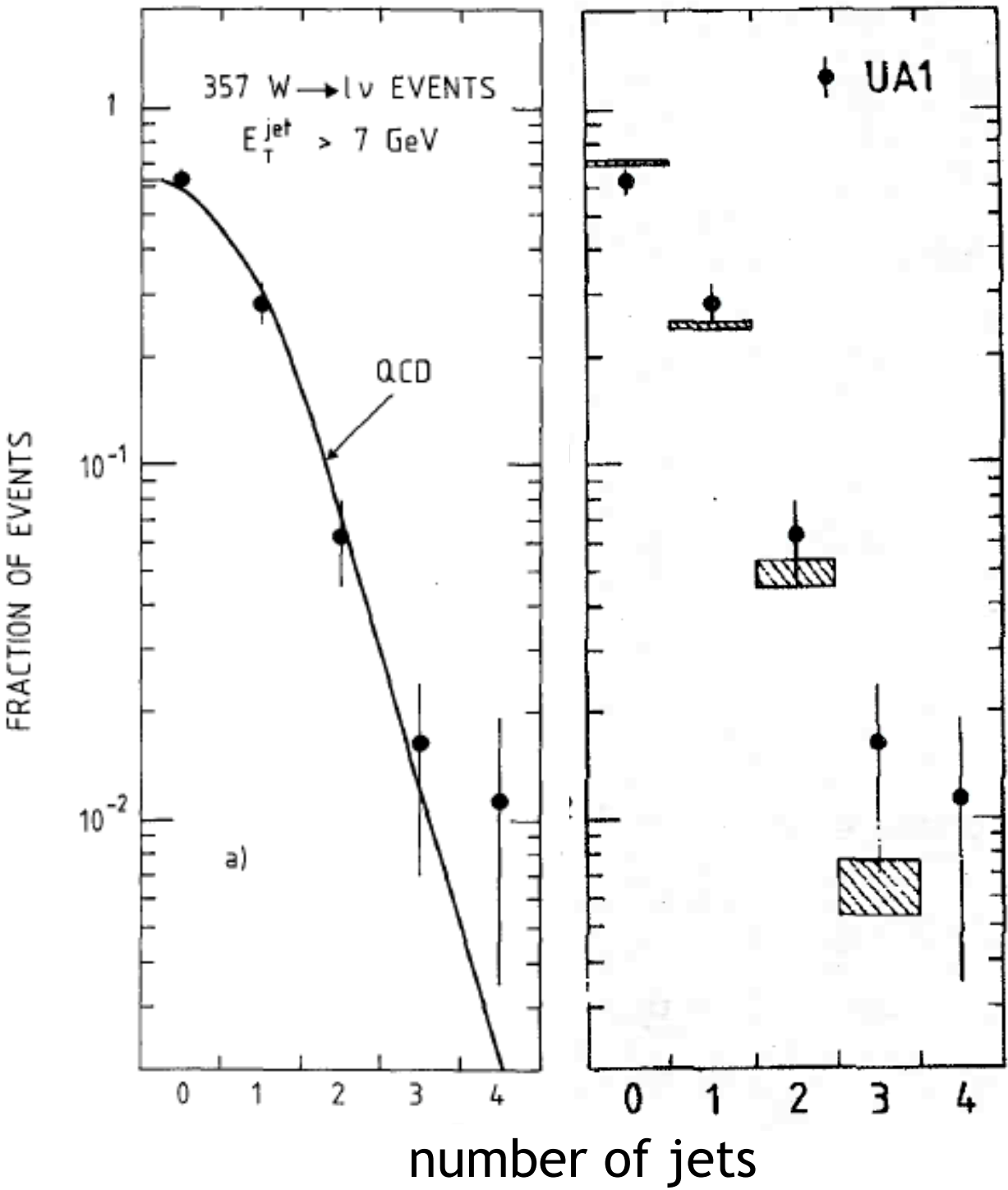
$$\text{no. of jets, } H_T = \sum_i E_{Ti}, \quad \cancel{E}_T$$

Such dependences were first recognized and computed by **Ellis, Kleiss, and Stirling** for the analysis of the data from the UA1 and UA2 experiments. I like to call such a relation a **staircase**.

## Ellis-Kleiss-Stirling staircase (1985)

compared to preliminary data from UA1.





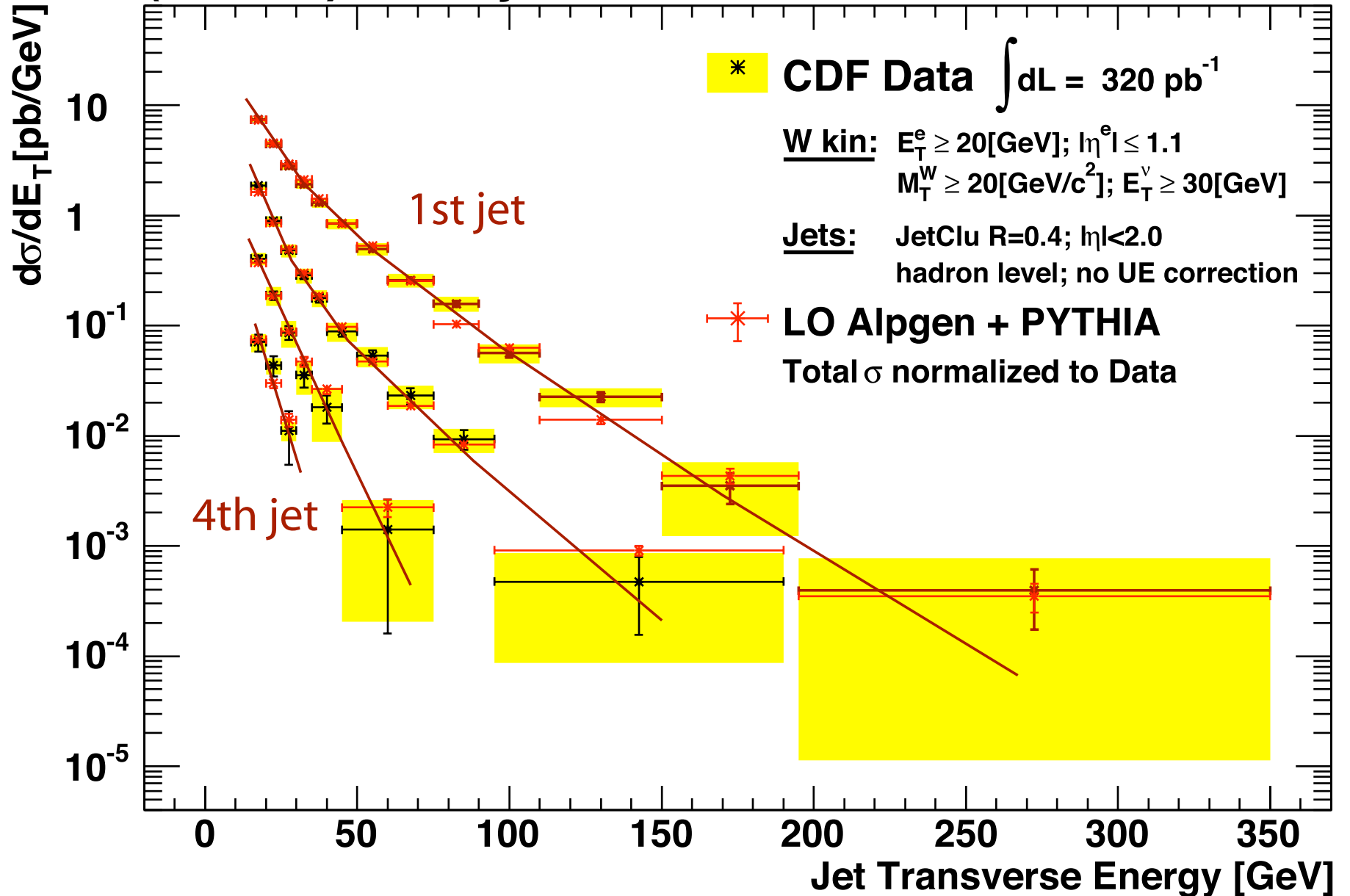
Here is the published UA1 data, compared to a calculation based on the **Berends-Giele** technology for multijet computation :

UA1  
 1988

Berends,  
 Giele, Kuijf,  
 Kleiss, Stirling  
 1989

# $(W \rightarrow e\nu) + \geq n$ jets

CDF Run II Preliminary





To look for top, we add to this analysis the requirement that at least one jet should have a b-tag.

Most events in the category:

$$p\bar{p} \rightarrow l + \cancel{E}_T + \text{b-tag jet} + \text{jets}$$

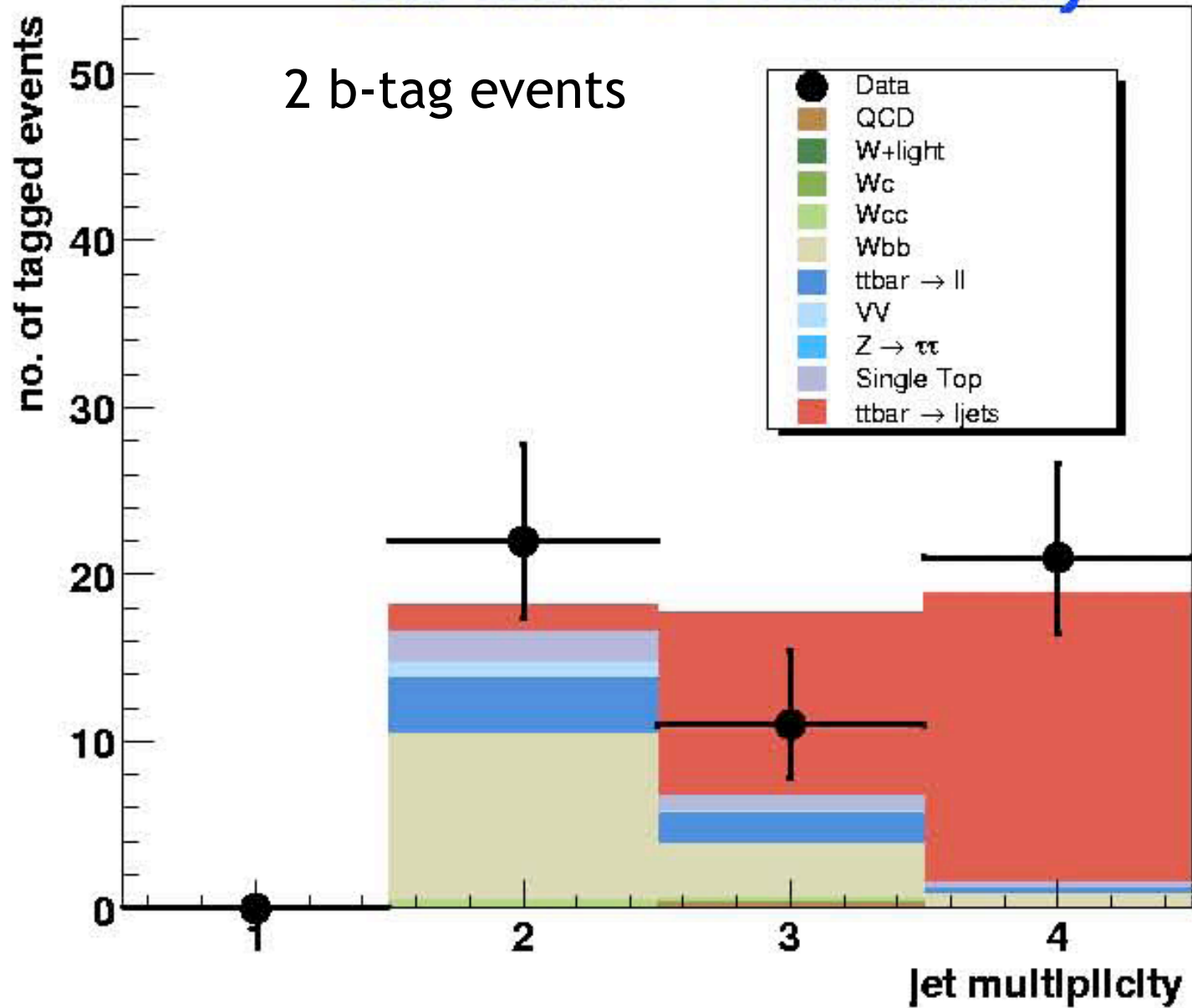
actually come from

$$p\bar{p} \rightarrow W + ng, \quad g \rightarrow c\bar{c}$$

However, as we extrapolate to large jet number, there are not enough W + jets events to explain the data.

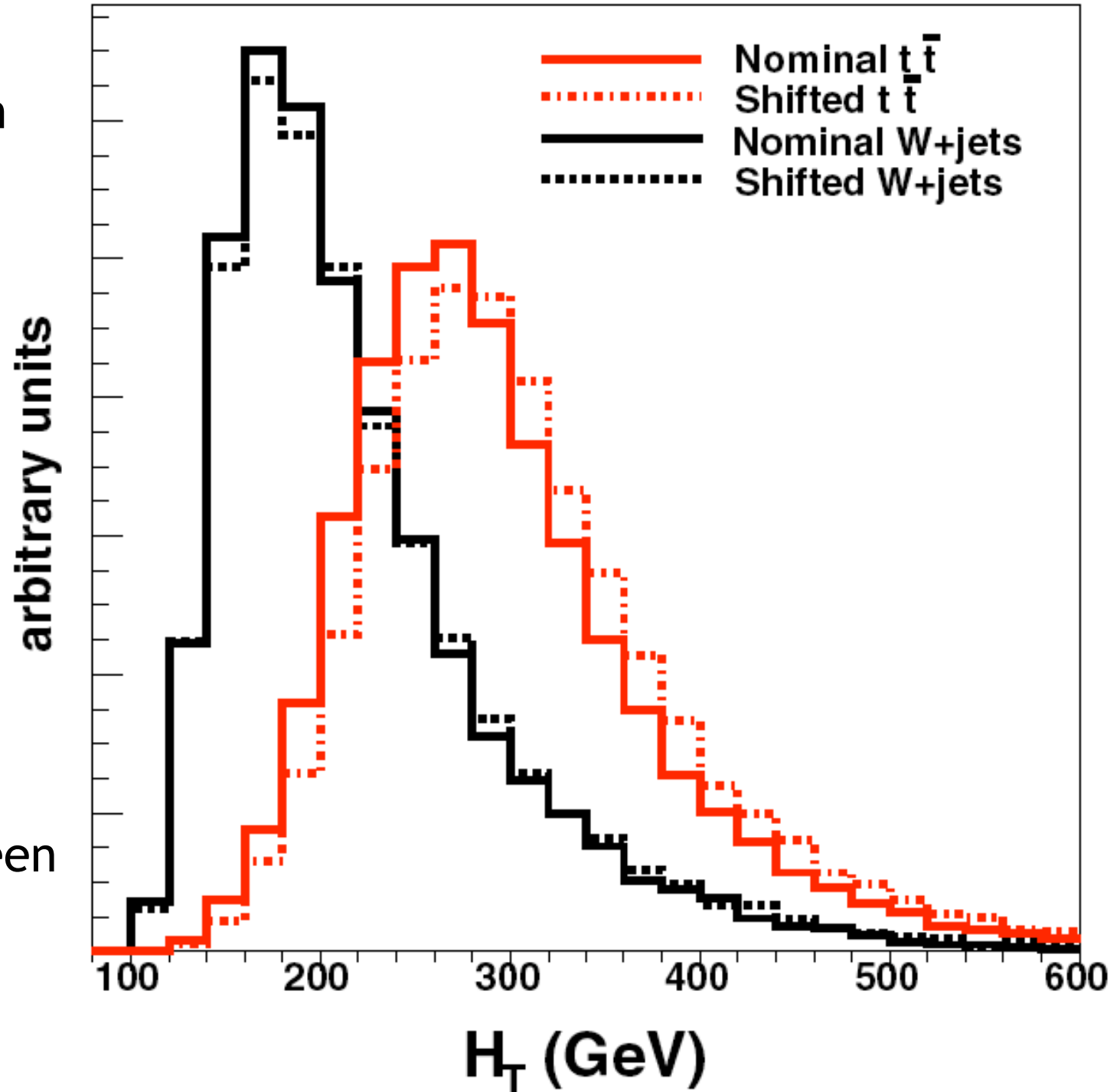


# DØ Run II Preliminary

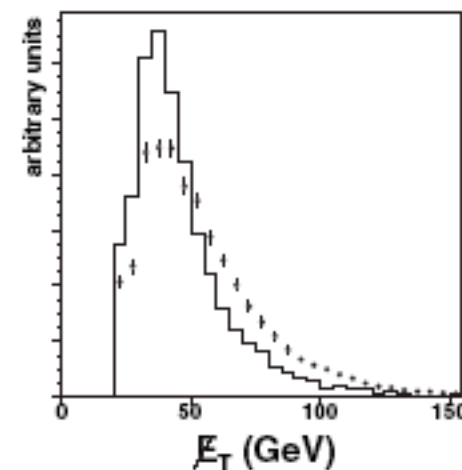
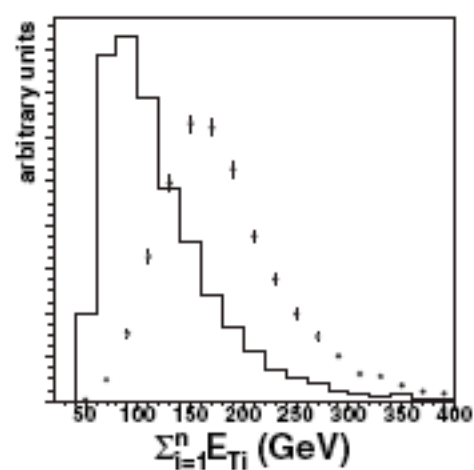
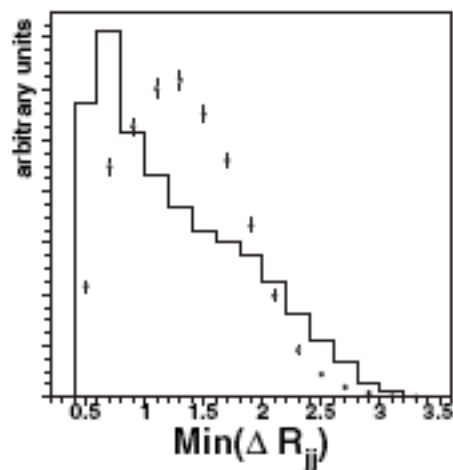
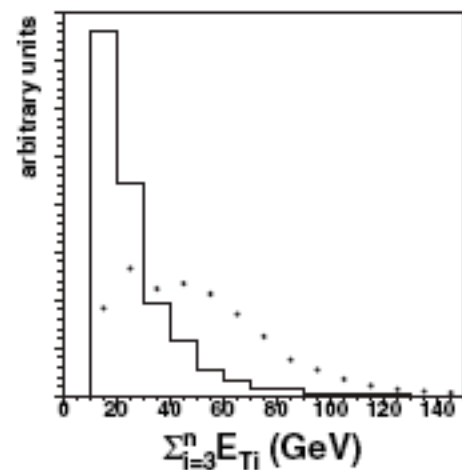
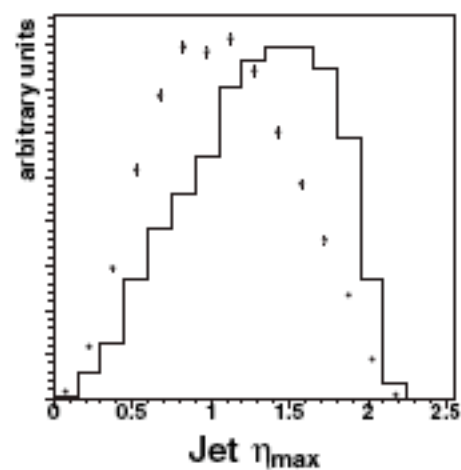
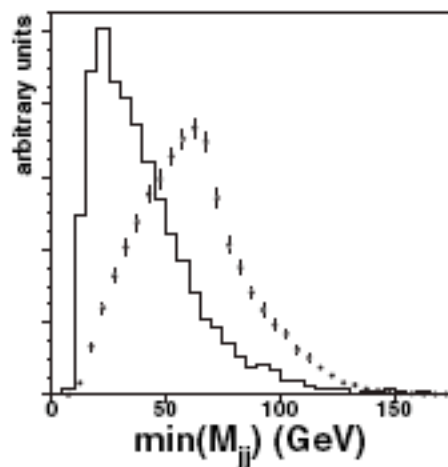
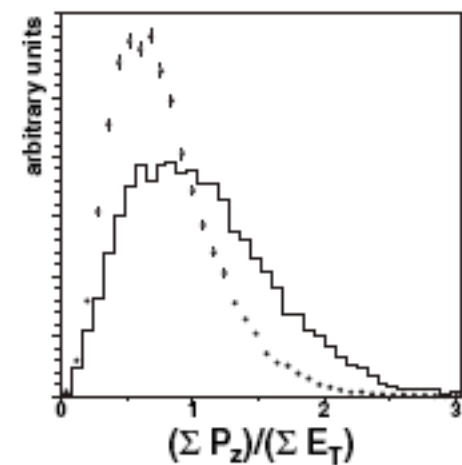
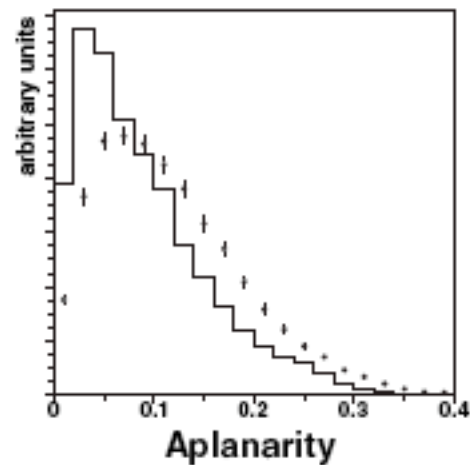
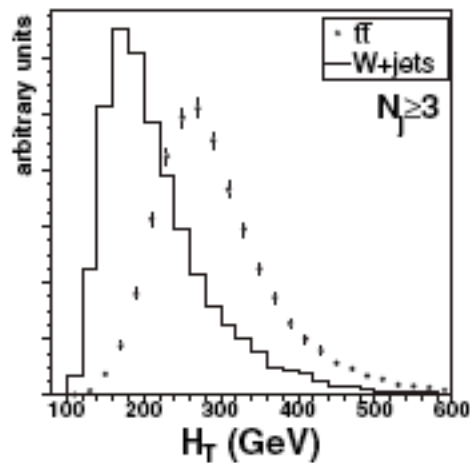


CDF has also shown that top quarks can be found by purely kinematic analysis.

Comparison of HT distributions between ttbar and W + jets events



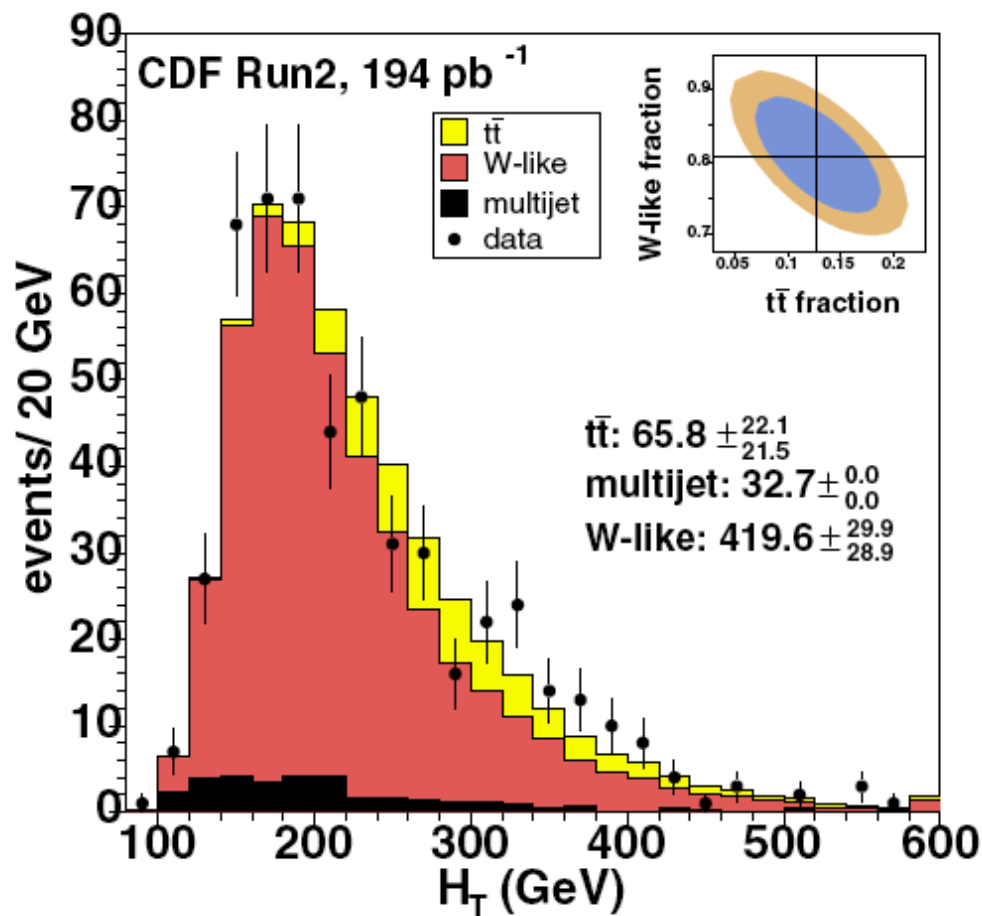
$t\bar{t}$ /W+jets  
shape  
comparisons  
for 9  
kinematic  
observables.



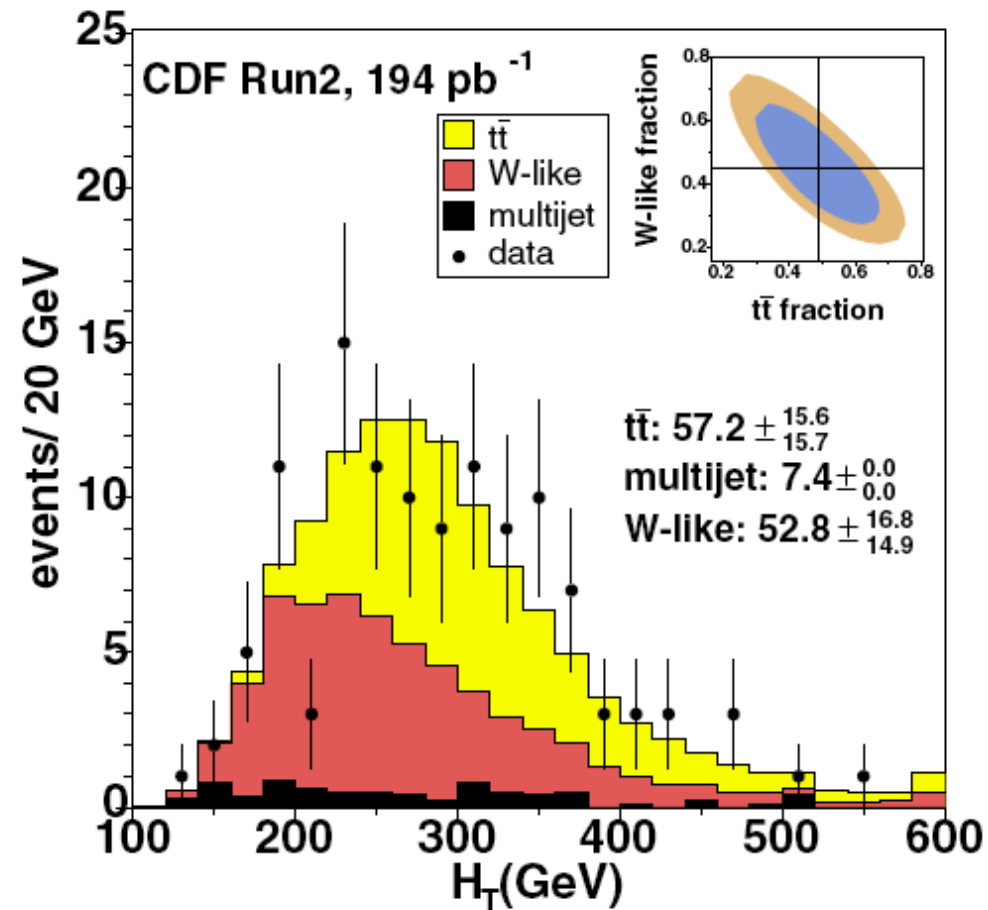
CDF

Using these and 10 more variables input to a neural network classifier, CDF has demonstrated the ability to observe  $t\bar{t}$  events **without b-tagging**. Here are the last two steps in the staircase in that analysis.

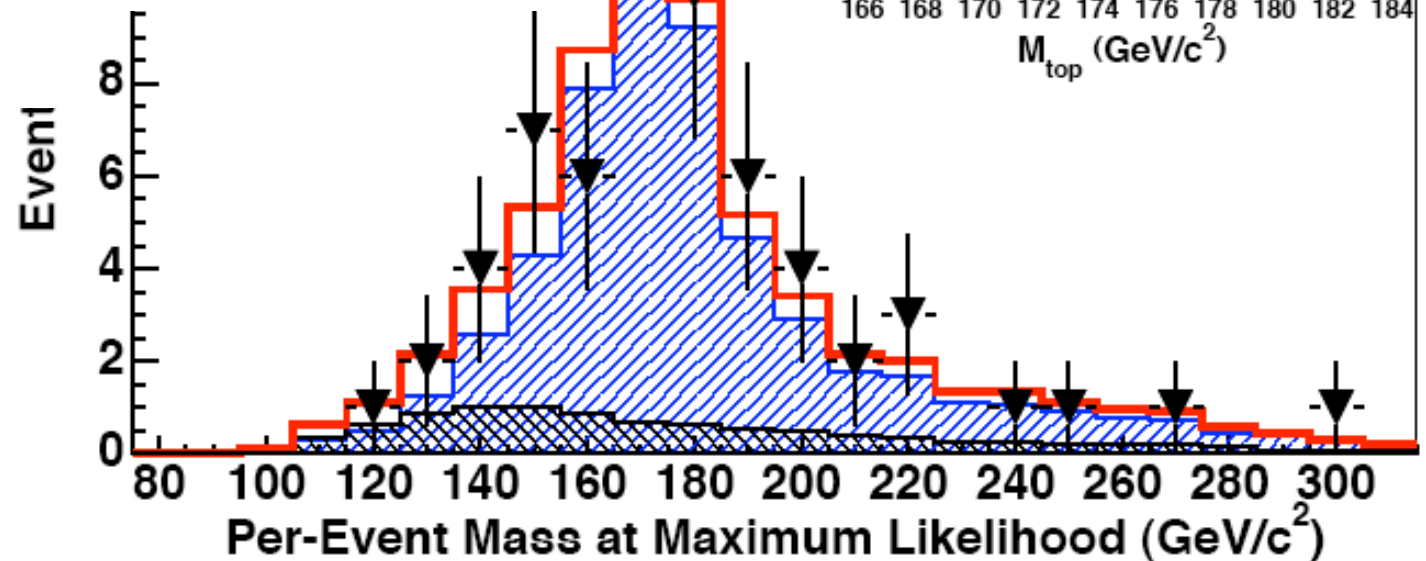
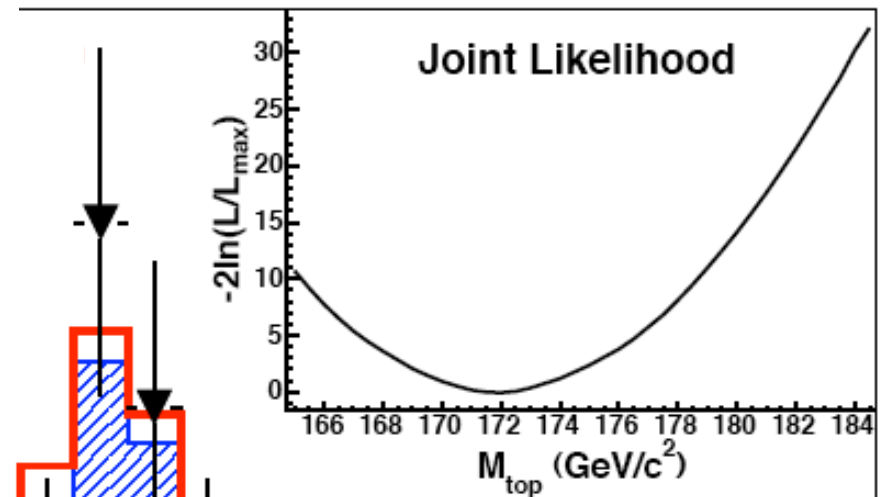
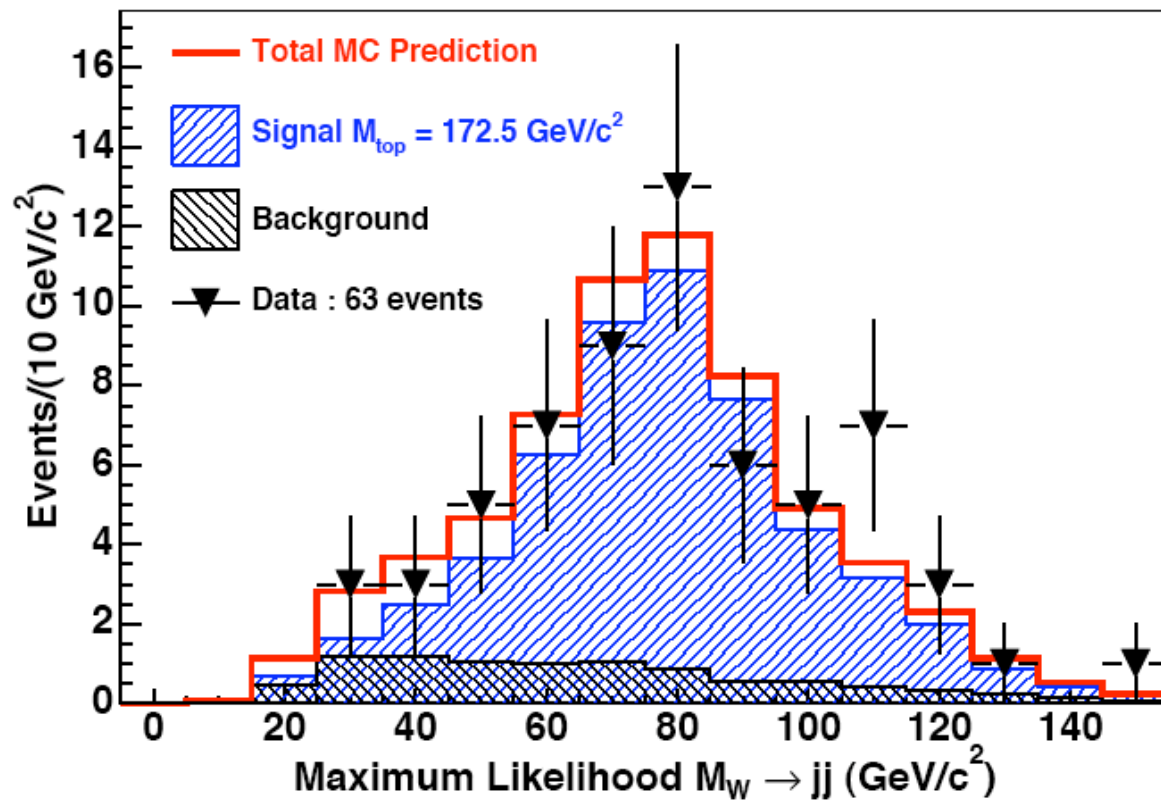
W+3 jets



W + 4 jets



$$H_T = \cancel{E}_T + E_{T\ell} + \sum_i E_{Ti}$$



Here are the  
**2-jet** and **3-jet**  
 mass distributions  
 in the final event  
 sample selected  
 by CDF.

In the remainder of this lecture, I would like to explain how one would actually calculate the rates for complex background processes with production of multiple jets.

A huge amount of work has been done in the area in the past few years. There are several dedicated codes - in particular, **ALPGEN**, **MADEVENT**, and **SHERPA**.

But the subject is not inaccessible, and it good for everyone to know how to do these calculations at some level.



In particular, some very interesting methods (not explained in Peskin and Schroeder) have been invented to make these calculations simpler. **New and important methods were invented here at IAS in the past few years.**

I will discuss only the calculation of tree diagrams. We might estimate backgrounds by

**computing tree diagrams to get the shapes of staircases,  
and normalizing to the data**

**using similar methods to do more accurately normalized  
1-loop calculations**

The final results should be validated by comparing these methods.

The computation of QED and QCD tree diagrams has undergone enormous development since 1981, when Berends, Wu, and their CALKUL collaboration pointed out that computations with **massless** particles could be dramatically simplified by the use of **spinors of lightlike momenta**

$$|1\rangle = u_R(1) \quad |1] = u_L(1) \quad \langle 1| = \bar{u}_L(1) \quad [1| = \bar{u}_R(1)$$

These objects are related to more familiar objects by

$$|1\rangle [1| = \frac{1}{2} (1 + \gamma^5) \not{A}$$

$$\langle 12| = \bar{u}_L(1) u_R(2) \quad [12| = \bar{u}_R(1) u_L(2)$$

The spinor products are square roots of Lorentz vector products:

$$|\langle 12|)^2 = |[12|)^2 = 2k_1 \cdot k_2$$

The major developments before 2004 from the application of spinor product methods to QCD are nicely reviewed by [Mangano and Parke](#), Physics Reports, 1991, and [Dixon](#), 1995 TASI lectures.

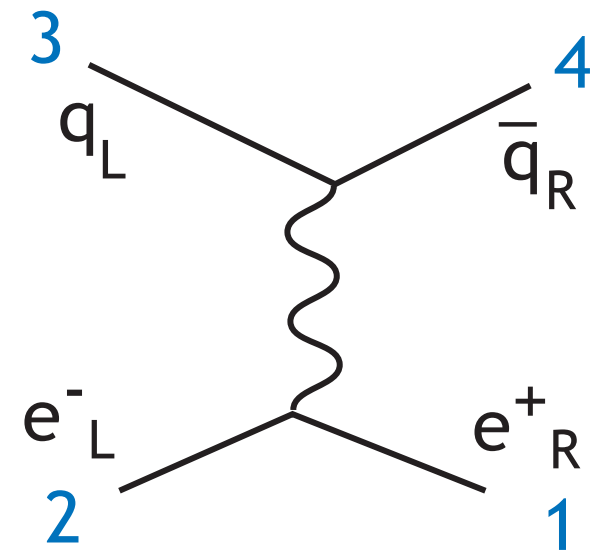
Here is a very simple example:  $e_L^- e_R^+ \rightarrow q_L \bar{q}_R$

Spinor products are antisymmetric and obey the Fierz identity

$$\langle 1\gamma^\mu 2 \rangle \langle 3\gamma_\mu 4 \rangle = 2 \langle 13 \rangle [42]$$

so

$$\begin{aligned} iM &= (-ie)^2 \langle 1\gamma^\mu 2 \rangle \frac{-i}{s_{34}} \langle 3\gamma_\mu 4 \rangle \\ &= 2ie^2 \frac{\langle 13 \rangle [42]}{\langle 34 \rangle [43]} \times \frac{\langle 31 \rangle}{\langle 31 \rangle} \\ &= -2ie^2 \frac{(\langle 13 \rangle)^2}{\langle 34 \rangle \langle 12 \rangle} \end{aligned}$$



Then

$$|M|^2 = 4e^4 \frac{u^2}{s^2} = e^4 (1 + \cos \theta)^2$$

which is correct !

Photon and gluon polarization vectors for momentum  $\mathbf{k}$  are conveniently written in terms of spinor products involving  $\mathbf{k}$  and a second lightlike “reference” vector  $\mathbf{r}$ :

$$\epsilon_+^\mu(k) = \frac{1}{\sqrt{2}} \frac{\langle r \gamma^\mu k \rangle}{\langle r k \rangle} \quad \epsilon_-^\mu(k) = -\frac{1}{\sqrt{2}} \frac{[r \gamma^\mu k]}{[r k]}$$

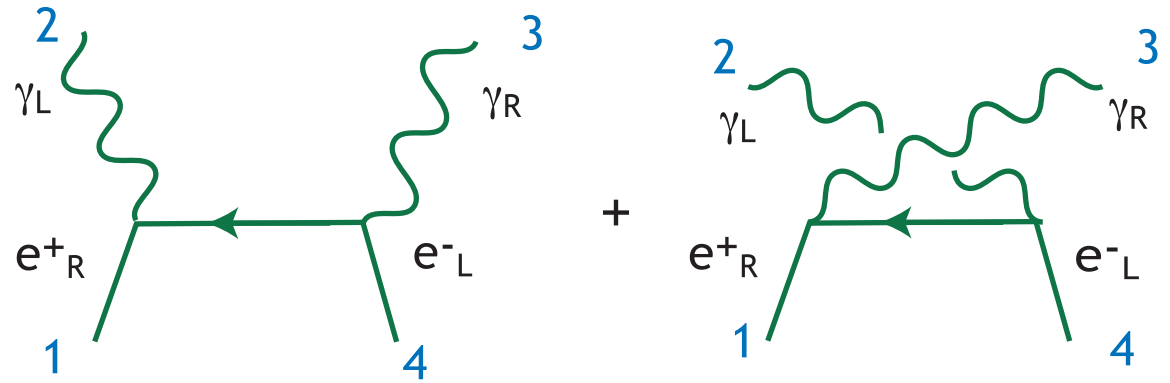
The logic of this choice is:

if  $\mathbf{k} = 1$  is parallel to  $\mathbf{z}$ ,  $\mathbf{r} = 2$  is parallel to  $-\mathbf{z}$ :

$$\langle 2 \gamma^\mu 1 \rangle = \bar{u}_L(2) \gamma^\mu u_L(1) = \sqrt{s_{12}} \cdot (0, 1, -i, 0)$$

a change in  $\mathbf{r}$  is like a change of gauge and has no physical effect. We can choose  $\mathbf{r}$  for maximum convenience, independently in each helicity amplitude.

Another example:



choose  $r = 1$  for 3  
 $r = 4$  for 2

second diagram:

$$\langle 1 \gamma^\mu \dots \frac{\langle 1 \gamma_\mu 3 \rangle}{\langle 13 \rangle} \dots = \langle 11 \rangle \dots = 0$$

first  
 diagram:

$$\begin{aligned} & (-ie^2) \langle 1 \gamma^\mu \frac{(1+2)}{s_{12}} \gamma^\nu 4 \rangle \cdot \frac{1}{\sqrt{2}} \frac{[4 \gamma_\mu 2 \rangle}{[42]} \cdot \frac{-1}{\sqrt{2}} \frac{\langle 1 \gamma_\mu 3 \rangle}{\langle 13 \rangle} \\ &= 2ie^2 \frac{\langle 12 \rangle [4(1+2)1 \rangle [34]}{[42] \langle 12 \rangle [21] \langle 13 \rangle} \\ &= 2ie^2 \frac{\langle 21 \rangle [34]}{[21] \langle 13 \rangle} \\ &= 2ie^2 \left( \frac{t}{u} \right)^{1/2} \quad \text{which is correct!} \end{aligned}$$

We can incorporate massive vector bosons such as W by decaying them to massless fermions ([Kleiss and Stirling](#)). Schematically, replace

$$\epsilon^\mu(Q) \rightarrow \bar{u}_L(1)\gamma^\mu u_L(2) = \langle 1\gamma^\mu 2 \rangle$$

To correctly normalize the result, consider

$$\int \frac{d\Omega}{4\pi} \langle 1\gamma^\mu 2 \rangle \langle 2\gamma^\nu 1 \rangle = A(\eta^{\mu\nu} - Q^\mu Q^\nu / Q^2)$$

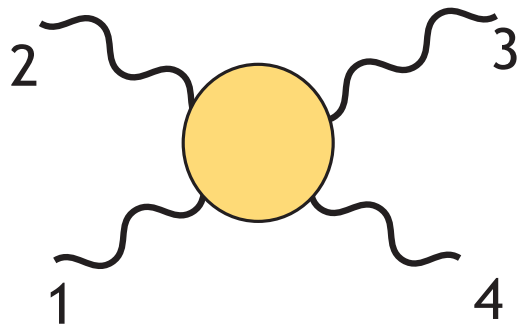
contract:  $\int \frac{d\Omega}{4\pi} 2\langle 12 \rangle [12] = A \cdot 3$

This shows that

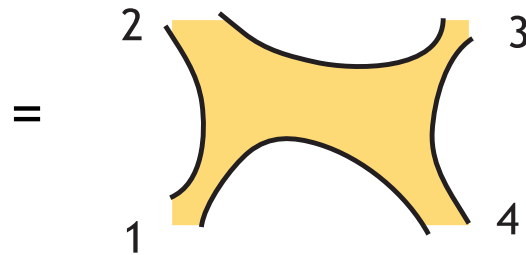
$$\frac{3}{2m_W^2} \int \frac{d\Omega}{4\pi} \langle 1\gamma^\mu 2 \rangle \langle 2\gamma^\nu 1 \rangle = \sum_i \epsilon^\mu(Q) \epsilon^{*\nu}(Q)$$

As a bonus, the generated final state fermions correctly represent the polarization of the W's. A similar trick works for top quarks.

Now think about QCD. It is very convenient to write QCD amplitudes in terms of color structures



$$= \text{tr}[T_1 T_2 T_3 T_4] A(1, 2, 3, 4) + \dots$$



+ other color structures

To do this, write

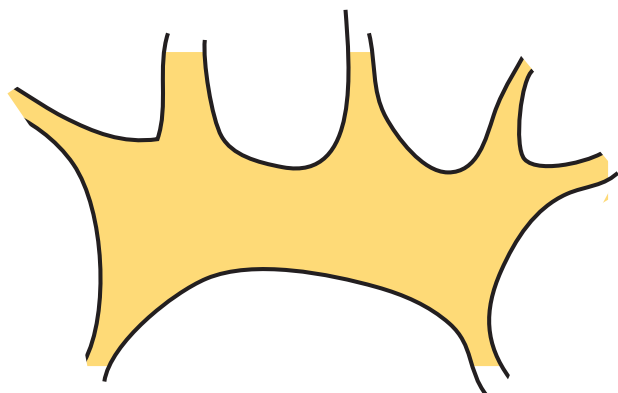
$$t^a = \frac{1}{\sqrt{2}} T^a \quad i f^{abc} = \frac{-i}{\sqrt{2}} \text{tr}[T^a T^b T^c - T^a T^c T^b]$$

To leading order in  $N_C$ , the color-ordered amplitudes  $A$  do not interfere. Square them and multiply by the power of  $N_C$  indicated by the number of color loops.

Color-ordered amplitudes can be computed with the color-ordered Feynman rules:

$$\begin{aligned}
 & \text{Vertex: } \begin{array}{c} \nearrow \\ \searrow \end{array} \text{---} \text{wavy line}^{\mu} = \frac{ig}{\sqrt{2}} \gamma^{\mu} \\
 & \text{Three-point vertex: } \begin{array}{c} \text{wavy line}^{\rho} \text{ (q)} \\ \text{wavy line}^{\nu} \text{ (p)} \end{array} \text{---} \text{wavy line}^{\mu} \text{ (k)} = \frac{ig}{\sqrt{2}} (\eta^{\mu\nu} (k-p)^{\rho} + \eta^{\nu\rho} (p-q)^{\mu} + \eta^{\rho\mu} (q-k)^{\nu}) \\
 & \text{Four-point vertex: } \begin{array}{c} \text{wavy line}^{\mu} \\ \text{wavy line}^{\nu} \end{array} \text{---} \text{wavy line}^{\rho} \text{---} \text{wavy line}^{\sigma} = \frac{ig^2}{2} (2\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})
 \end{aligned}$$

A given color-ordered amplitude has only the singularities from collinear singularities in its color-ordering:



$$\sim \frac{1}{(2k_1 \cdot k_2)(2k_2 \cdot k_3) \cdots (2k_n \cdot k_1)}$$

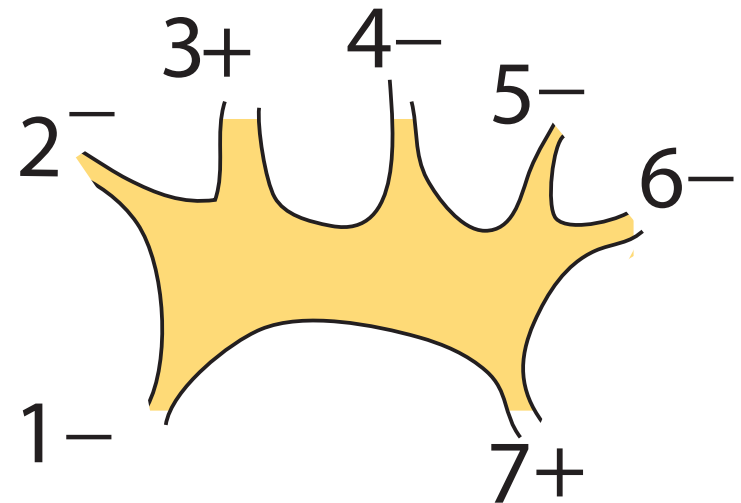


Every initial and final fermion or boson has a definite helicity R or L. From now on, I will label all helicities as if the particles were **outgoing**, and use the notation  $+$  or  $-$ .

Certain amplitudes are very simple after all diagrams are summed. Notate, for example

$$i\mathcal{M}(1^-, 2^-, 3^+, 4^-, 5^-, 6^-, 7^+) =$$

where the figure denotes the color-ordered amplitude appropriate to large  $N_C$ .



**Then tree amplitudes with zero or one  $+$  or  $-$  vanish!** This vanishing turns out to be a consequence of supersymmetry, even for QCD.

Parke and Taylor found that the amplitudes with two - take a very simple form. For example,

$$i\mathcal{M}(1^+, 2^-, 3^+, 4^-, 5^+, 6^+, 7^+) = ig^5 \frac{\langle 24 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \cdots \langle 71 \rangle}$$

These are called **MHV amplitudes**. The amplitudes with two + (anti-MHV) take a similar form in terms of square brackets.

Notice that the square of this amplitude gives the most general set of singular factors appropriate to a color-ordered amplitude.

In 2004, Witten interpreted this result as being **holomorphic in twistor space** and set out to develop a **twistor string theory** that would compute it.

## Exercise:

Derive the color-ordered Feynman rules for QCD from the usual QCD Feynman rules.

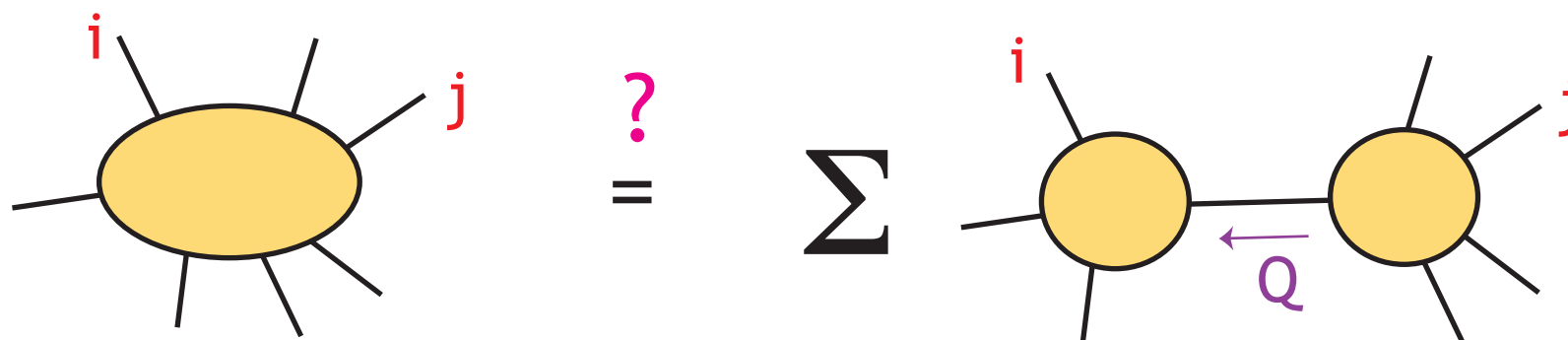
Derive the Parke-Taylor amplitudes

$$i\mathcal{M}(1^+, \dots, i^-, \dots, j^-, \dots, n^+) = ig^{n-2} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$
$$i\mathcal{M}(1^-, \dots, i^+, \dots, j^+, \dots, n^-) = i(-1)^n g^{n-2} \frac{[ij]^4}{[12][23] \dots [n1]}$$

using Feynman diagrams for the cases  $n = 3, 4$ . For  $n = 3$ , keep the reference momenta  $r$  general. For  $n = 4$ , a specific set of choices for the reference momenta will simplify the computation.

For practical purposes, we need all helicity amplitudes, not just MHV. However, Witten's program led to a remarkable method for the computation of non-MHV amplitudes, discovered by Britto, Cachazo, and Feng.

It would be wonderful to build up non-MHV amplitudes from the simple MHV expressions. But it is not so obvious how to do this. MHV amplitudes are on-shell expressions, but if we cut the more complex amplitudes, we will have to evaluate them off-shell.



Or do we ? BCF suggested that we pick legs  $i$  and  $j$  and shift

$$i] \rightarrow i] + z j] \quad j\rangle \rightarrow j\rangle - z i\rangle$$

for  $z$  a complex variable.

Now consider

$$\oint \frac{dz}{2\pi i} \frac{i\mathcal{M}(z)}{z} = i\mathcal{M}(z=0) + (\text{other poles}) = (\text{contour at } \infty)$$

The first term is the amplitude that we wish to evaluate. The contour at  $\infty$  vanishes if we choose **i** and **j** correctly (e.g **i** a - gluon and **j** a + gluon). The additional poles result when a momentum on an intermediate line satisfies

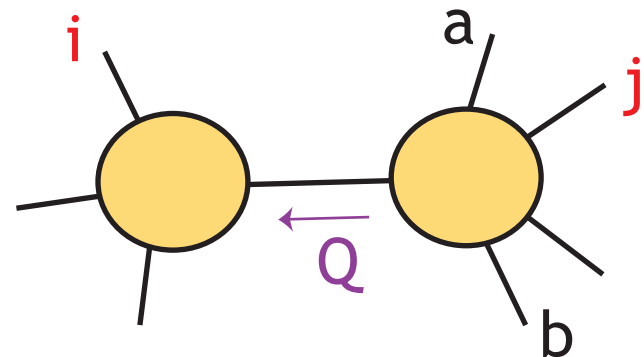
$$Q(z)^2 = 0$$

Looking again at the diagram,

$$Q^\mu(z)\gamma_\mu = \sum_{k=a}^b \langle k | [k - z | i \rangle [j$$

and so

$$z_* = \frac{S_{a\dots b}}{\langle i(\sum k) [k) j ]}$$



Tidying up the formula, one finds the following relation:

$$i\mathcal{M}(1 \cdots n) = \sum_{splits} i\mathcal{M}(b+1 \cdots \hat{i} \cdots a-1 - \hat{Q}) \cdot \frac{1}{s_{a \cdots b}} \cdot i\mathcal{M}(a \cdots \hat{j} \cdots b \hat{Q})$$

called the **Britto-Cachazo-Feng (BCF) recursion formula**.

Momenta with hats have the shift with  $z_*$ . The hatted momenta are complex but satisfy  $\hat{Q}^2 = 0$ , so the amplitudes on the right-hand side are to be evaluated on shell!

This allows the n-point amplitudes to be recursively evaluated in terms of amplitudes with fewer legs. We can stop when we reach MHV. At 5 points all amplitudes are MHV or anti-MHV.

The technique applies not only to multi-gluon amplitudes but also to other amplitudes of interest. Here are two more MHV formulae:

$$i\mathcal{M}(q_L g_2^+ g_3^+ \cdots g_i^- \cdots g_{n-1}^+ \bar{q}_R) = ig^{n-2} \frac{\langle 1i \rangle^3 \langle ni \rangle}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

$$i\mathcal{M}(e_L^- e_R^+ q_L g_4^+ g_5^+ \cdots g_{n-1}^+ \bar{q}_R) = -ig_w^2 g^{n-4} \frac{\langle 13 \rangle^2}{\langle 12 \rangle \langle 34 \rangle \langle 45 \rangle \cdots \langle n-1 n \rangle}$$

The second formula crosses into an MHV formula for

$$u\bar{d} \rightarrow W^+ + ng \rightarrow \ell^+ \nu + ng$$

and we can use this to compute heavy particle production at LHC. **Badger, Glover, Khoze,** and **Svrcek** have shown that the technique applies to even more general heavy-particle amplitudes, such as those of top quark production.

## Exercise:

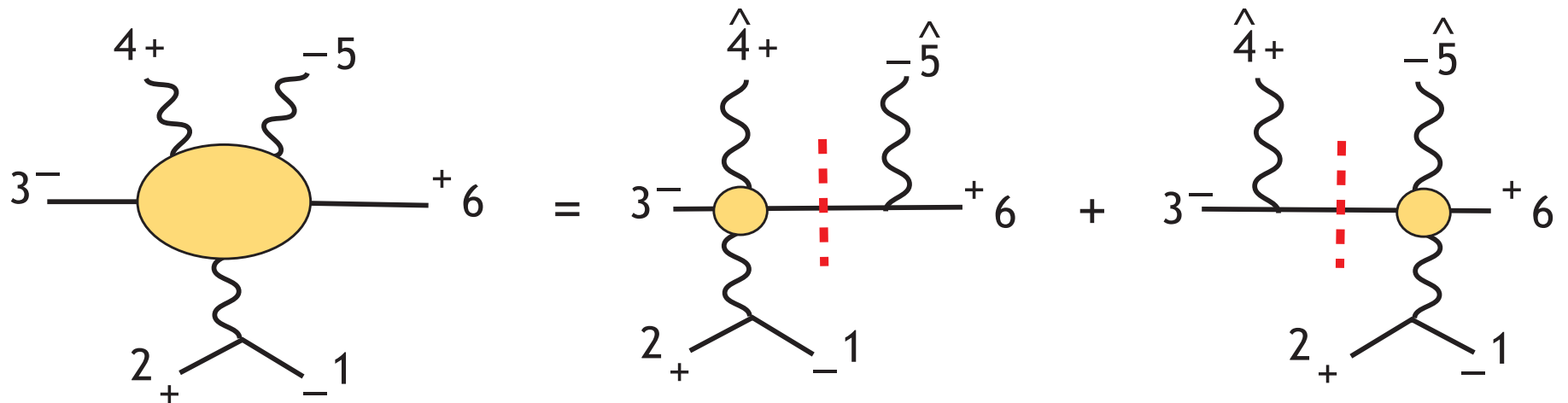
Using the results of the previous exercise for  $n = 3, 4$  and the BCF recursion formula, prove the validity of the general MHV formula for the  $n$ -gluon case by induction on the number of external gluons.



I would like to illustrate the use of the BCF recursion formula by computing a non-MHV contribution to  $e^+e^- \rightarrow \bar{q}q + 2$  gluons. This crosses into a contribution to  $W + 2$  jets, which we would need to compute the mass distribution of the hadronic system recoiling against a  $W$  or  $Z$ .

Using the shift  $\hat{5}] = 5] + z 4]$   $\hat{4}\rangle = 4\rangle - z 5\rangle$

we find



I will evaluate the first of these BCF cuts.

$$\begin{aligned}
& ig_w^2 g^2 \frac{\langle 13 \rangle^2}{\langle 12 \rangle \langle 3\hat{4} \rangle \langle \hat{4}(-\hat{Q}) \rangle} \frac{i}{s_{56}} ig \frac{\langle \hat{Q}5 \rangle^3 \langle 56 \rangle}{\langle \hat{Q}5 \rangle \langle 56 \rangle \langle 6\hat{Q} \rangle} \\
& = -ig_w^2 g^2 \frac{\langle 13 \rangle^2 \langle 5\hat{Q} \rangle^2}{\langle 12 \rangle \langle 3\hat{4} \rangle \langle \hat{4}\hat{Q} \rangle \langle 6\hat{Q} \rangle s_{56}}
\end{aligned}$$

where  $\hat{Q} = -5 \rangle [5 - 6 \rangle [6 - z 5 \rangle [4$

$$\hat{Q}^2 = 0 \text{ for } z = -\frac{s_{56}}{\langle 5(5+6)4 \rangle} = -\frac{[65]}{[64]}$$

Multiply the formula top and bottom by  $[\hat{Q}4]^2$  and work out the pieces:

$$\begin{aligned}
\langle 5\hat{Q} \rangle [\hat{Q}4] & = -\langle 56 \rangle [64] \\
\langle 6\hat{Q} \rangle [\hat{Q}4] & = -\langle 65 \rangle [54] \\
\langle \hat{4}\hat{Q} \rangle [\hat{Q}4] & = -s_{456} \\
\langle 3\hat{4} \rangle & = \langle 3(4+5)6 \rangle / [46]
\end{aligned}$$

in all

$$ig_w^2 g^2 \frac{\langle 13 \rangle^2 \langle 56 \rangle^2 [46]^3}{\langle 12 \rangle [45] s_{56} s_{456} \langle 3(4+5)6 \rangle}$$

An even nicer property of this formalism is that you do not need to work out these expressions directly. Instead, it is possible to compute directly with the complex spinors !

Once we have derived expressions for multi-parton tree level cross sections, there is still problems in interpreting these formulae correctly.

As Steve Ellis explained, jets are built from multiple parton radiation. Our explicit formulae account only the first few radiated quarks or gluons in the cascade.

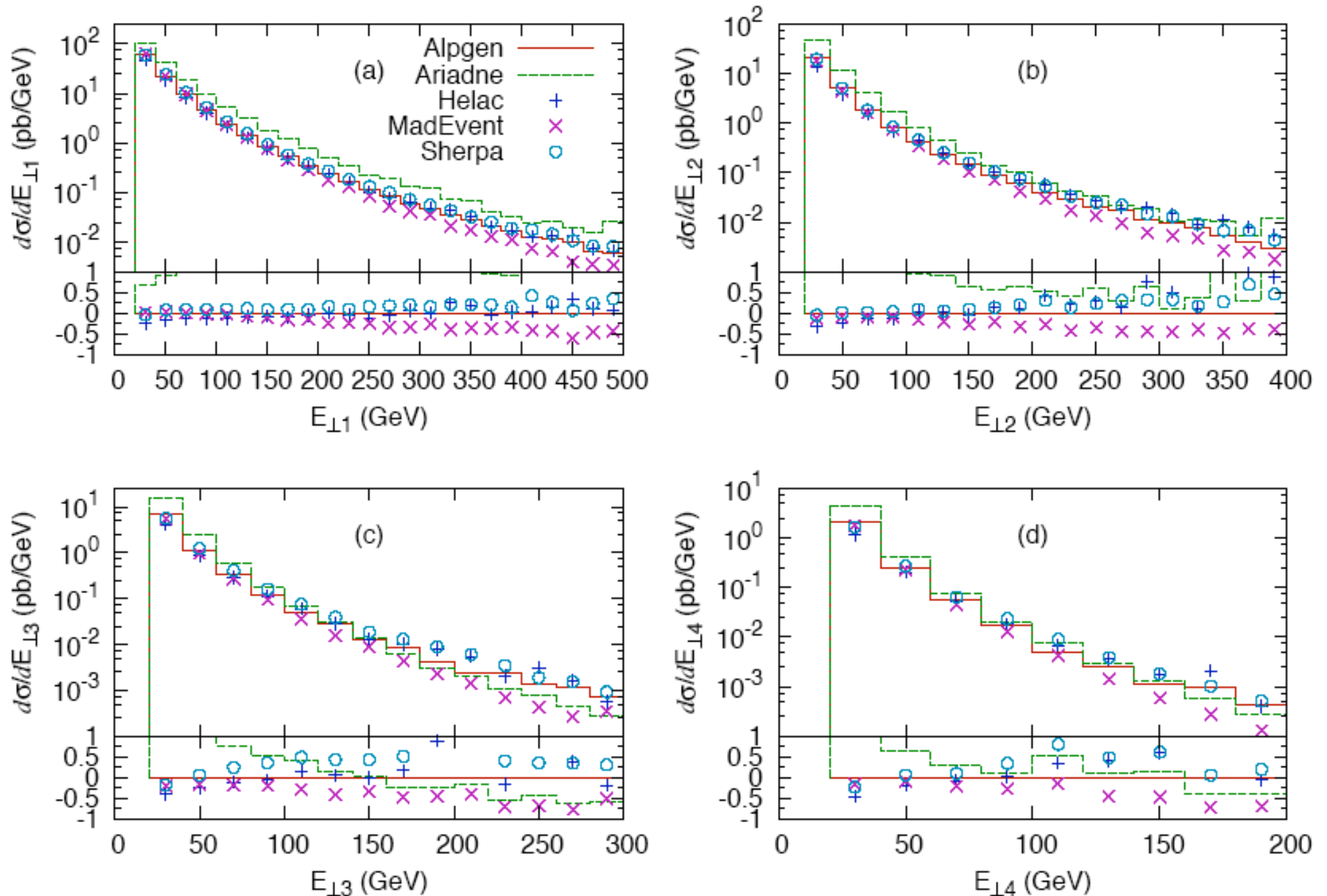
Often, we rely on a parton shower Monte Carlo program such as PYTHIA or HERWIG to supply the additional quarks and gluons. But then **there is a problem of double-counting**. We do not want partons to be radiated by our explicit calculation and radiated again by the parton shower MC.

Explicit formulae for multi-parton radiation give very large cross sections if we integrate down to soft partons. Loop diagrams provide compensating factors that restore unitarity. It is a **problem how to include the important effects without a full loop computation**.

The authors of codes for generating multijet processes have tried out different ways to solve these problems.

To the extent that different codes agree, we might have confidence (or not) that we understand these issues. Ultimately, comparison to deeper and higher-order QCD calculations is needed.

predictions of the ET spectrum in  $W+$  jets at the LHC for the 4 hardest jets, from J. Alwall et al. arXiv:0706.2569



In this lecture, I have introduced the issues involved in the prediction of multi-jet Standard Model backgrounds to new particle production at the LHC.

Much work is being done. I am optimistic that it will be enough to make strong statements about whether new particle production is seen at the LHC.

Once we have observed violation of the Standard Model, we can try to interpret the anomalies. This will be the subject of my third lecture.