Homework Problems: PITP 2007

Aneesh Manohar

I.

Estimate using dimensional analysis, the contribution of the hadronic vacuum polarization and hadronic light by light scattering amplitudes to $g-2$ of the muon. How accurately must these amplitudes be known so that the theoretical uncertainty equals the current experimental uncertainty?

$$\mu = 1.0011596521859(38)$$

II.

Compute the dependence of α and m_p on m_t.

III.

Estimate the proton lifetime assuming it is due to a dimension six operator induced at the GUT scale of 10^{15} GeV. Repeat, assuming the operator is dimension five.

IV.

In a theory with baryon number violation, but no lepton number violation, one can have a $\Delta B = 2$ operator that produces $n - \bar{n}$ oscillations. The oscillation time limit has a lower bound of 0.86×10^8 s. Put a lower limit on the scale of this operator.

V.

Compute the free-quark decay rate for $b \to c\bar{u}d$ assuming the weak Lagrangian has the form

$$L = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \ [C_1 O_1 + C_2 O_2] \tag{1}$$

where

$$\begin{aligned} O_1 &= (\bar{c}^\alpha \gamma^\mu P_L b_\alpha) (\bar{d}^\beta \gamma_\mu P_L u_\beta) \\ O_2 &= (\bar{c}^\alpha \gamma^\mu P_L b_\beta) (\bar{d}^\beta \gamma_\mu P_L u_\alpha) \end{aligned} \tag{2}$$

[See last chapter of the book on heavy quark physics with Mark Wise, Eq. (6.106), coefficient of the leading order term only.]

VI.

Compute the one-loop mixing matrix for the operators in Eq. (2). [See first chapter of the book on heavy quark physics with Mark Wise, Eq. (1.135)]

VII.

Use field redefinitions to eliminate the $D_0^2/2m$ term in the Lagrangian

$$L = \bar{h} \left(iD_0 + \frac{\mathbf{D}^2 - D_0^2}{2m} \right) h \tag{3}$$

Compute the matrix element of $\bar{h}h$ between incoming and outgoing heavy quarks at one loop using both forms of the Lagrangian, and verify that they give the same value on-shell.

VIII.

Compute the one-loop matching for the HQET Lagrangian. The calculations are given in A.M., Phys. Rev. D56 (1977) 230. Verify Eqs. (20,30,31). This should show you how to regulate the IR using dimensional regularization, and compute matching coefficients.