Introduction to the Standard Model

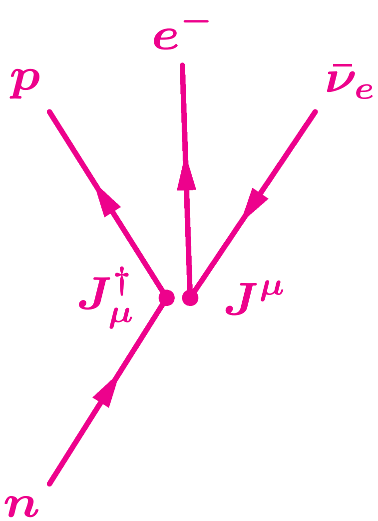
- Origins of the Electroweak Theory
- Gauge Theories
- The Standard Model Lagrangian
- Spontaneous Symmetry Breaking
- The Gauge Interactions
- Problems With the Standard Model

("Structure Of The Standard Model," hep-ph/0304186)

The Weak Interactions

- Radioactivity (Becquerel, 1896)
- β decay appeared to violate energy (Meitner, Hahn; 1911)
- Neutrino hypothesis (Pauli, 1930)
  - ν_e (Reines, Cowan; 1953)
  - ν_μ (Lederman, Schwartz, Steinberger; 1962)
  - ν_τ (DONUT, 2000) (τ, 1975)

- **Fermi theory (1933)**
  - **Loosely like QED, but zero range (non-renormalizable) and non-diagonal (charged current)**



$$H \sim G_F J_\mu^\dagger J^\mu$$

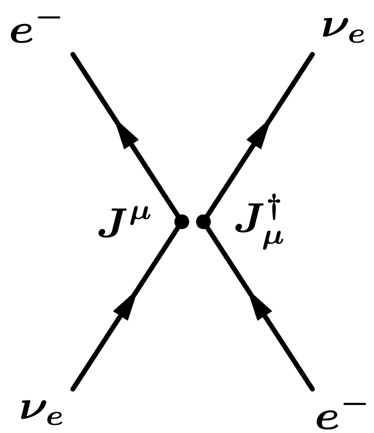
$$J_\mu^\dagger \sim \bar{p}\gamma_\mu n + \bar{\nu}_e \gamma_\mu e^- \qquad [n \to p,\ e^- \to \nu_e]$$

$$J_\mu \sim \bar{n}\gamma_\mu p + \bar{e}\gamma_\mu \nu_e \qquad [p \to n,\ \nu_e \to e^- \ (\ \times \to e^- \bar{\nu}_e)]$$

$$G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2} \qquad [\text{Fermi constant}]$$

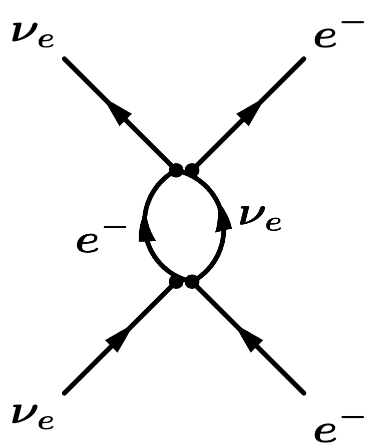
- Fermi theory modified to include
  - $\mu, \ \tau$ decay
  - strangeness (Cabibbo)
  - quark model
  - heavy quarks (CKM)
  - ν mass and mixing
  - parity violation $(V - A)$ (Lee, Yang; Wu; Feynman-Gell-Mann)
- Fermi theory correctly describes (at tree level)
  - Nuclear/neutron β decay $(n \to p e^- \bar{\nu}_e)$
  - $\mu, \ \tau$ decays $(\mu^- \to e^- \bar{\nu}_e \nu_\mu; \quad \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau, \ \nu_\tau \pi^-, \ \cdots)$
  - $\pi, \ K$ decays $(\pi^+ \to \mu^+ \nu_\mu, \ \pi^0 e^+ \nu_e; \quad K^+ \to \mu^+ \nu_\mu, \ \pi^0 e^+ \nu_e, \ \pi^+ \pi^0)$
  - hyperon decays $(\Lambda \to p \pi^-; \quad \Sigma^- \to n \pi^-; \quad \Sigma^+ \to \Lambda e^+ \nu_e)$
  - heavy quark decays $(c \to s e^+ \nu_e; \quad b \to c \mu^- \bar{\nu}_\mu, \ c \pi^-)$
  - ν scattering $(\nu_\mu e^- \to \mu^- \nu_e; \quad \underbrace{\nu_\mu n \to \mu^- p}_{\text{“elastic''}}; \quad \underbrace{\nu_\mu N \to \mu^- X}_{\text{deep}-\text{inelastic}})$

- **Fermi theory violates unitarity at high energy** (non-renormalizable)



  - $\sigma(\nu_e e^- \to e^- \nu_e) \to \frac{G_F^2 s}{\pi}, \quad s \equiv E_{CM}^2$
  - pure S-wave unitarity: $\sigma < \frac{16\pi}{s}$
  - fails for $\frac{E_{CM}}{2} \geq \sqrt{\frac{\pi}{G_F}} \sim 500$ GeV

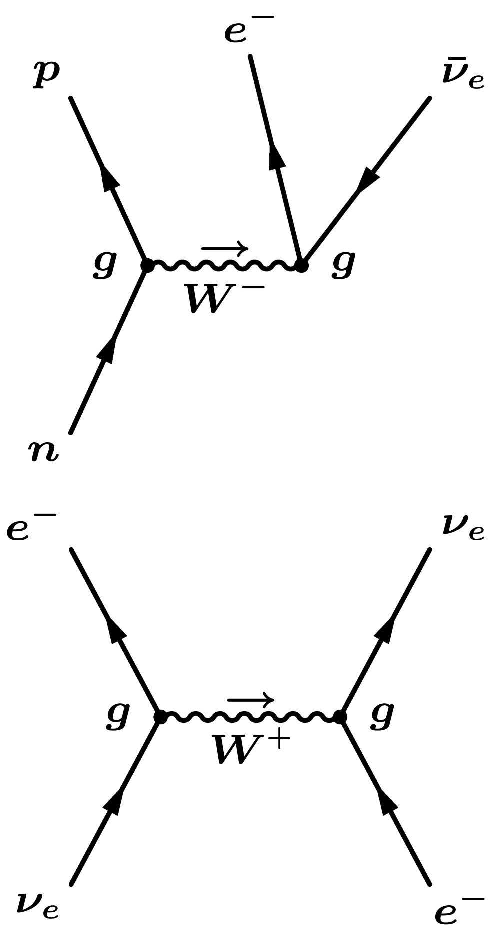
  - Born not unitary; often restored by H.O.T.



  - Fermi theory: divergent integrals

$$\int d^4k \frac{\not{k} + m_e}{k^2 - m_e^2} \frac{\not{k}}{k^2}$$

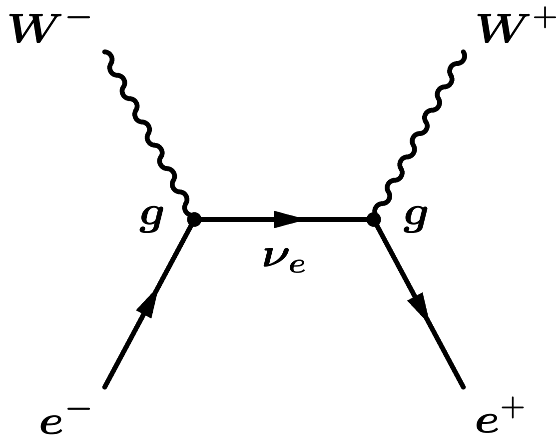
- **Intermediate vector boson theory** (Yukawa, 1935; Schwinger, 1957)



$$\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{8M_W^2} \text{ for } M_W \gg Q$$

– no longer pure S-wave $\Rightarrow$

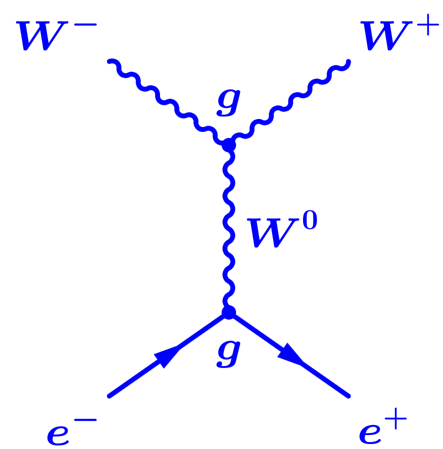
– $\nu_e e^- \rightarrow \nu_e e^-$ better behaved



– *but*, $e^+e^- \rightarrow W^+W^-$ violates unitarity for $\sqrt{s} \gtrsim 500$ GeV

– $\epsilon_\mu \sim k_\mu/M_W$ for longitudinal polarization (non-renormalizable)

– introduce W^0 to cancel

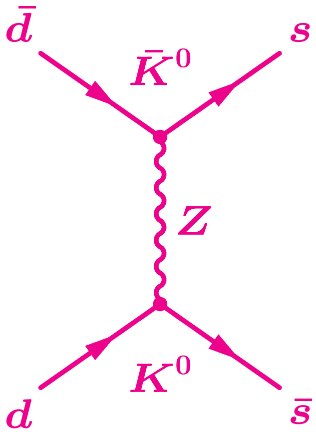


– fixes $W^0W^+W^-$ and $e^+e^-W^0$ vertices

– requires $\left[J, J^\dagger\right] \sim J^0$ (like $SU(2)\times U(1)$)

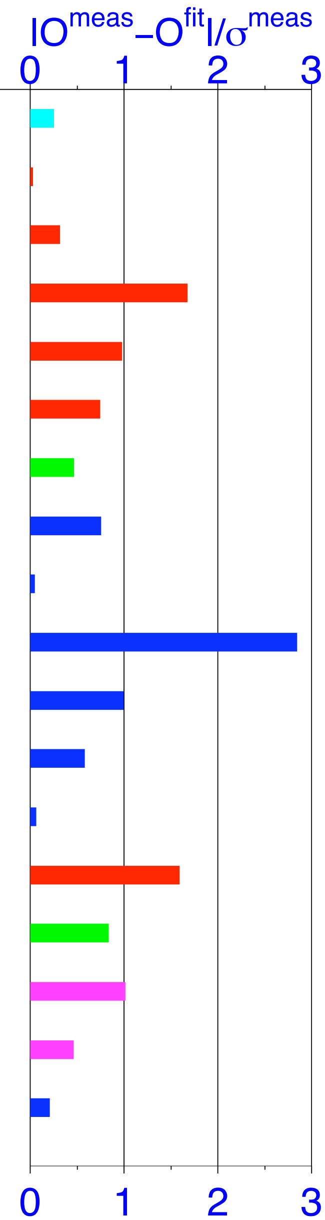
– not realistic

- **Glashow model (1961)** ($W^\pm, Z, \gamma$, but no mechanism for $M_{W,Z}$)
- **Weinberg-Salam (1967): Higgs mechanism $\longrightarrow M_{W,Z}$**
- **Renormalizable (1971)** ('t Hooft, $\cdots$)
- **Flavor changing neutral currents (FCNC)**
  - very large $K^0 \leftrightarrow \bar{K}^0$ mixing
  - GIM mechanism (c quark) (1970)
  - c discovered (1974)



- **Weak neutral current (1973)**
- **QCD (1970's)**
- **W, Z (1983)**
- **Precision tests (1989-2000)**
- **CKM unitarity ($\sim$ 1995-)**
- **t quark (1995)**
- **ν mass (1998-2002)**

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.410 ± 0.032	80.377
Γ_W [GeV]	2.123 ± 0.067	2.092
m_t [GeV]	172.7 ± 2.9	173.3



Gauge Theories

Standard Model is remarkably successful gauge theory of the microscopic interactions

- Gauge symmetry $\Rightarrow$ (apparently) massless spin-1 (vector, gauge) bosons
- Interactions $\Leftrightarrow$ group, representations, gauge coupling
- Like QED ($U(1)$), but gauge self interactions for non-abelian
- Application to strong (short range) $\Rightarrow$ confinement
- Application to weak (short range) $\Rightarrow$ spontaneous symmetry breaking (Higgs or dynamical)
- Unique renormalizable field theory for spin-1

QED

- Free electron equation,

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right)\psi = 0,$$

is invariant under $U(1)$ (phase) transformations,

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right)\psi' = 0, \text{ where } \psi' \equiv e^{-i\beta}\psi$$

- *Not* invariant under local (gauge) transf.,

$$\psi \rightarrow \psi' \equiv e^{-i\beta(x)}\psi, \quad x \equiv (\vec{x}, t)$$

- **Introduce vector field $A^\mu \equiv (\vec{A}, \phi)$:**

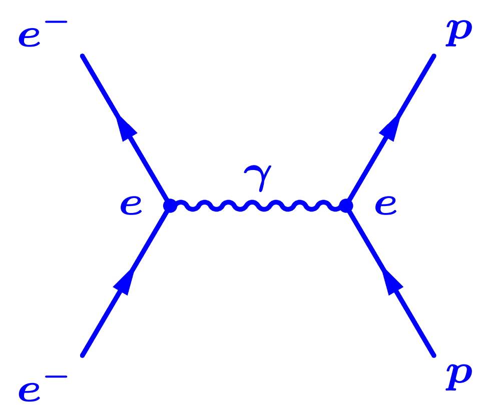
$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} + e\gamma^\mu A_\mu - m \right) \psi = 0,$$

($e > 0$ is gauge coupling) **is invariant under**

$$\psi \rightarrow e^{-i\beta(x)}\psi, \quad A_\mu \rightarrow A_\mu - \frac{1}{e}\frac{\partial \beta}{\partial x^\mu}$$

- **Quantization of $A_\mu \Rightarrow$ massless gauge boson**

- **Gauge invariance $\Rightarrow \gamma$, long range force, prescribed (up to e) amplitude for emission/absorption**



Non-Abelian

- n non-interacting fermions of same mass m:

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right)\psi_a = 0, \quad a = 1 \cdots n,$$

invariant under (global) $SU(n)$ group,

$$\begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \rightarrow \exp(i\sum_{i=1}^{N} \beta^i L^i) \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}.$$

L^i are $n \times n$ generator matrices ($N = n^2 - 1$); β^i are real parameters

$$[L^i, L^j] = i c_{ijk} L^k$$

(c_{ijk} are structure constants)

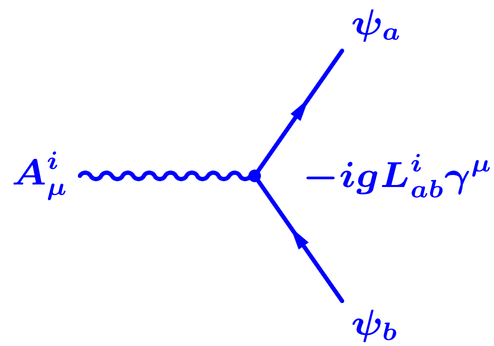
- **Gauge (local) transformation:** $\beta^i \rightarrow \beta^i(x) \Rightarrow$

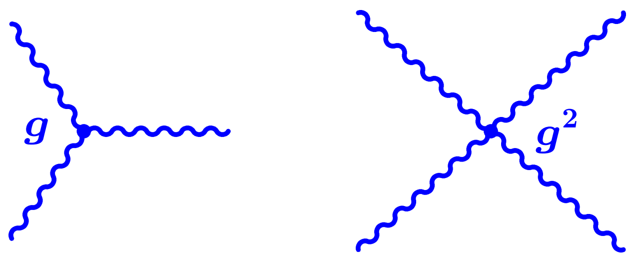
$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu}\delta_{ab} - g\sum_{i=1}^{N} \gamma^\mu A^i_\mu L^i_{ab} - m\delta_{ab} \right)\psi_b = 0$$

- **Invariant under**

$$\begin{aligned}
\Phi &\equiv \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \rightarrow \Phi' \equiv U\Phi \\
\vec{A}_\mu \cdot \vec{L} &\rightarrow \vec{A}'_\mu \cdot \vec{L} \equiv U \vec{A}_\mu \cdot \vec{L} U^{-1} + \frac{i}{g}\left(\partial_\mu U\right) U^{-1} \\
U &\equiv e^{i\vec{\beta}\cdot\vec{L}}
\end{aligned} \qquad (1)$$

- Gauge invariance implies:
  - N (apparently) massless gauge bosons A_μ^i
  - Specified interactions (up to gauge coupling g, group, representations), including self interactions





- Generalize to other groups, representations, chiral ($L \neq R$)
  - Chiral Projections: $\psi_{L(R)} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi$
    (Chirality = helicity up to $O(m/E)$)

The Standard Model

- Gauge group $SU(3)\times SU(2)\times U(1)$; gauge couplings g_s, g, g'

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

$$\begin{matrix} u_R & u_R & u_R & \nu_{eR}(?) \\ d_R & d_R & d_R & e^-_R \end{matrix}$$

(L = left-handed, R = right-handed)

- $SU(3)$: $u \leftrightarrow u \leftrightarrow u$, $d \leftrightarrow d \leftrightarrow d$ (8 gluons)
- $SU(2)$: $u_L \leftrightarrow d_L$, $\nu_{eL} \leftrightarrow e^-_L$ $(W^\pm)$; phases (W^0)
- $U(1)$: phases (B)
- Heavy families (c, s, ν_μ, μ^-), (t, b, ν_τ, τ^-)

Quantum Chromodynamics (QCD)

$$\mathcal{L}_{SU(3)} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \sum_r \bar{q}_{r\alpha} i \not{D}^\alpha_\beta \, q^\beta_r$$

F^2 term leads to three and four-point gluon self-interactions.

$$F^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_s f_{ijk} \, G^j_\mu \, G^k_\nu$$

is field strength tensor for the gluon fields G^i_μ, $i = 1, \cdots, 8$.
g_s = QCD gauge coupling constant. No gluon masses.

Structure constants f_{ijk} $(i, j, k = 1, \cdots, 8)$, defined by

$$[\lambda^i, \lambda^j] = 2 i f_{ijk} \lambda^k$$

where λ^i are the Gell-Mann matrices.

$$\boldsymbol{\lambda}^i = \begin{pmatrix} \boldsymbol{\tau}^i & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3$$

$$\boldsymbol{\lambda}^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \boldsymbol{\lambda}^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{\lambda}^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \boldsymbol{\lambda}^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\boldsymbol{\lambda}^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The $SU(3)$ (Gell-Mann) matrices.

Quark interactions given by $\bar{q}_{r\alpha} i \not{D}^{\alpha}_{\beta} q^{\beta}_{r}$

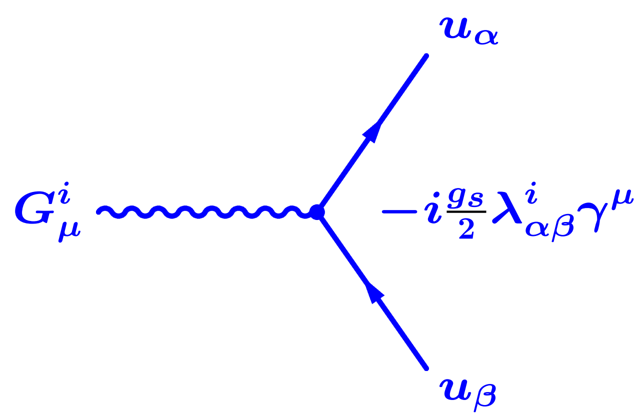
$q_r = r^{\text{th}}$ quark flavor; $\alpha, \beta = 1, 2, 3$ are color indices

Gauge covariant derivative

$$D^{\alpha}_{\mu\beta} = (D_{\mu})_{\alpha\beta} = \partial_{\mu}\delta_{\alpha\beta} + i g_s \, G^{i}_{\mu} \, L^{i}_{\alpha\beta},$$

for triplet representation matrices $L^i = \lambda^i/2$.

Quark color interactions:



Diagonal in flavor

Off diagonal in color

Purely vector (parity conserving)

Bare quark mass allowed by QCD, but forbidden by chiral symmetry of $\mathcal{L}_{SU(2)\times U(1)}$ (generated by spontaneous symmetry breaking)

Additional ghost and gauge-fixing terms

Can add (unwanted) CP-violating term

$$\mathcal{L}_\theta = \frac{\theta g_s^2}{32\pi^2} F^i_{\mu\nu} \tilde{F}^{i\mu\nu}, \quad \tilde{F}^{i\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F^i_{\alpha\beta}$$

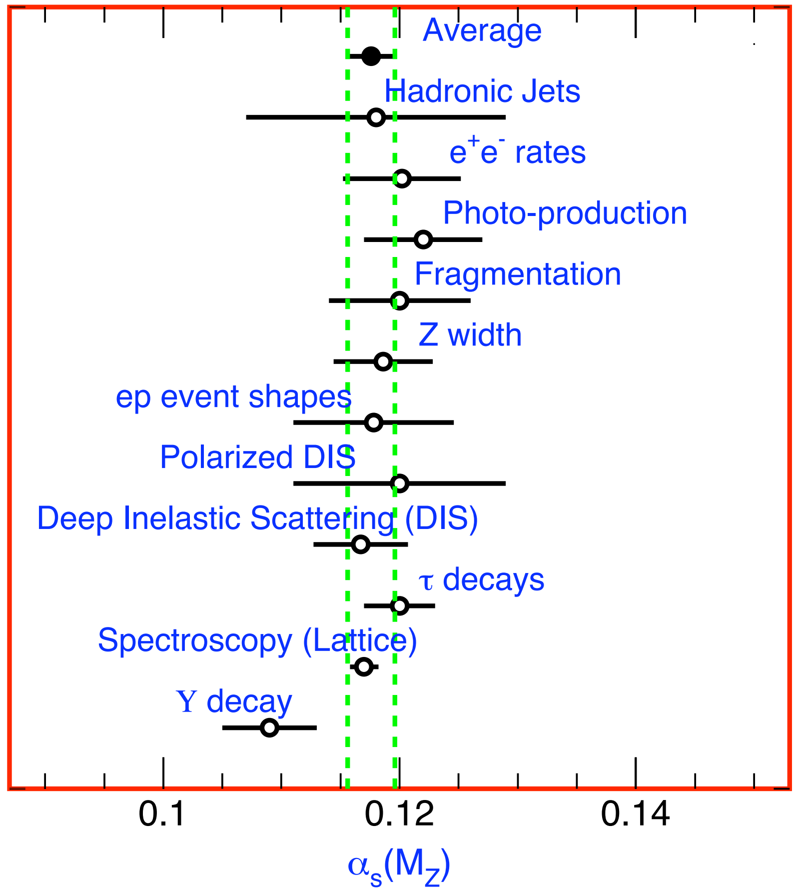
QCD now very well established

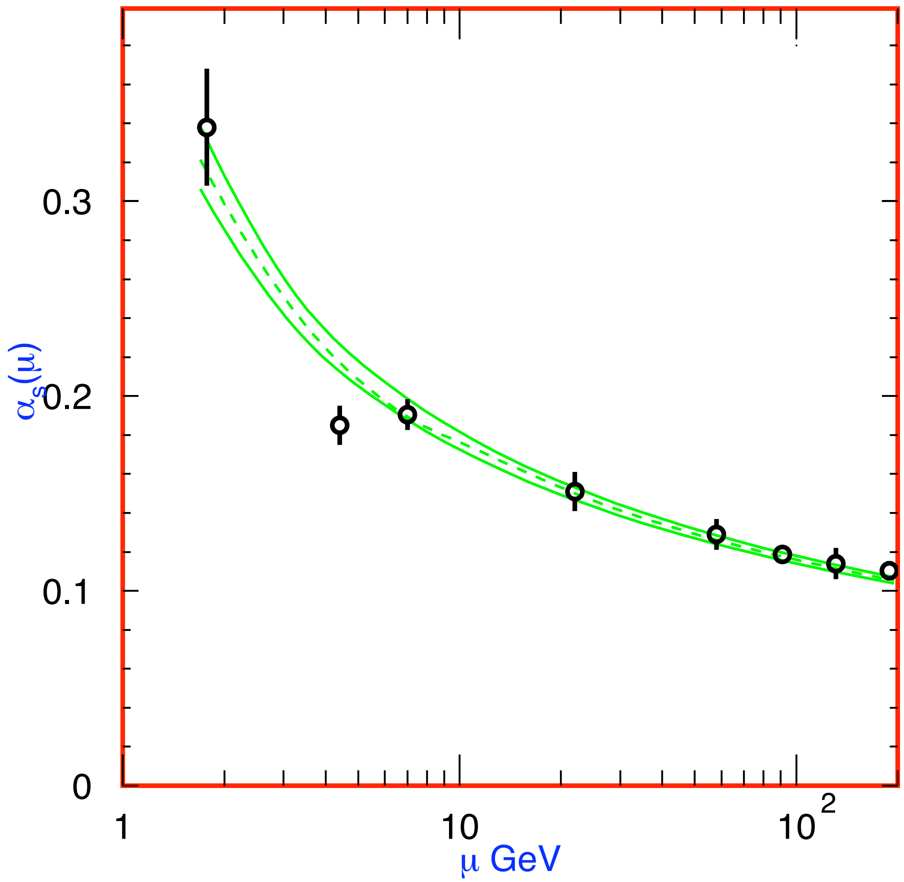
- Short distance behavior (asymptotic freedom)

- Confinement, light hadron spectrum (lattice)
  - $g_s = O(1)$ $(\alpha_s(M_Z) = g_s^2/4\pi \sim 0.12)$
  - Strength $+$ gluon self-interactions$\Rightarrow$ confinement
  - Yukawa model $\Rightarrow$ dipole-dipole

- Approximate global $SU(3)_L \times SU(3)_R$ symmetry and breaking (π, K, η are pseudo-goldstone bosons)

- Unique field theory of strong interactions


Average
Hadronic Jets
e^+e^- rates
Photo-production
Fragmentation
Z width
ep event shapes
Polarized DIS
Deep Inelastic Scattering (DIS)
τ decays
Spectroscopy (Lattice)
Υ decay
0.1
0.12
0.14
$\alpha_s(M_Z)$


$\alpha_s(\mu)$
0.3
0.2
0.1
0
1
10
10^2
μ GeV

The Electroweak Sector

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\varphi} + \mathcal{L}_{f} + \mathcal{L}_{\rm Yukawa}$$

Gauge part

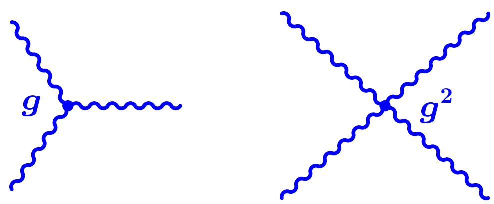
$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

Field strength tensors

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ F^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu, \quad i = 1\cdots 3 \end{aligned}$$

$g(g')$ is $SU(2)$ $(U(1))$ gauge coupling; ϵ_{ijk} is totally antisymmetric symbol

Three and four-point self-interactions for the W_i



B and W_3 will mix to form γ, Z

Scalar part

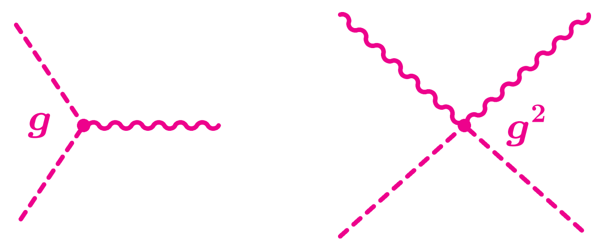
$$\mathcal{L}_\varphi = (D^\mu \varphi)^\dagger D_\mu \varphi - V(\varphi)$$

where $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ is the (complex) Higgs doublet.
Gauge covariant derivative:

$$D_\mu \varphi = \left(\partial_\mu + ig \frac{\tau^i}{2} W^i_\mu + \frac{ig'}{2} B_\mu \right) \varphi$$

where τ^i are the Pauli matrices

Three and four-point interactions between gauge and scalar fields



Higgs potential

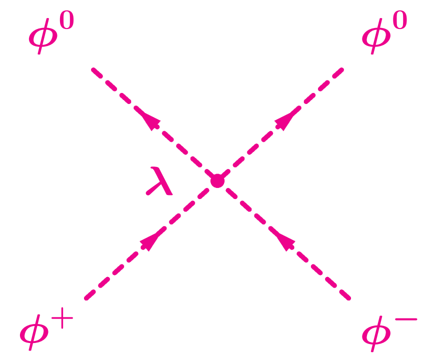
$$V(\varphi) = +\mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2$$

Allowed by renormalizability and gauge invariance

Spontaneous symmetry breaking for $\mu^2 < 0$

Vacuum stability: $\lambda > 0$.

Quartic self-interactions



Fermion part

$$\mathcal{L}_F = \sum_{m=1}^{F} \left(\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL} \right.$$
$$\left. + \; \bar{u}^0_{mR} i \not{D} u^0_{mR} + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR} + \bar{\nu}^0_{mR} i \not{D} \nu^0_{mR} \right)$$

L-doublets

$$q^0_{mL} = \begin{pmatrix} u^0_m \\ d^0_m \end{pmatrix}_L \qquad l^0_{mL} = \begin{pmatrix} \nu^0_m \\ e^{-0}_m \end{pmatrix}_L$$

R-singlets

$$u^0_{mR}, \; d^0_{mR}, \; e^{-0}_{mR}, \; \nu^0_{mR}(?)$$

($F \geq 3$ families; $m = 1 \cdots F$ = family index;
^{0} = weak eigenstates (definite $SU(2)$ rep.), mixtures of mass eigenstates (flavors);
quark color indices $\alpha = r, \; g, \; b$ suppressed (e.g., $u^0_{m\alpha L}$).)

Can add gauge singlet ν^0_{mR} for Dirac neutrino mass term

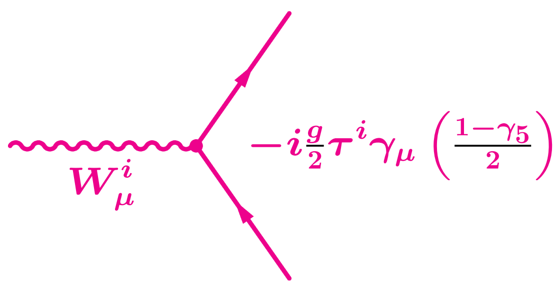
Different (chiral) L and R representations lead to parity and charge conjugation violation (maximal for $SU(2)$)

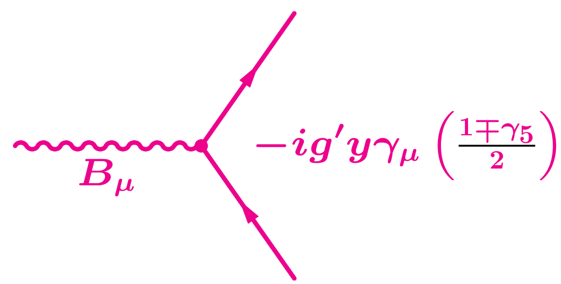
Fermion mass terms forbidden by chiral symmetry

Gauge covariant derivatives

$$
\begin{aligned}
D_\mu q^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu + i\frac{g'}{6}B_\mu\right) q^0_{mL} \\
D_\mu l^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu - i\frac{g'}{2}B_\mu\right) l^0_{mL} \\
D_\mu u^0_{mR} &= \left(\partial_\mu + i\frac{2}{3}g' B_\mu\right) u^0_{mR} \\
D_\mu d^0_{mR} &= \left(\partial_\mu - i\frac{g'}{3}B_\mu\right) d^0_{mR} \\
D_\mu e^0_{mR} &= \left(\partial_\mu - ig' B_\mu\right) e^0_{mR}
\end{aligned}
$$

Read off W and B couplings to fermions





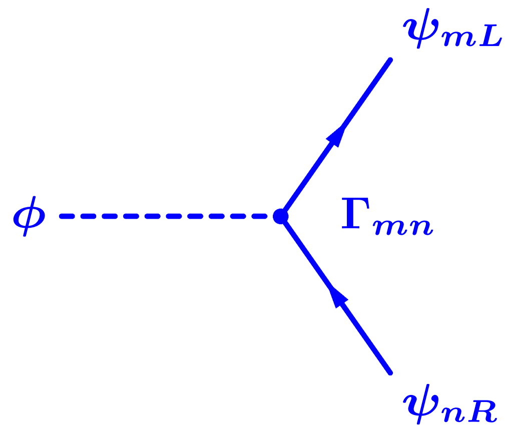
Yukawa couplings (couple L to R)

$$-\mathcal{L}_{\text{Yukawa}} = \sum_{m,n=1}^{F} \left[\Gamma^u_{mn}\bar{q}^0_{mL}\tilde{\varphi}u^0_{mR} + \Gamma^d_{mn}\bar{q}^0_{mL}\varphi d^0_{nR} + \Gamma^e_{mn}\bar{l}^0_{mn}\varphi e^0_{nR}\ (+\Gamma^\nu_{mn}\bar{l}^0_{mL}\tilde{\varphi}\nu^0_{mR})\right] + \text{H.C.}$$

Γ_{mn} are completely arbitrary Yukawa matrices, which determine fermion masses and mixings

d, e terms require doublet $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ with $Y_\varphi = 1/2$

u (and ν) terms require doublet $\Phi = \begin{pmatrix} \Phi^0 \\ \Phi^- \end{pmatrix}$ with $Y_\Phi = -1/2$



In $SU(2)$ the 2 and 2^* are similar $\Rightarrow \tilde{\varphi} \equiv i\tau^2 \varphi^\dagger = \begin{pmatrix} \varphi^{0^\dagger} \\ -\varphi^- \end{pmatrix}$ transforms as a 2 with $Y_{\tilde{\varphi}} = -\frac{1}{2} \Rightarrow$ only one doublet needed.

Does not generalize to $SU(3)$, most extra $U(1)'$, supersymmetry, $SO(10)$ etc $\Rightarrow$ need two doublets.

(Does generalize to $SU(2)_L \times SU(2)_R \times U(1)$ and $SU(5)$)

Spontaneous Symmetry Breaking

Gauge invariance implies massless gauge bosons and fermions

Weak interactions short ranged $\Rightarrow$ spontaneous symmetry breaking for mass; also for fermions

Color confinement for QCD $\Rightarrow$ gluons remain massless

Allow classical (ground state) expectation value for Higgs field

$$v = \langle 0|\varphi|0\rangle = \text{constant}$$

$\partial_\mu v \neq 0$ increases energy, but important for monopoles, strings, domain walls, phase transitions (e.g., EWPT, baryogenesis)

Minimize $V(v)$ to find v and quantize $\varphi' = \varphi - v$

$SU(2)\times U(1)$: introduce Hermitian basis

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2) \\ \frac{1}{\sqrt{2}}(\varphi_3 - i\varphi_4 \end{pmatrix},$$

where $\varphi_i = \varphi_i^\dagger$.

$$V(\varphi) = \frac{1}{2}\mu^2 \left(\sum_{i=1}^4 \varphi_i^2\right) + \frac{1}{4}\lambda \left(\sum_{i=1}^4 \varphi_i^2\right)^2$$

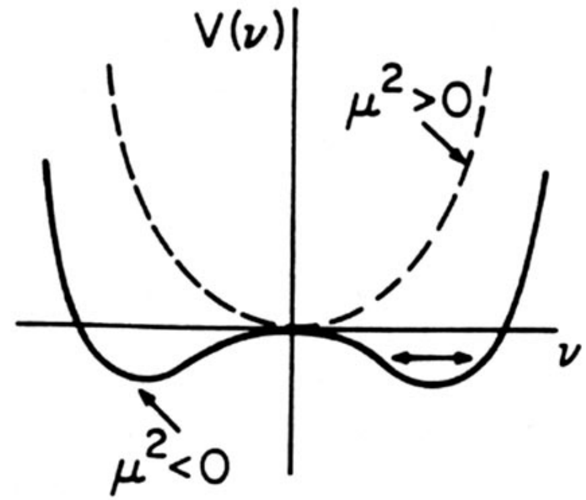
is $O(4)$ invariant.

w.l.o.g. choose $\langle 0|\varphi_i|0\rangle = 0, \quad i = 1, 2, 4$ and $\langle 0|\varphi_3|0\rangle = \nu$

$$V(\varphi) \rightarrow V(v) = \frac{1}{2}\mu^2\nu^2 + \frac{1}{4}\lambda\nu^4$$

For $\mu^2 < 0$, minimum at

$$V'(\nu) = \nu(\mu^2 + \lambda\nu^2) = 0$$
$$\Rightarrow \nu = (-\mu^2/\lambda)^{1/2}$$



SSB for $\mu^2 = 0$ also; must consider loop corrections

$\varphi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \equiv v \Rightarrow$ the generators L^1, L^2, and $L^3 - Y$ spontaneously broken, $L^1 v \neq 0$, etc $(L^i = \frac{\tau^i}{2},\ Y = \frac{1}{2} I)$

$Qv = (L^3 + Y)v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} v = 0 \Rightarrow U(1)_Q$ unbroken $\Rightarrow$ $SU(2) \times U(1)_Y \rightarrow U(1)_Q$

Quantize around classical vacuum

- Kibble transformation: introduce new variables ξ^i for rolling modes

$$\varphi = \frac{1}{\sqrt{2}} e^{i \sum \xi^i L^i} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

- $H = H^\dagger$ is the Higgs scalar
- No potential for $\xi^i \Rightarrow$ massless Goldstone bosons for global symmetry
- Disappear from spectrum for gauge theory ("eaten")
- Display particle content in unitary gauge

$$\varphi \rightarrow \varphi' = e^{-i \sum \xi^i L^i} \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

+ corresponding transformation on gauge fields

Rewrite Lagrangian in New Vacuum

Higgs covariant kinetic energy terms

$$
\begin{aligned}
(D_\mu \varphi)^\dagger D^\mu \varphi &= \frac{1}{2}(0\ \nu)\left[\frac{g}{2}\tau^i W_\mu^i + \frac{g'}{2} B_\mu\right]^2 \begin{pmatrix} 0 \\ \nu \end{pmatrix} + H \text{ terms} \\
&\to M_W^2 W^{+\mu} W_\mu^- + \frac{M_Z^2}{2} Z^\mu Z_\mu \\
&+ H \text{ kinetic energy and gauge interaction terms}
\end{aligned}
$$

Mass eigenstate bosons: W, Z, and A (photon)

$$
\begin{aligned}
W^\pm &= \frac{1}{\sqrt{2}}(W^1 \mp i W^2) \\
Z &= -\sin\theta_W B + \cos\theta_W W^3 \\
A &= \cos\theta_W B + \sin\theta_W W^3
\end{aligned}
$$

Masses

$$M_W = \frac{g\nu}{2}, \quad M_Z = \sqrt{g^2 + g'^2}\frac{\nu}{2} = \frac{M_W}{\cos\theta_W}, \quad M_A = 0$$

(Goldstone scalar transformed into longitudinal components of $W^\pm$, Z)

Weak angle: $\tan\theta_W \equiv g'/g$

Will show: Fermi constant $G_F/\sqrt{2} \sim g^2/8M_W^2$, **where** $G_F = 1.16637(1)\times 10^{-5}\ GeV^{-2}$ **from muon lifetime**

Electroweak scale

$$\nu = 2M_W/g \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246\ GeV$$

Will show: $g = e/\sin\theta_W$, where $\alpha = e^2/4\pi \sim 1/137.036 \Rightarrow$

$$M_W = M_Z \cos\theta_W \sim \frac{(\pi\alpha/\sqrt{2}G_F)^{1/2}}{\sin\theta_W}$$

Weak neutral current: $\sin^2\theta_W \sim 0.23 \Rightarrow M_W \sim 78\ GeV$, and $M_Z \sim 89\ GeV$ (increased by $\sim 2\ GeV$ by loop corrections)

Discovered at CERN: UA1 and UA2, 1983

Current:

$$\begin{aligned} M_Z &= 91.1876 \pm 0.0021 \\ M_W &= 80.403 \pm 0.029 \end{aligned}$$

The Higgs Scalar H

Gauge interactions: $ZZH, ZZH^2, W^+W^-H, W^+W^-H^2$

$$\varphi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_\varphi &= (D^\mu \varphi)^\dagger D_\mu \varphi - V(\varphi) \\ &= \tfrac{1}{2} (\partial_\mu H)^2 + M_W^2 W^{\mu +} W_\mu^- \left(1 + \frac{H}{\nu}\right)^2 \\ &+ \tfrac{1}{2} M_Z^2 Z^\mu Z_\mu \left(1 + \frac{H}{\nu}\right)^2 - V(\varphi) \end{aligned}$$

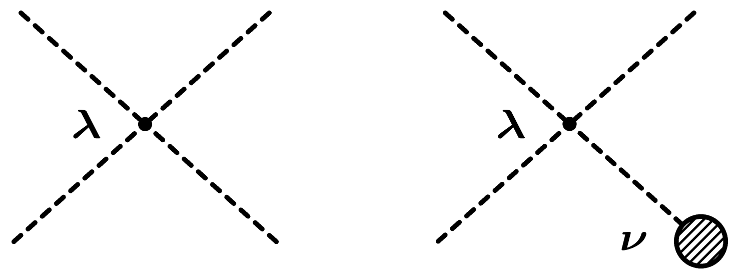
Higgs potential:

$$\begin{aligned} V(\varphi) &= +\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \\ &\to -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda \nu H^3 + \frac{\lambda}{4} H^4 \end{aligned}$$

Fourth term: Quartic self-interaction

Third: Induced cubic self-interaction

Second: (Tree level) H mass-squared, $M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\nu$

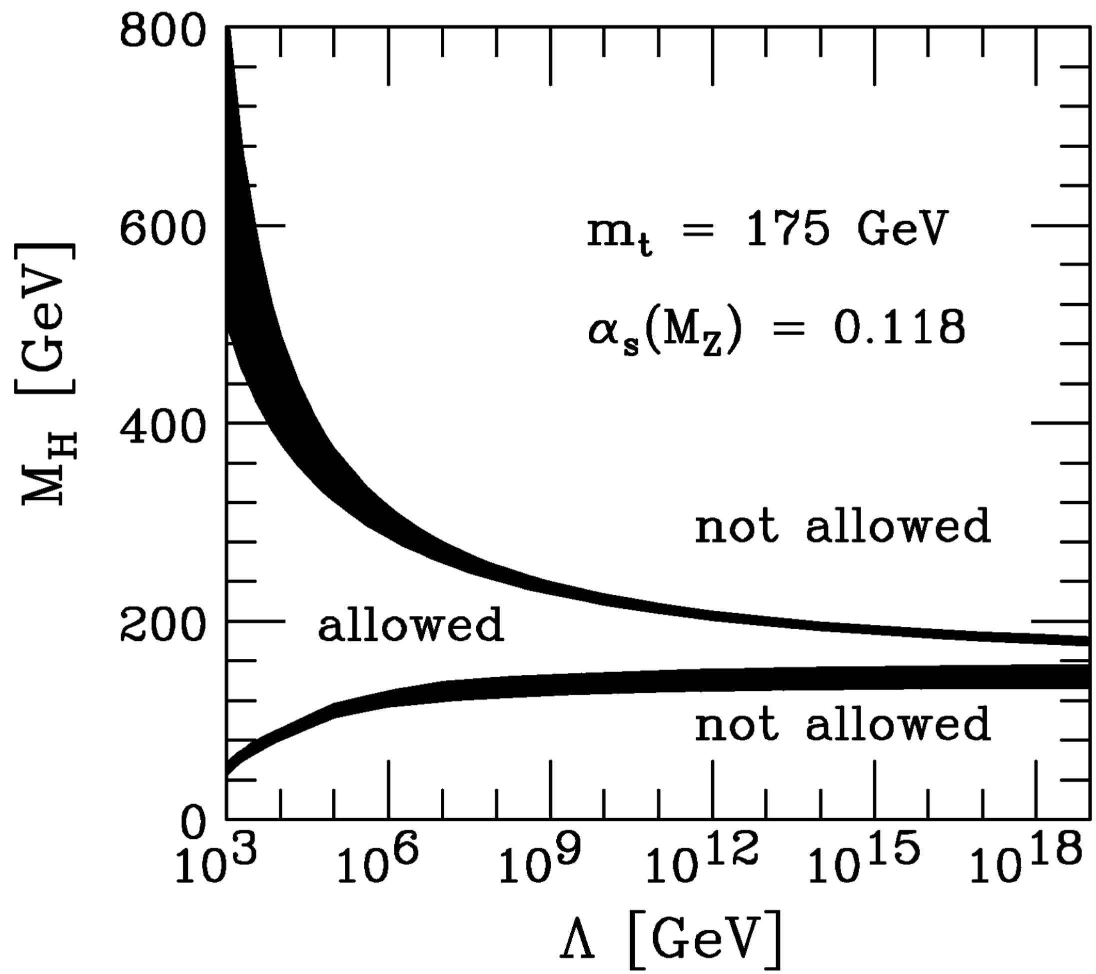


No a priori constraint on λ except vacuum stability ($\lambda > 0 \Rightarrow 0 < M_H < \infty$), but

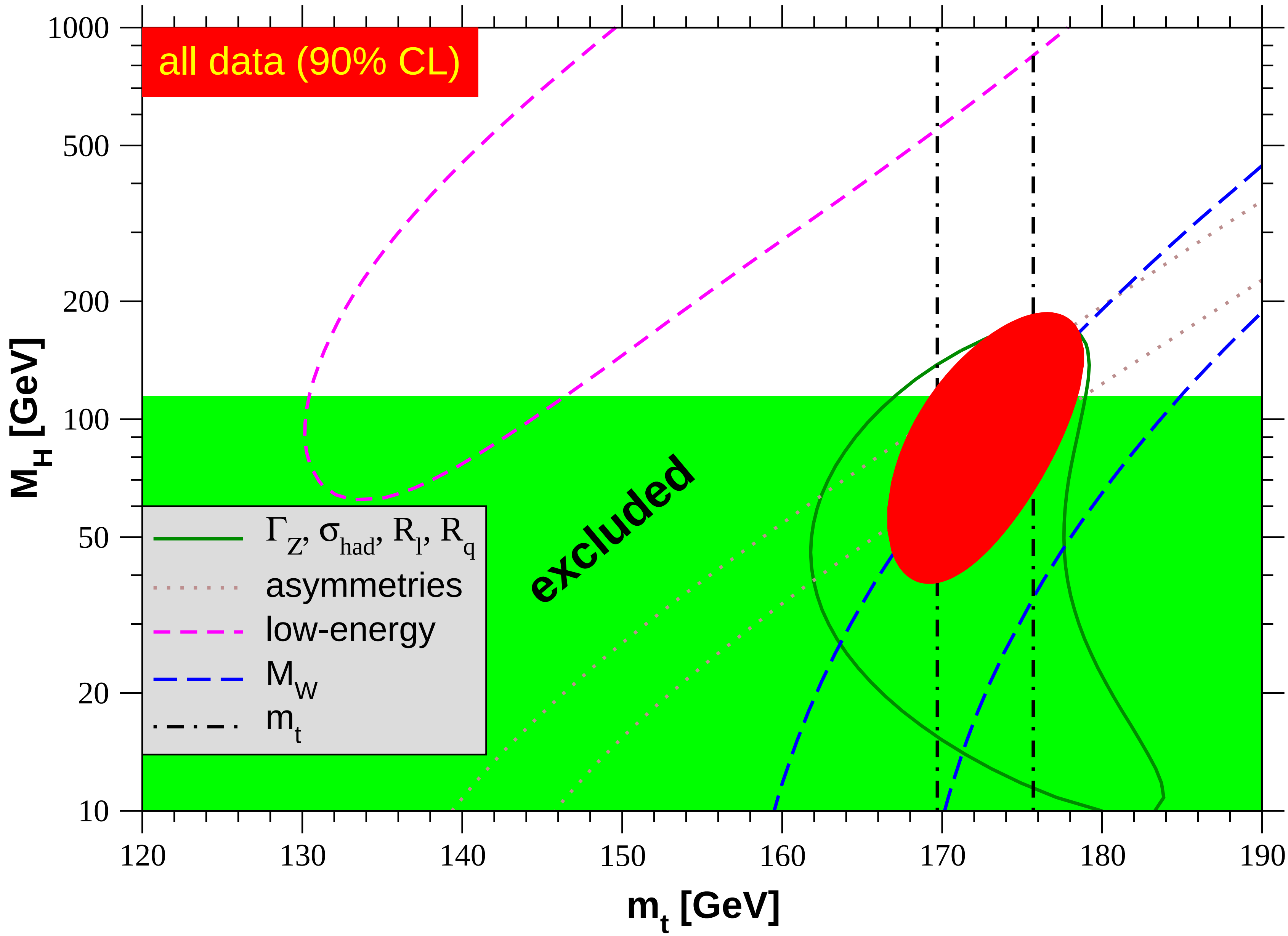
- t quark loops destabilize vacuum unless $M_H \gtrsim 130$ GeV (Depends on Λ. Doesn't hold in supersymmetry)
- Strong coupling for $\lambda \gtrsim 1 \Leftrightarrow M_H \gtrsim 1$ TeV
- Triviality: running λ should not diverge below scale Λ at which theory breaks down $\Rightarrow$

$$M_H < \begin{cases} O(200)\ \text{GeV}, & \Lambda \sim M_P = G_N^{-1/2} \sim 10^{19}\ \text{GeV} \\ O(750)\ \text{GeV}, & \Lambda \sim 2M_H \end{cases}$$

- Experimental bound (LEP 2), $e^+e^- \to Z^* \to ZH \Rightarrow M_H \gtrsim 114.5$ GeV at 95% cl (can evade with singlet or in MSSM)
- Hint of signal at 115 GeV
- Indirect (precision tests): $M_H < 189$ GeV, 95% cl
- MSSM: much of parameter space has standard-like Higgs with $M_H < 130$ GeV



Theoretical M_H limits, Hambye and Riesselmann, hep-ph/9708416


all data (90% CL)
excluded
Γ_Z, σ_{had}, R_l, R_q
asymmetries
low-energy
M_W
m_t
1000
500
200
100
50
20
10
120
130
140
150
160
170
180
190
M_H [GeV]
m_t [GeV]

Decays: $H \rightarrow \bar{b}b$ **dominates for** $M_H \lesssim 2M_W$ ($H \rightarrow W^+W^-$, ZZ **dominate when allowed because of larger gauge coupling)**

Production:

LEP: Higgstrahlung ($e^+e^- \rightarrow Z^* \rightarrow ZH$)

Tevatron, LHC: GG-fusion ($GG \rightarrow H$ **via top loop), WW fusion** ($WW \rightarrow H$), **or associated production** ($\bar{q}q \rightarrow WH$, ZH)

First term in V: vacuum energy

$$\langle 0|V|0\rangle = -\mu^4/4\lambda$$

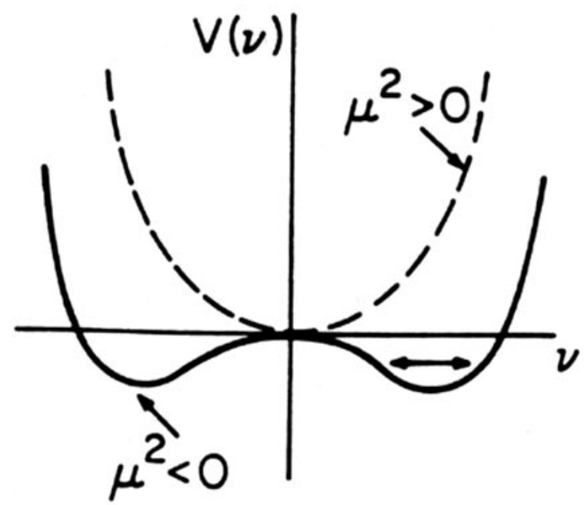
No effect on microscopic interactions, but gives *negative* contribution to cosmological constant

$$|\Lambda_{\rm SSB}| = 8\pi G_N|\langle 0|V|0$$

Require fine-tuned cancellation

$$\Lambda_{\rm cosm} = \Lambda_{\rm bare} + \Lambda_{\rm SSB}$$

Also, QCD contribution from SSB of global chiral symmetry



Yukaka Interactions

$$-\mathcal{L}_{\text{Yukawa}} \rightarrow \sum_{m,n=1}^{F} \bar{u}^0_{mL} \Gamma^u_{mn} \left(\frac{\nu + H}{\sqrt{2}} \right) u^0_{mR} + (d, e) \text{ terms } + \text{ H.C.}$$

$$= \bar{u}^0_L \left(M^u + h^u H \right) u^0_R + (d, e, \nu) \text{ terms } + \text{H.C.}$$

$u^0_L = \left(u^0_{1L} u^0_{2L} \cdots u^0_{FL} \right)^T$ is F-component column vector

M^u is $F \times F$ fermion mass matrix $M^u_{mn} = \Gamma^u_{mn} \nu / \sqrt{2}$ (need not be Hermitian, diagonal, symmetric, or even square)

$h^u = M^u / \nu = g M^u / 2 M_W$ is the Yukawa coupling matrix

Diagonalize M by separate unitary transformations A_L and A_R (($A_L = A_R$) for Hermitian M)

$$A_L^{u\dagger} M^u A_R^u = M_D^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

is diagonal matrix of physical masses of the charge $\frac{2}{3}$ quarks. Similarly

$$\begin{aligned} A_L^{d\dagger} M^d A_R^d &= M_D^d \\ A_L^{e\dagger} M^e A_R^e &= M_D^e \\ (A_L^{\nu\dagger} M^\nu A_R^\nu &= M_D^\nu) \end{aligned}$$

(may also be Majorana masses for ν_R)

Find A_L and A_R by diagonalizing Hermitian matrices $MM^\dagger$ and $M^\dagger M$, e.g., $A_L^\dagger M M^\dagger A_L = M_D^2$

Mass eigenstate fields

$$\begin{aligned}
u_L &= A_L^{u\dagger} u_L^0 = (u_L \; c_L \; t_L)^T \\
u_R &= A_R^{u\dagger} u_R^0 = (u_R \; c_R \; t_R)^T \\
d_{L,R} &= A_{L,R}^{d\dagger} d_{L,R}^0 = (d_{L,R} \;\; s_{L,R} \;\; b_{L,R})^T \\
e_{L,R} &= A_{L,R}^{e\dagger} e_{L,R}^0 = (e_{L,R} \;\; \mu_{L,R} \;\; \tau_{L,R})^T \\
\nu_{L,R} &= A_{L,R}^{\nu\dagger} \nu_{L,R}^0 = (\nu_{1L,R} \;\; \nu_{2L,R} \;\; \nu_{3L,R})^T
\end{aligned}$$

(For $m_\nu = 0$ or negligible, define $\nu_L = A_L^{e\dagger} \nu_L^0$, so that $\nu_i \equiv \nu_e, \; \nu_\mu, \; \nu_\tau$ are the weak interaction partners of the $e, \; \mu$, and τ.)

Typical estimates: $m_u = 1.5 - 4$ MeV, $m_d = 4 - 8$ MeV, $m_s = 80 - 130$ MeV, $m_c \sim 1.3$ GeV, $m_b \sim 4.2$ GeV, $m_t = 170.9 \pm 1.8$ GeV

Implications for global $SU(3)_L \times SU(3)_R$ of QCD

These are current quark masses. $M_i = m_i + M_{dyn}$, $M_{dyn} \sim \Lambda_{\overline{MS}} \sim 300$ **MeV from chiral condensate** $\langle 0|\bar{q}q|0\rangle \neq 0$

m_t **is pole mass; others, running masses at m or at 2 GeV**^{2}

Yukawa couplings of Higgs to fermions

$$L_{\text{Yukawa}} = \sum_i \bar{\psi}_i \left(-m_i - \frac{g m_i}{2 M_W} H \right) \psi_i$$

Coupling $g m_i / 2 M_W$ is flavor diagonal and small except t quark

$H \rightarrow \bar{b} b$ dominates for $M_H \lesssim 2 M_W$ ($H \rightarrow W^+ W^-$, ZZ dominate when allowed because of larger gauge coupling)

Flavor diagonal because only one doublet couples to fermions $\Rightarrow$ fermion mass and Yukawa matrices proportional

Often flavor changing Higgs couplings in extended models with two doublets coupling to same kind of fermion (*not* MSSM)

Stringent limits, e.g., tree-level Higgs contribution to $K_L - K_S$ mixing (loop in standard model) $\Rightarrow h_{\bar{d}s} / M_H < 10^{-6}\ \text{GeV}^{-1}$