Warped Conifolds and Their Applications to Cosmology

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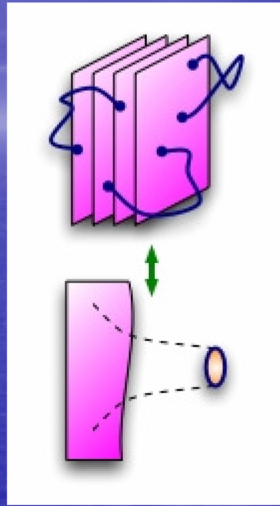
PiTP Seminar
IAS, July 19, 2007

From D-branes to AdS/CFT

- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved 10-d background of closed type IIB superstring theory (artwork by E.Imeroni)



$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

The AdS/CFT duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS_d space is a hyperboloid

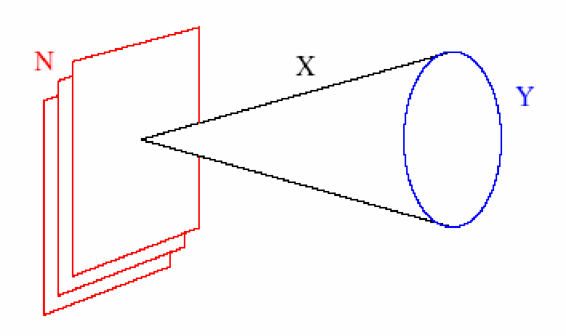
$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 \, .$$

- Its metric is $$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2}(dx^i)^2\right)$$

Cone-Brane Dualities

- To reduce the number of supersymmetries in AdS/CFT, we may place the stack of N D3-branes at the tip of a 6-d Ricci-flat cone X whose base is a 5-d Einstein space Y:

$$ds_X^2 = dr^2 + r^2 ds_Y^2$$



- Taking the near-horizon limit of the background created by the N D3-branes, we find the space AdS_5 x Y, with N units of RR 5-form flux, whose radius is given by

$$L^4 = \frac{\sqrt{\pi}\kappa N}{2\,\mathrm{Vol}(Y)} = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\mathrm{Vol}(Y)}$$

- This type IIB background is conjectured to be dual to the IR limit of the gauge theory on N D3-branes at the tip of the cone X.

Kachru, Silverstein; …

D3-branes on the Conifold

- The conifold is a Calabi-Yau 3-fold cone X described by the constraint $\sum_{a=1}^{4} z_a^2 = 0$ on 4 complex variables.
- Its base is called $T^{1,1}$; it has symmetry $SO(4) \sim SU(2)_A \times SU(2)_B$ that rotates the z's, and also $U(1)_R$: $z_a \to e^{i\theta} z_a$
- The Sasaki-Einstein metric on $T^{1,1}$ is

$$\mathrm{d}s^2_{T^{1,1}} = \frac{1}{9}\Big(\mathrm{d}\psi + \sum_{i=1}^{2} \cos\theta_i \mathrm{d}\phi_i\Big)^2 + \frac{1}{6}\sum_{i=1}^{2}\Big(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\Big)$$

  where $\theta_i \in [0,\pi], \phi_i \in [0,2\pi], \psi \in [0,4\pi]$

- The topology of $T^{1,1}$ is $S^2 \times S^3$.

- To `solve' the conifold constraint det Z = 0 we introduce another set of convenient coordinates:

$$Z = \begin{pmatrix} z^3 + iz^4 & z^1 - iz^2 \\ z^1 + iz^2 & -z^3 + iz^4 \end{pmatrix} = \begin{pmatrix} w_1 & w_3 \\ w_4 & w_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix}$$

- The action of global symmetries is

$$\begin{aligned} SU(2) \times SU(2) \text{ symmetry} \quad &: \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \to L \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \qquad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \to R \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ \text{R-symmetry} \quad &: \quad (a_i, b_j) \to e^{i\frac{\alpha}{2}} (a_i, b_j) \ , \end{aligned}$$

- There is a redundancy under

$$a_i \to \lambda\, a_i \quad , \quad b_j \to \frac{1}{\lambda}\, b_j \quad (\lambda \in \mathbf{C})$$

which is partly fixed by imposing

$$|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2 = 0$$

- It remains to quotient the space by the phase rotation $a \sim e^{i\alpha}a, b \sim e^{-i\alpha}b$ (in the gauge theory, this will have the meaning of the U(1) baryon number symmetry).
- In the IR gauge theory on D3-branes at the apex of the conifold, the coordinates a_1, a_2, b_1, b_2 are replaced by chiral superfields. For a single D3-brane it is necessary to introduce gauge group U(1) x U(1).

- The $\mathcal{N}$=1 SCFT on N D3-branes at the apex of the conifold has gauge group SU(N)xSU(N) coupled to bifundamental chiral superfields A_1, A_2, in $(\overline{\mathbf{N}}, \mathbf{N})$, and B_1, B_2 in $(\mathbf{N}, \overline{\mathbf{N}})$. IK, Witten
- The R-charge of each field is ½. This insures $U(1)_R$ anomaly cancellation.
- The unique $SU(2)_A \times SU(2)_B$ invariant, exactly marginal quartic superpotential is added:

$$W = \epsilon^{ij}\epsilon^{kl} \,\mathrm{tr}\, A_i B_k A_j B_l$$

- This theory also has a baryonic U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$, and a Z_2 symmetry which interchanges the A's with the B's and implements charge conjugation.

Resolution and Deformation

- There are two well-known Calabi-Yau blow-ups of the conifold singularity.
- The `deformation' replaces the constraint on the z-coordinates by

$$z_1^2+z_2^2+z_3^2+z_4^2=\epsilon^2$$

- This replaces the singularity by a finite 3-sphere.
- In the `small resolution' the singularity is replaced by a finite 2-sphere. This is implemented by modifying the constraint on the a and b variables

$$|b_1|^2 + |b_2|^2 - |a_1|^2 - |a_2|^2 = u^2$$

Warped Resolved Conifold

- In the gauge theory the resolution is achieved by giving VEV's to the chiral superfields. IK, Witten
- For example, we may give a VEV to only one of the four superfields: $B_2 = u 1_{N\times N}$
- The dual of such a gauge theory is a resolved conifold, which is warped by a stack of N D3-branes placed at the north pole of the blown up 2-sphere.

$$ds_{10}^2 = \sqrt{H^{-1}(y)}dx^\mu dx_\mu + \sqrt{H(y)}ds_6^2$$

- The explicit CY metric on the resolved conifold is Pando Zayas, Tseytlin

$$\kappa(r) = \frac{r^2 + 9u^2}{r^2 + 6u^2}$$

$$\begin{aligned} ds_6^2 &= \kappa^{-1}(r) dr^2 + \frac{1}{9}\kappa(r) r^2 \left(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\right)^2 \\ &\quad + \frac{1}{6} r^2 (d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{6}(r^2 + 6u^2)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) \end{aligned}$$

- The warp factor is the Green's function on this space IK, Murugan

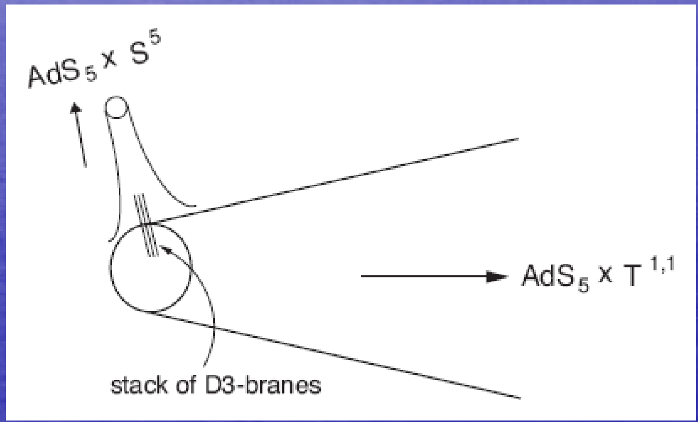
$$H(r, \theta_2) = L^4 \sum_{l=0}^{\infty} (2l+1) H_l^A(r) P_l(\cos\theta_2)$$

- The radial functions are hyper-geometric:

$$H_l^A(r) = \frac{2}{9u^2} \frac{C_\beta}{r^{2+2\beta}} \; {}_2F_1\left(\beta, 1+\beta; 1+2\beta; -\frac{9u^2}{r^2}\right)$$

$$C_\beta = \frac{(3u)^{2\beta}\Gamma(1+\beta)^2}{\Gamma(1+2\beta)}, \qquad \beta = \sqrt{1 + (3/2)l(l+1)}$$

- We get an explicit `localized' solution which describes SU(2)xU(1)xU(1) symmetric holographic RG flow to the $\mathcal{N}$=4 SU(N) SYM.



- A previously known `smeared' solution corresponds to taking just the l=0 harmonic. This solution is singular Pando Zayas, Tseytlin

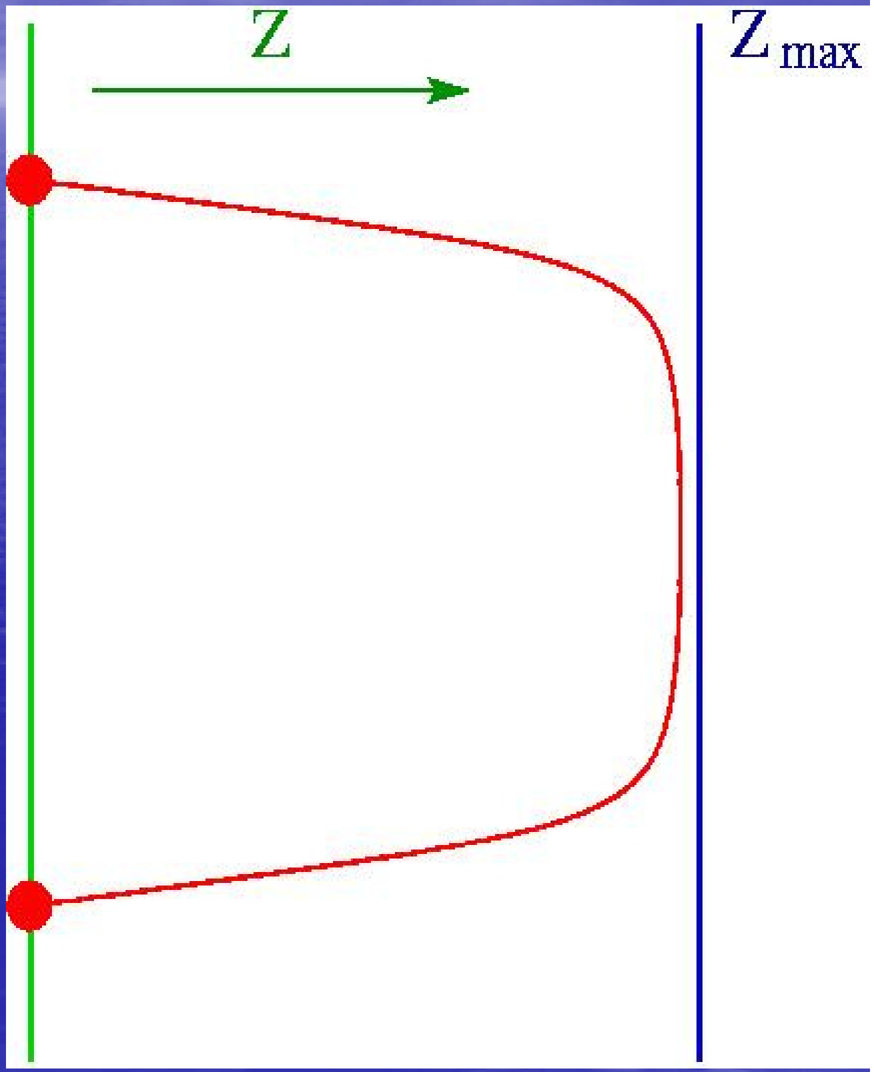
$$\frac{2}{9u^2r^2}+\frac{4\beta^2}{81u^4}\ln r+\mathcal{O}(1) \quad \xleftarrow{0\leftarrow r} \quad H_l^A(r) \quad \xrightarrow{r\rightarrow\infty} \quad \frac{2C_\beta}{9u^2r^{2+2\beta}}$$

String Theoretic Approaches to Confinement

- It is possible to generalize the AdS/CFT correspondence so that the quark-antiquark potential is linear at large distance and nearly logarithmic at small.
- A "cartoon" of the necessary metric is

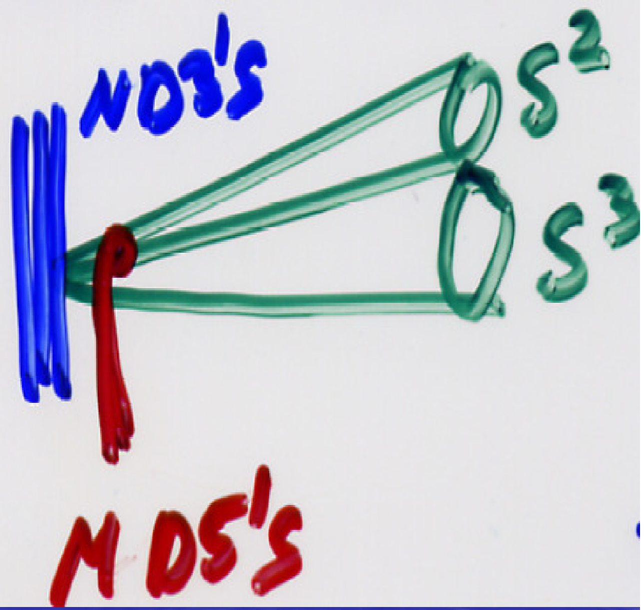
$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is $\frac{a^2(z_{\rm max})}{2\pi\alpha'}$



Warped Deformed Conifold

- A useful tool is to add to the N D3-branes M D5-branes wrapped over the S^2 at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)



$$ds^2_{10} = h^{-1/2}(t)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(t) ds^2_6$$

- ds^2_6 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The warp factor is finite at the `end of space' t=0, as required for the confinement: h(t)= $2^{-8/3}\, \gamma\, I(t)$

$$I(t) = \int_t^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh 2x - 2x)^{1/3} , \qquad \gamma = 2^{10/3} (g_s M \alpha')^2 \varepsilon^{-8/3}$$

- The standard warp factor a^2 , which measures the string tension, is identified with h(t) $^{-1/2}$ and is minimized at t=0. It blows up at large t (near the boundary).
- The dilaton is exactly constant due to the self-duality of the 3-form background

$$\star_6 G_3 = i G_3 , \qquad G_3 = F_3 - \frac{i}{g_s} H_3$$

- The radius-squared of the S^3 at $t=0$ is $\sim g_s M$ in string units.
- When $g_s M$ is large, the curvatures are small everywhere, and the SUGRA solution is reliable in `solving' this confining gauge theory.
- Even when $g_s M$ is small, the curvature gets small at large t (in the UV). The SUGRA description of the duality cascade is robust.

Log running of couplings in UV

- The large radius asymptotic solution is characterized by logarithmic deviations from $AdS_5 \times T^{1,1}$ IK, Tseytlin
- The near-AdS radial coordinate is $r \sim \varepsilon^{2/3} e^{t/3}$
- The NS-NS and R-R 2-form potentials:

$$F_3 = \frac{M\alpha'}{2}\omega_3 \ , \qquad B_2 = \frac{3g_s M\alpha'}{2}\omega_2 \ln(r/r_0)$$

$$\omega_2 = \frac{1}{2}(g^1 \wedge g^2 + g^3 \wedge g^4) = \frac{1}{2}(\sin\theta_1 d\theta_1 \wedge d\phi_1 - \sin\theta_2 d\theta_2 \wedge d\phi_2)$$

$$\omega_3 = \frac{1}{2} g^5 \wedge (g^1 \wedge g^2 + g^3 \wedge g^4)$$

- This translates into log running of the gauge couplings through

$$\frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} = \frac{\pi}{g_s e^{\Phi}} ,$$

$$\left[\frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2}\right] g_s e^{\Phi} = \frac{1}{2\pi\alpha'}\left(\int_{S^2} B_2\right) - \pi$$

$$\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2} = 6M \ln(r/r_s)$$

- This agrees with the β-functions in the gauge theory

$$\frac{d}{d\log(\Lambda/\mu)} \frac{8\pi^2}{g_1^2} = 3(N+M) - 2N(1-\gamma)$$

$$\frac{d}{d\log(\Lambda/\mu)} \frac{8\pi^2}{g_2^2} = 3N - 2(N+M)(1-\gamma)$$

$$\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2} = M \ln(\Lambda/\mu)[3 + 2(1-\gamma)]$$

- In the UV the anomalous dimension of operators $\mathrm{Tr} A_i B_j$ is $\gamma \sim -1/2$

- The warp factor deviates from the M=0 solution logarithmically.

$$h(r) = \frac{27\pi(\alpha')^2[g_s N + a(g_s M)^2 \ln(r/r_0) + a(g_s M)^2/4]}{4r^4}$$

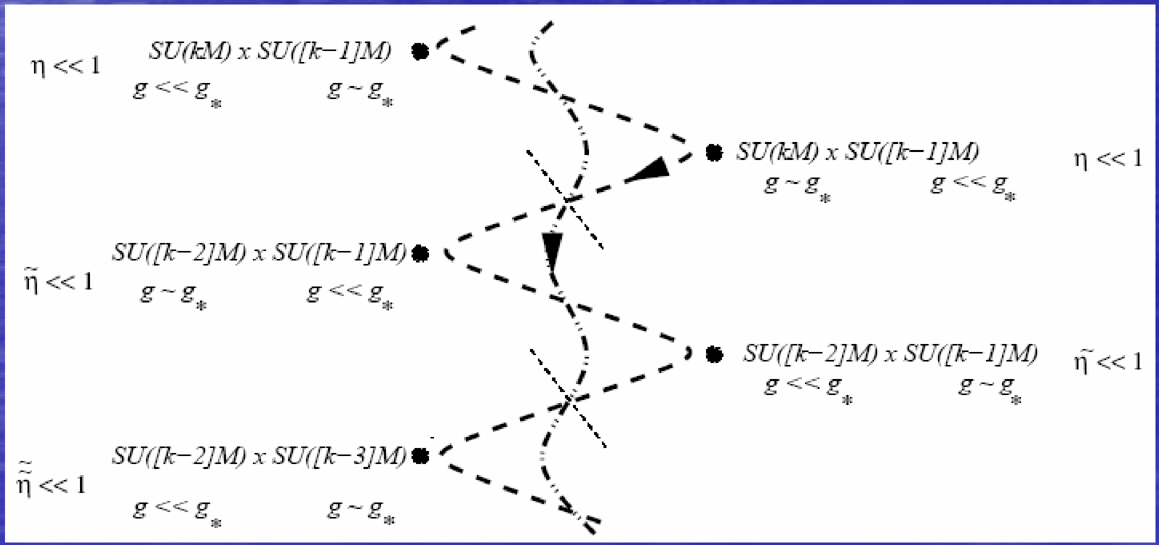
- Remarkably, the 5-form flux, dual to the number of colors, also changes logarithmically with the RG scale.

$$\tilde{F}_5 = \mathcal{F}_5 + \star\mathcal{F}_5 \ , \qquad \mathcal{F}_5 = 27\pi\alpha'^2 N_{eff}(r)\mathrm{vol}(\mathrm{T}^{1,1})$$

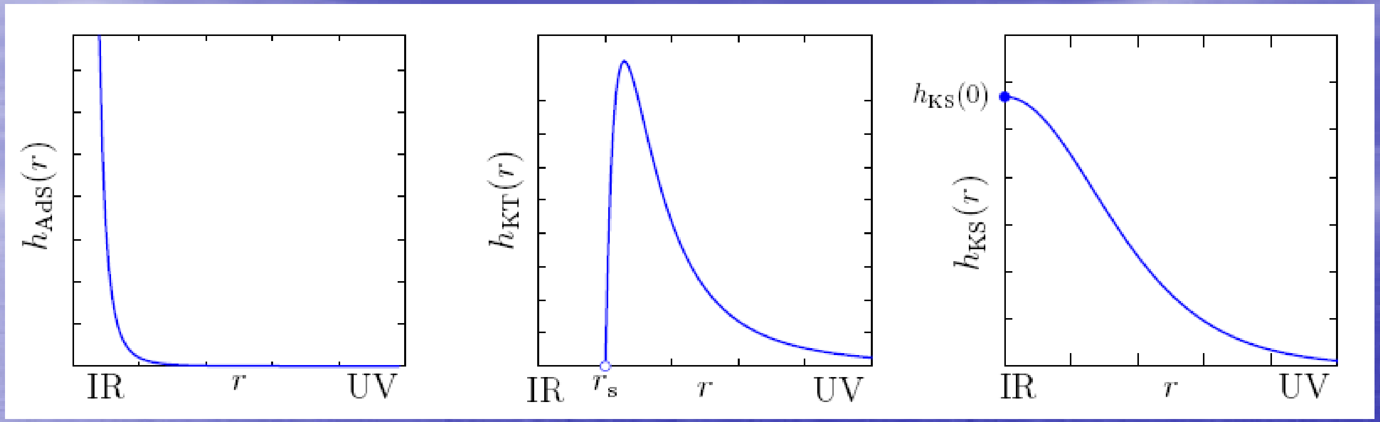
$$N_{eff}(r) = N + \frac{3}{2\pi} g_s M^2 \ln(r/r_0)$$

- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a duality cascade, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler

(diagram of RG flows from a review by M. Strassler)



- There is a scale where the SU(kM) coupling becomes very strong. This gauge group has $N_f=2(k-1)M$ flavors.
- To understand further RG flow, perform Seiberg duality to $SU(N_f-N_c)=SU((k-2)M)$.
- The resulting SU((k-1)M)xSU((k-2)M) theory has the same structure as the original one, but k is reduced by 1.
- The flow proceeds in a quasi-periodic fashion until k becomes O(1) in the IR.



- Comparison of warp factors in the AdS, warped conifold, and warped deformed conifold cases. The warped conifold (KT) solution, which has a naked singularity, should be interpreted as asymptotic (UV) approximation to the correct solution.

- The graph of quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD. The upper graph, from the recent Senior Thesis of V. Cvicek shows the string theory result for the warped deformed conifold.
- The lower graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm.

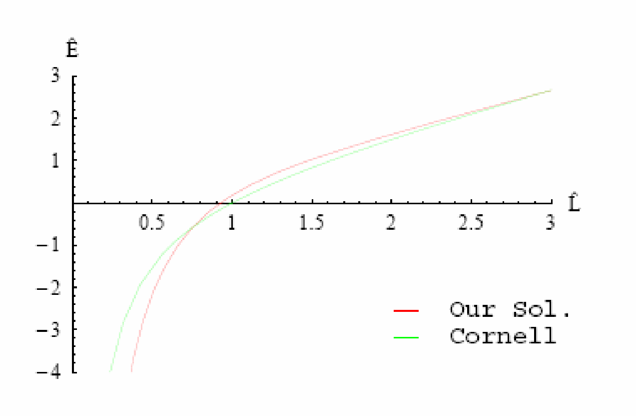
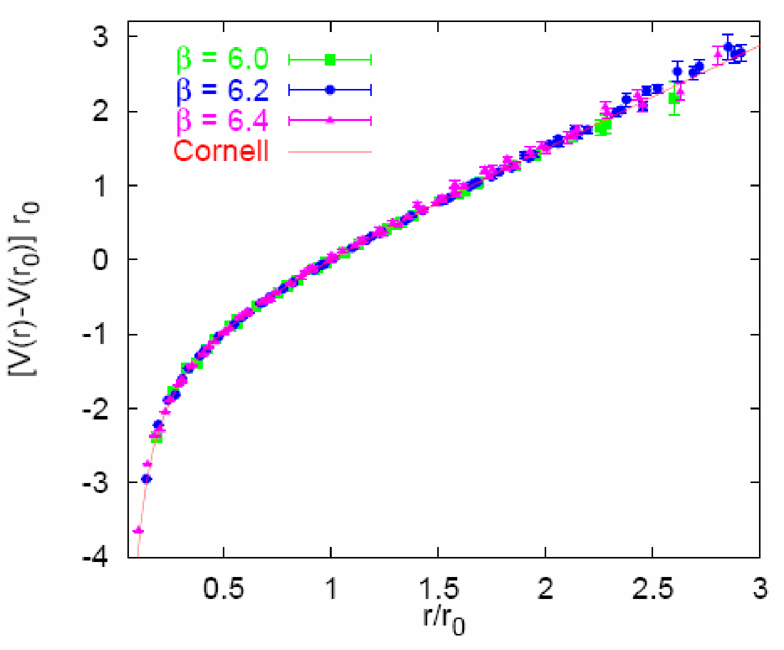


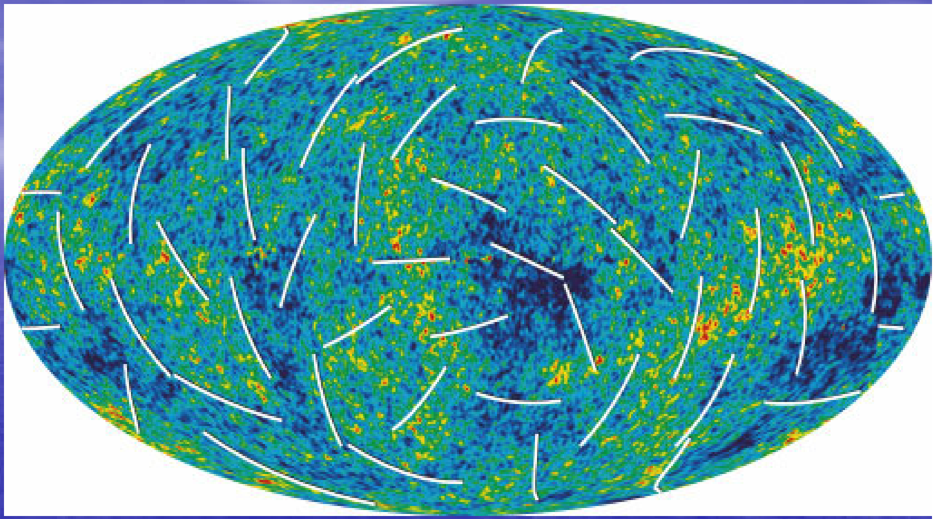
Figure 11: Comparison to the Cornell model



- All of this provides us with an exact solution of a 4-d large N confining supersymmetric gauge theory.
- This should be a good playground for testing various ideas about strongly coupled gauge theory.
- Some results on glueball spectra are already available, and further calculations are ongoing. Krasnitz; Caceres, Hernandez; Dymarsky, Melnikov; Berg, Haack, Muck
- Could there be applications of these models to new physics?

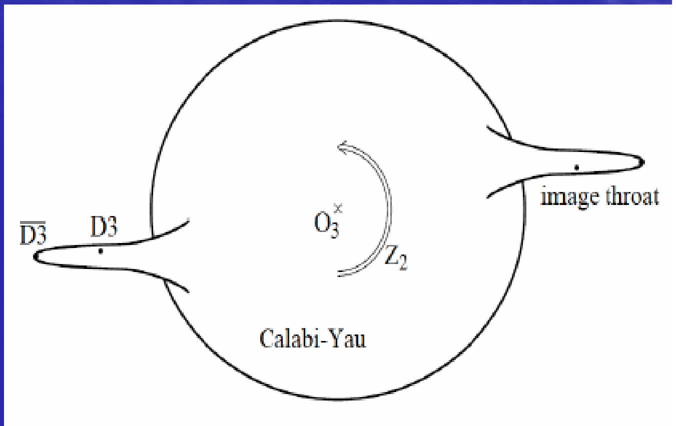
D-Brane Inflation

- The Inflationary Universe (Guth; Linde; Albrecht, Steinhardt) is a very promising idea for generating the CMB anisotropy spectrum observed by the WMAP.
- Finding slow-roll models has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, …



- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton.

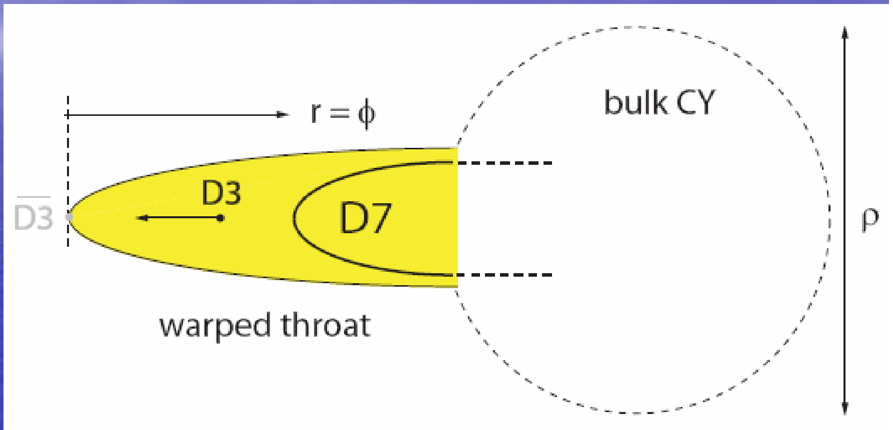
Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi



Cosmic Strings

- Brane anti-brane annihilation produces various kinds of long strings at the bottom of the throat: fundamental, D-strings and their (p,q) bound states. Sarangi, Tye; Copeland, Myers, Polchinski
- Their tension μ is `warped down' by the factor $h_0^{-1/2} \sim \exp(-4\pi K/3 g_s M)$ if K cascade steps take place in the throat. It is not hard to attain $G\mu < 10^{-7}$ dictated by present experimental bounds.
- The F-strings are dual to confining strings in the gauge theory; D-strings to certain solitonic strings. Gubser, IK, Herzog

Slow roll D-brane inflation?



- Effects of D7-branes and of compactification generically spoil the flatness of the potential. Non-perturbative effects introduce the KKLT-type superpotential $W = W_0 + A(X)e^{-a\rho}$ where X denotes the D3-brane position. In any warped throat D-brane inflation model, it is important to calculate A(X).

- The gauge theory on D7-branes wrapping a 4-cycle Σ_4 has coupling $\frac{1}{g^2} = \frac{V^w_{\Sigma_4}}{g_7^2} = \frac{T_3 V^w_{\Sigma_4}}{8\pi^2}$
- The non-perturbative superpotential $\propto \exp\left(-\frac{T_3 V^w_{\Sigma_4}}{N_{D7}}\right)$

  depends on the D3-brane location through the warped volume $V^w_{\Sigma_4} \equiv \int_{\Sigma_4} \mathrm{d}^4\xi \sqrt{g^{ind}}\, h(X)$
- In the throat approximation, the warp factor can be calculated and integrated over a 4-cycle explicitly. Baumann, Dymarsky, IK, Maldacena, McAllister, Murugan.
- If the D7-brane emedding is specified by $f(z_\alpha) = 0$

$$A(z_\alpha) = A_0 \left(\frac{f(z_\alpha)}{f(0)}\right)^{1/n}$$

- The F-term potential in $\mathcal{N}$=1 SUGRA is

$$V_F = e^{\kappa^2 \mathcal{K}} \left[D_\Sigma W \mathcal{K}^{\Sigma\overline{\Omega}} \overline{D_\Omega W} - 3\kappa^2 W\overline{W} \right]$$

$$\kappa^2 = M_P^{-2} \equiv 8\pi G$$

- Using the DeWolfe-Giddings Kaehler potential for the volume modulus ρ and the three D3-brane coordinates z_α on the conifold

$$\kappa^2 \mathcal{K}(\rho, \overline{\rho}, z_\alpha, \overline{z_\alpha}) = -3\log[\rho + \bar{\rho} - \gamma k(z_\alpha, \overline{z_\alpha})] \equiv -3\log U$$

$$k = \frac{3}{2}\left(\sum_{i=1}^{4} |z_i|^2\right)^{2/3} = \frac{3}{2} r^2$$

the F-term potential is found to be

Burgess, Cline, Dasgupta, Firouzjahi; Baumann, Dymarsky, IK, McAllister, Steinhardt

$$\frac{\kappa^2}{3U^2}\left[\left(\rho + \overline{\rho} + \gamma(k_\gamma k^{\gamma\overline{\delta}} k_{\overline{\delta}} - k)\right)|W_{,\rho}|^2 - 3(\overline{W} W_{,\rho} + c.c.) \right.$$
$$\left. + \underbrace{(k^{\alpha\overline{\delta}} k_{\overline{\delta}} \overline{W_{,\rho}} W_{,\alpha} + c.c.) + \frac{1}{\gamma} k^{\alpha\overline{\beta}} W_{,\alpha} \overline{W_{,\beta}}}_{\Delta V_F}\right].$$

- This generally gives Hubble-scale corrections to the inflaton potential, so fine-tuning is needed.
- The `uplifting' is accomplished by the D3 anti-D3 potential

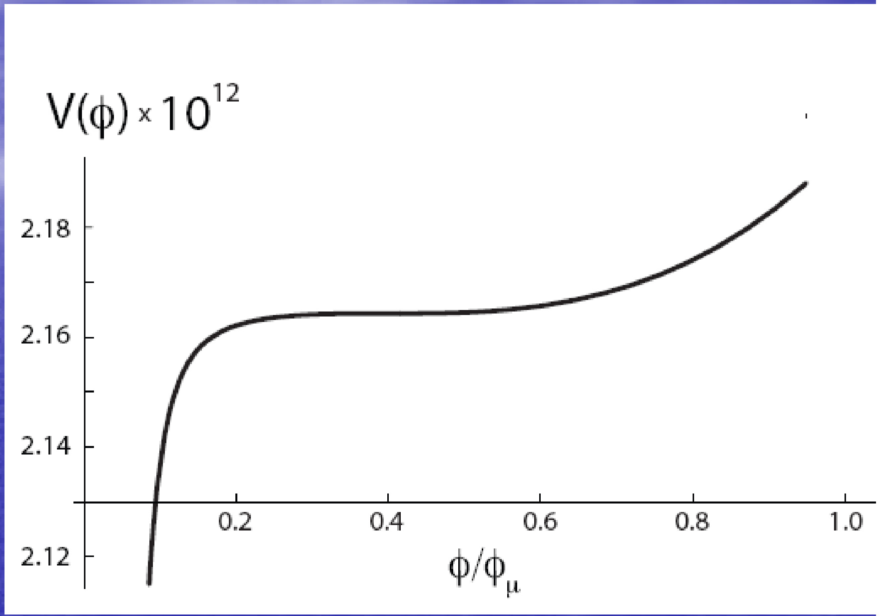
$$V_D = D(r)U^{-2}\,, \qquad D(r) \equiv D\left(1 - \frac{3D}{16\pi^2}\frac{1}{(T_3 r^2)^2}\right)$$

- For the KKLMMT model with anti-D3 brane, $D = 2T_3/h_0$ where h_0 is the warp factor at the bottom of the throat.
- We have studied a simple and symmetric Kuperstein embedding $z_1 = \mu$
- The stable trajectory for positive μ is

$$z_1 = -\frac{1}{\sqrt{2}}r^{3/2}$$

$$\phi \equiv r\sqrt{\tfrac{3}{2}T_3}$$

- The effective potential for the inflaton generically has a local maximum and minium. It can be fine-tuned to have an inflection point.



- Motion near the inflection point can produce enough e-folds of inflation.
- But cosmological predictions are very sensitive, e.g. $n_s - 1 = (2\eta - 6\epsilon)|_{\phi_{\rm CMB}} \approx 2\eta(\phi_{\rm CMB})$
- The sign of $\eta(\phi_{\rm CMB})$ depends on its position relative to inflection point. This is a `Delicate Universe.' Baumann, Dymarsky, IK, McAllister, Steinhardt

$$\eta \equiv M_P^2 \frac{\mathbb{V}_{,\phi\phi}}{\mathbb{V}}$$

Conclusions

- Placing D3-branes at the tip of a CY cone, such as the conifold, leads to AdS/CFT dualities with $\mathcal{N}=1$ SUSY. Symmetry breaking in the gauge theory produces warped resolved conifolds.
- Adding wrapped D5-branes at the apex produces a cascading confining gauge theory whose dual is the warped deformed conifold.
- This example of gauge/string duality gives a new geometrical view of such important phenomena as dimensional transmutation, chiral symmetry breaking, and quantum deformation of moduli space.

- Embedding gauge/string dualities into string compactifications offers new possibilities for modeling inflation and cosmic strings.
- Calculation of non-perturbative corrections to the inflaton potential is important for determining if these models can be fine-tuned to produce slow-roll inflation.