1. Consider an FRW metric

$$ds^2 = -dt^2 + a^2(t)dx^i dx^i \ .$$

Let

$$\frac{\dot a}{a} = \frac{1}{2t} \ , \quad 10^{10} M_P^{-1} < t < 10^{5.5} \text{ yr} \ ; \quad \frac{\dot a}{a} = \frac{2}{3t} \ , \quad 10^{5.5} \text{ yr} < t < 10^{10} \text{ yr} \ .$$

That is, we start after inflation (at a Hubble scale near the current upper bound), and we neglect the recent acceleration. Consider a string loop whose present length is 10^3 ly. When the physical string length l is less than the horizon length ($\sim t$) it is constant. When l is greater than t, the comoving length (i.e. in the coordinate x^i) is constant. Assuming that the string formed at the initial time $10^{10} M_P^{-1}$, what was the ratio of its length to the horizon length at that time? At what time and temperature did l equal the horizon length?

2. Consider an Abelian gauge field coupled to a complex scalar, with the usual symmetry breaking potential. Consider a configuration in which the fields and gradients are nonvanishing only in the 1-2 directions (as for a straight string). Show that when the parameters are such that the vector mass is equal to the Higgs mass, the energy density can be written as the sum of two squares, plus a total derivative. Show that for string boundary conditions ($\phi \to v e^{i\theta}$, $D_i \phi \to 0$ exponentially) you can evaluate the surface term [there is a sign choice such that it is positive]. Inserting the string ansatz $\phi = f(r) e^{i\theta}$, $A_\theta = a(r)$, show that the vanishing of the squares gives first order equations for f and a. These cannot be solved in closed form, but can be shown to have nonsingular solutions.

3. Consider a gauge theory with two $U(1)$s, the first broken by a Higgs of charge $(2, 0)$ and the second unbroken. There is another charged field, charges $(1, 1)$, and one can form composites out of arbitrary integer combinations of this and the Higgs. Find the spectrum of monopole charges, assuming that it saturates the Dirac quantization. Show that the string obtained from a singly-wound Higgs field is unstable, and show that it is quasi-Aharonov-Bohm.

Now suppose that the second charged field also has a vev. Strings are now indexed by two integers, the winding numbers for each Higgs. For each (n_1, n_2) classify the string as AB or local (determine the fluxes of the gauge fields by the requirement that the covariant derivatives fall exponentially at infinity), and independently check the stability.