Problem 3

Igor Klebanov

Consider warped metrics of the form

$$ds^2 = h^{-1/2}(r)dx_4^2 + h^{1/2}(r)(dr^2 + r^2 d\Omega_Y^2)$$

where Y is a compact space, such as the $T^{1,1}$.

We would like to calculate the quark-antiquark potential in the dual gauge theory. For this purpose, consider the quark and antiquark at a very large value of r (call it r_{max}), at the same point in Y, and separated by distance l along direction x within $R^{3,1}$.

a) Derive the equation for the shape of the string connecting the quark and the antiquark, $r(x)$.

b) For the AdS warp factor, $h = R^4/r^4$, calculate the energy of the string as a function of l. Show that, after a subtraction of a term linear in r_{max}, there is an attractive Coulombic potential.

c) Try to repeat this calculation for the cascade warp factor, $h = R^4 \ln(r/r_s)/r^4$. How does the quark antiquark potential $V(l)$ behave for small l.