Problems for the String Vacuum Construction Tutorial

1. (a) From the ten-dimensional string action for the gravitational and the Dp-brane part, determine the four-dimensional gauge coupling g_{YM} and Planck mass M_{Pl} in terms of the string coupling g_{st}, the string scale M_{st}, the volume of the compactified space V_6 and the volume of the compactified part of the Dp-brane world-volume V_{p-3}. (Assume that the Dp-brane fills the four-dimensional Minkowski space-time.)

(b) For a D6-brane, and $M_{st} = 1$ TeV, determine an average radius R_{p-3} and R_{trans} of V_{p-3} and V_{trans}, respectively. V_{trans} is the volume of the compactified space transverse to the D6-brane world-volume. Take: $g_{YM} = 0.5$, $M_{p]} = 10^{19}$ GeV, and $g_{st} = 1$. Repeat the exercise for a D3-brane.

2. Consider two intersecting D6-branes on a six-torus T^6 which is factorized as a product of three two-tori $T^5 = T^2 \times T^2 \times T^2$. The two D6-branes wrap three-cycles $[a]$ and $[b]$ which are chosen to be a product of one cycles $[a_i]$ and $[b_i]$ on each two-torus T_i^2. At a particular intersection of two D6-branes, the relative angles are $0 \leq \pi\nu_i^{ab} \leq \pi$ $(i = 1, 2, 3)$. Employing the conformal field theory (or level matching conditions), determine conditions on ν_i that determine the mass-square m^2 for the lowest lying scalar field to be $m^2 > 0$,$m^2 = 0$ and $m^2 < 0$, respectively.