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LECTURE BY

JUAN MALDACENA

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SCHOOL

"GIANT MAGNONS"

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- CONSIDER OPERATORS WITH LARGE CHARGE
- THEY ARE SIMPLER ~ "NEAR" BPS.
- WE CONSIDER PLANAR LIMIT

- SIMPLEST

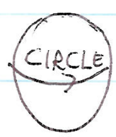
$$\mathcal{O}_J = \mathrm{Tr}[Z^J] \qquad Z = \phi^5 + i\phi^6 \; ; \; \phi^i, \; i=1,\dots,6$$

6- SCALARS.

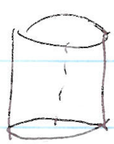
$\frac{1}{2}$ BPS. , $\Delta - J = 0$

$1 \ll J \ll N.$

- $\mathcal{O}_J \leftrightarrow$ MASSLESS MODE IN 10-d. $\to$ ANGULAR MOMENTUM ON S^5.

S^5 

$$ds^2 = R^2\left[\cos^2\theta \, d\varphi^2 + d\theta^2 + \sin^2\theta \, d\Omega_3^2\right]$$

AdS_5  SITS AT THE "CENTER"

$\varphi - t$ = CONST

- OTHER OPERATORS

$\mathrm{Tr}[\phi^1 Z^J] \to$ ALSO BPS.

$\hookrightarrow \approx J_{1,5+i6} \, \mathrm{Tr}[Z^{J+1}]$, $J_{i,j}$ = SO(6) GENERATOR.

$\uparrow$ ACTION OF BROKEN SYMMETRY.

$\Delta - J = 1$

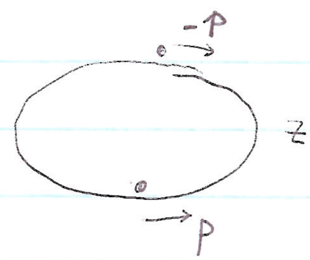
$\boxed{2}$ ACT TWICE

$$\sum_\ell \mathrm{Tr}[\phi^1 Z^\ell \phi^1 Z^{J-\ell}] \quad ; \quad \Delta - J = 2.$$

+ TERM WHEN Z-ϕ's ARE COINCIDENT

- WE CAN CONSIDER.

$$\sum_\ell e^{ip\ell}\, \mathrm{Tr}[\phi^1 Z^\ell \phi^1 Z^{J-\ell}]$$



- QUANTIZATION CONDITION FOR p:

  IGNORING INTERACTION $\rightarrow$ $pJ \sim 2\pi n$

  $\rightarrow$ IN GENERAL $\rightarrow$ INCLUDE IT $\rightarrow$ BETHE EQNS.

- DISPERSION RELATION

  $\varepsilon(p)$

  $\rightarrow$ INFINITE CHAIN

$$\sum_\ell e^{ip\ell} \ldots ZZ \cdot \underset{\uparrow \ell}{\phi} Z \cdots Z$$

- IT TURNS OUT THAT THE FORM OF $\varepsilon(p)$ IS CONSTRAINED BY THE SYMMETRY

FOR EXAMPLE $\mathcal{E}(P=0) = 1$.

- SYMMETRIES PRESERVED BY Z.
  - $\Delta - J = \mathcal{E}$
  - $\underset{\uparrow AdS_5}{SO(4)} \times \underset{\uparrow S^5}{SO(4)}$.
  - 16 SUSY

$\{Q, Q\} = \Delta - J + SO(4) + SO(4)$.

SPLITS INTO $SU(2|2) \times SU(2|2)$.

$SU(2|2) \to$ BOSONIC $\to$ - $U(1) \sim \mathbb{R} \to \mathcal{E}$.
- $SU(2) \times SU(2)$.
- 8 Fermionic.

- $\mathcal{E}$ = EQUAL IN THE TWO $SU(2|2)$ FACTORS.

- REPRESENTATIONS OF $SU(2|2)$

<u>ROUGHLY</u>:
8-FERMIONIC $\sim$ 4 CREATION & 4 ANIHILLATION
$\sim 2^4 \sim 16. \times$ (dimension of $SU(2)$ REPRESENT.)

- $\exists$ SHORT REPRESENTATIONS.

$\mathcal{E} = 2j_2$ $\quad (0, j_2)$ UNDER $SU(2) \times SU(2)$

FOR HIGHEST WEIGHT STATE.

$j_2 = \frac{1}{2} \to \mathcal{E} = 1 \to$ is OUR CASE.

$\to$ REPRES. $2_{\text{BOSONS}} + 2_{\text{FERMIONS}} = 4$ STATES.

$SU(2|2)^2 \to 4 \times 4 = 16$ STATES.

(4)

8 BOSONS $\rightarrow$ $\phi_1\ \phi_2\ \phi_3\ \phi_4\quad \partial_1 Z \ldots \partial_4 Z$

$+$ 8 FERMIONS.

- $P \neq 0 \rightarrow \mathcal{E}(p) > 1.$

$\rightarrow$ BUT # OF STATES CANNOT JUMP.

$\Rightarrow$ REPRESENT. MUST BE SOMEHOW BPS.

$\exists$ CENTRAL CHARGES.

. BEISERT

$$\{Q_{\frac{1}{2}}, Q_{-\frac{1}{2}}\} = \mathcal{E} + SU(2) + SU(2).$$

$$\left.\begin{aligned} \{Q_{\frac{1}{2}}, Q_{\frac{1}{2}}\} &= K_1 \\ \{Q_{-\frac{1}{2}}, Q_{-\frac{1}{2}}\} &= \underbrace{K_{-1}}_{\text{CHARGE } \pm 1 \text{ UNDER } J.} \end{aligned}\right\} SU(2) \text{ SINGLETS.}$$

$$K_1 \left(Z \ldots Z \underset{\overrightarrow{p}}{\phi} \ldots Z \right) \sim K_1(p)\ Z \ldots Z\,(\underset{\substack{\uparrow \\ \text{EXTRA } Z.}}{Z}\phi)\,Z \ldots$$

- CLOSED CHAIN $\rightarrow \sum p_i = 2\pi m \rightarrow K_{TOTAL}$ SHOULD VANISH.

$$\Rightarrow K_1 \sim \beta(e^{iP} - 1) \qquad K_{-1} \sim \beta^*(e^{-iP} - 1)$$

- FINAL ALGEBRA $\rightarrow$ $\mathcal{E}$, K_{-1}, K_1 $\rightsquigarrow$ like k^0, k^1, k^2 OF 2+1 POINCARE SUPER ALGEBRA.

8.6 PLANE WAVE LIMIT.

$\lambda \to$ LARGE

$J \to$ LARGE

$J/R^2 \sim J/\sqrt{\lambda}$ = FIXED

$p \cdot J = 2\pi m$ = FIXED

$$\mathcal{E}(p) \rightsquigarrow \mathcal{E}(m) = \sqrt{1 + \frac{\lambda}{J^2} m^2} = \sqrt{1 + \lambda' m^2}$$

- STRING SIDE → VERY NEAR THE MASSLESS GEODESIC.

→ EXPAND THE METRIC.

$$ds^2 = -2dx^+ dx^- - r^2 (dx^+)^2 + d\vec{r}^{\,2}$$

$$x^+ = t \qquad x^- = (t-\varphi)R^2$$

→ QUANTIZE STRING IN LIGHT CONE GAUGE

$$X^+ = \tau$$

→ MASSIVE SCALARS (AND FERMIONS) ON THE WORLDSHEET

→ REPRODUCE THE SPECTRUM

$\boxed{5}$

- SHORT REPRESENTATION.

$p=0 \to \varepsilon = 1 \quad k^0 = 1 \quad , k_1 + i k_2 = 0$

$p \neq 0$

$$\varepsilon = \sqrt{1 + \vec{k}^2} = \sqrt{1 + f(\lambda) \sin^2 p/2}$$

$f(\lambda) = \frac{\lambda}{\pi^2}$ AT WEAK COUPLING & STRONG COUPLING.

- OTHER THEORIES WITH $SU(2|2)$.

PLANE WAVE MATRIX MODES

$$SU(2|4) \xrightarrow[Z]{} SU(2|2) \times SU(2)_{Bos}$$

$\downarrow$

$f(\lambda)$ = NONTRIVIAL.

- SANTAMBROGIO - ZANON
- BEISERT STAUDACHER
- BERENSTEIN, CORREA, VAZQUEZ

- $\varepsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 p/2}$

- WE CAN NOW TAKE λ LARGE.

$$\varepsilon = \frac{\sqrt{\lambda}}{\pi} |\sin p/2|$$

$\to$ SHOULD CORRESPOND TO SOME STRING CONFIGURATION IN ADS

— o —

- WE WANT TO IDENTIFY THESE STATES ON THE STRING THEORY SIDE.

- LARGE J LIMIT
  - FIRST 't HOOFT LIMIT
  - $J \to \infty$
  - λ = FIXED
  - p = FIXED

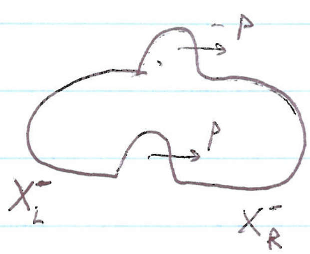
- ∞ CHAIN OF Z's

MASSLESS STRING STATE

- CONSIDER A STRING IN FLAT SPACE IN LIGHT CONE GAUGE

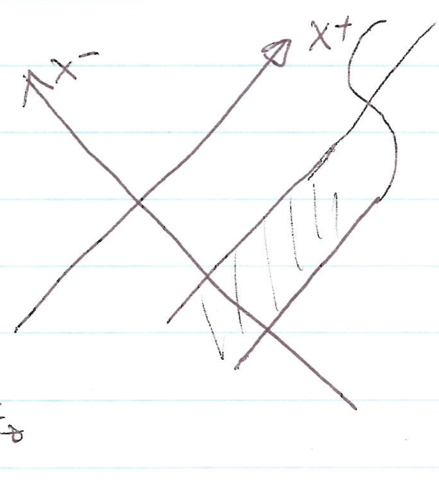
$X^+ = \tau.$

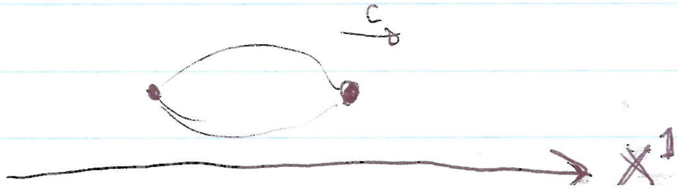
$P_-(\sigma) = 1$ = CONSTANT DENSITY OF P_-



2 LOCALIZED EXCITATIONS

$\Delta X^- = p$





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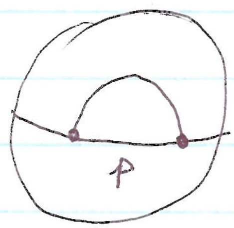
$\Delta \Psi = P$

↑ PERIODICITY

- SOLUTION
  - NAMBU ACTION $\to$ MINIMIZE $\mathcal{E}$ FOR FIXED P



$p < \pi$

$P = \pi$

- PROJECT S^5 ON A DISK

- CENTRAL CHARGES $\to$ IN SUGRA

$$[\delta_{\xi_1}, \delta_{\xi_2}] = \delta_{p^\mu} + \delta_{\Lambda}^{B^{\mu\nu}} + \cdot$$

$$\hookrightarrow \Lambda_\mu = \xi_1 \Gamma_\mu \xi_2 \;\cdots$$

- SAME ALGEBRA FROM THE SUGRA POINT OF VIEW.
- COLLECTIVE COORDINATE QUANTIZATION → SHOULD GIVE THE FULL MULTIPLET.

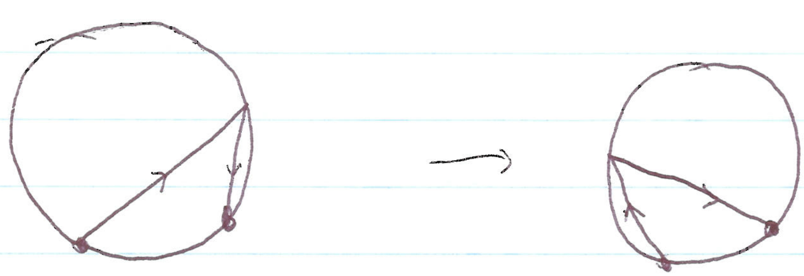
NE →

- ON THE GAUGE THEORY SIDE THESE ARE THE ELEMENTARY EXCITATIONS.
  - OTHERS → DECAY

$$ZZZZZ\,\partial^2 Z\,ZZ\ldots \longrightarrow \ldots ZZ\partial Z\,ZZZZ\,\partial Z\,ZZ\ldots$$

- THESE ARE STABLE ASYMPTOTIC STATES.
- ASYMPTOTIC SCATTERING

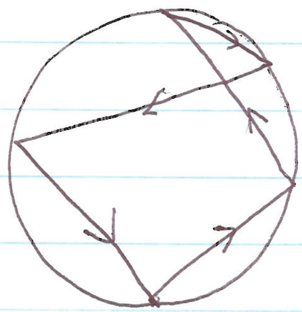
$$\overset{\xrightarrow{p_1}}{Z W_{A_1} Z}\cdots\cdots \overset{\xleftarrow{p_2}}{Z W_{B_2} Z} \rightarrow S^{CD}_{AB}(p_1,p_2)\ \underset{p_2}{Z W_{C_1} Z}\cdots \underset{p_1}{Z W_{D_2} Z}\cdots$$



$\boxed{8.5}$

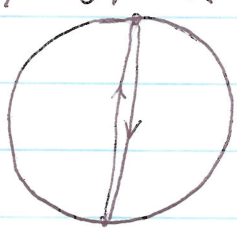
- TAKE J FINITE BUT LARGE.

  $\rightarrow$ CLOSED STRINGS.



ENERGY OF A PARTICULAR SPINNING

GUBSER
KLEBANOV
POLYAKOV.

- 

ROTATING
STRING.

$$\Delta - J \approx 2\frac{\sqrt{\lambda}}{\pi}$$

- SUPERPOSITION OF 2 MAGNONS WITH $p=\pi$.

$$E = 2\,\mathcal{E}(p=\pi) = 2\sqrt{1+\frac{\lambda}{\pi^2}}$$

- MATRIX STRUCTURE OF THE S-MATRIX IS CONSTRAINED BY SUPERSYMMETRY (IT IS IMPORTANT TO INCLUDE THE CENTRAL CHARGES)

$$\underset{\substack{\uparrow \\ 1-16}}{S_{AB}{}^{CD}} = S_0(p_1, p_2)\, \underset{\substack{\uparrow \\ \text{KNOWN MATRIX}}}{M_{AB}{}^{CD}}$$

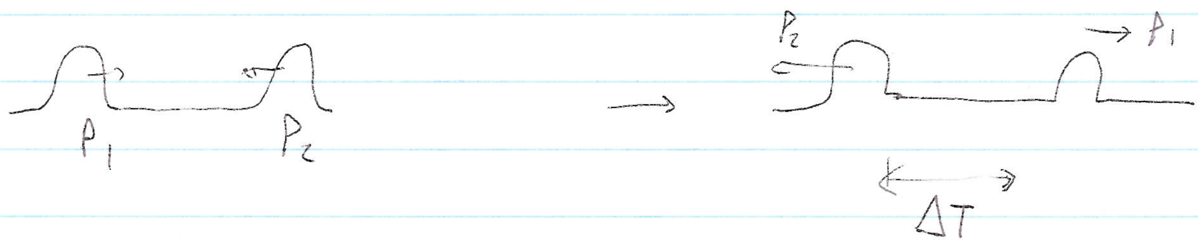
- FROM INTEGRABILITY WE EXPECT THAT THE FULL $n \rightarrow m$ PARTICLE S-MATRIX FACTORIZES IN TERMS OF $2 \rightarrow 2$ S-MATRICES (IN PARTICULAR $n = m$)

  ↳ NO PARTICLE CREATION

- $S_0(p_1, p_2, \lambda) \rightarrow$ SHOULD BE DETERMINED.

- CLASSICAL SCATTERING OF MAGNONS.

→ CLASSICAL SOLUTION REPRESENTING $2 \to 2$ SCATTERING



ΔT = TIME DELAY

$$S = e^{i\delta} \quad ; \quad \frac{\partial \delta}{\partial E_1} = \Delta T(E_1, E_2)$$

$$\delta(p_1, p_2) = -\frac{\sqrt{\lambda}}{\pi}\left(\cos\frac{p_1}{2} - \cos\frac{p_2}{2}\right)\log\left[\frac{\sin^2\frac{p_1-p_2}{4}}{\sin^2\frac{p_1+p_3}{4}}\right]$$

- AGREES WITH A FORM PROPOSED BY ARYUTUNOV-FROLOV & STAUDACHER.

$\boxed{11}$

- BOUND STATES

  → TIME DEPENDENT LOCALIZED SOLUTIONS.

  → SEMICLASSICAL QUANTIZATION

$$\left.\begin{array}{l}\text{SEMICLASSICAL QUANTIZATION}\\ P_1 = p + iq \qquad P_2 = p - iq\end{array}\right\} \rightarrow \text{DETERMINE } q_m$$

$$\mathcal{E}_B(p) = \mathcal{E}(p+iq) + \mathcal{E}(p-iq) = \sqrt{m^2 + \frac{4\lambda}{\pi^2}\sin^2 p/2}$$

→ OTHER BPS BOUND STATES.

- BPS MASS FORMULA FOR $SU(2|2)$

$$\mathcal{E} = 2j_2 \qquad (p=0)$$

- $\mathcal{E} = \sqrt{(2j_2)^2 + \sin^2 p/2}$

  $\hookrightarrow J_2$

→ DEPEND ON POLES IN THE MATRIX STRUCTURE OF THE S-MATRIX

→ PRESENT ALREADY IN THE XXX SPIN CHAIN.

→ BOUND STATE OF J_2 MAGNONS.

→ CORRESPONDING CLASSICAL SOLUTIONS.

$\boxed{12}$ WHAT IS $S_0(p_1, p_2)$?

- WE KNOW IT UP TO 3 LOOPS FOR SMALL λ
- WE KNOW THE LEADING & SUBLEADING CORRECTIONS AT STRONG COUPLING
- IT IS CONJECTURED TO OBEY JANIK'S EQUATION: CROSSING SYMMETRY.

CROSSING: INCOMING PARTICLE WITH p^μ

$\rightarrow$ OUTGOING ANTIPARTICLE WITH $-p^\mu$

- INVOLVES ANALYTIC CONTINUATION OF THE AMPLITUDES.
- DISPERSION RELATION IN OUR CASE

$$\varepsilon = \pm\sqrt{1 + \frac{\lambda}{\pi^2}\sin^2 p/2}$$

CROSSING: (JANIK)

$$S_0(\bar{1}, 2)\, S_0(1, 2) = f(1,2)$$

: KNOWN FUNCTION RELATED TO THE MATRIX STRUCTURE OF THE S-MATRIX.

$\bar{1}$ $\uparrow$ 2nd SOLUTION. $\left\{ \begin{array}{l} \varepsilon \to -\varepsilon \\ p \to -p \end{array} \right.$

$$S_0(1,2)\, S_0(2,1) = 1$$

- ANALYTIC STRUCTURE ?