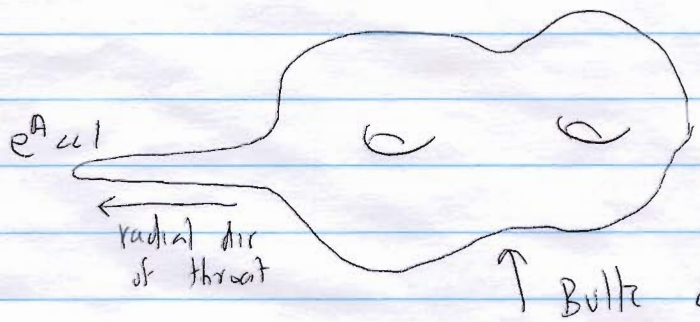
IAS Lecture III (Kachru)

Today, we'll discuss flux vacua where the flux → **significant** warping:

$$ds_{10}^2 = e^{2A(y)} \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn}(y)\, dy^m dy^n$$

$$e^{2A}\Big|_{\text{bulk}} \sim \mathcal{O}(1)$$

$$e^{2A}\Big|_{\substack{\text{end of}\\ \text{warped throat}}} \ll \underline{1}$$



One way to think of such models uses the gravity duals of the "cone-branes" that Igor discussed -- the IR scale ($\leftrightarrow e^A \ll 1$) can be the confinement scale of the dual.

Potential applications:

- Can try to explain $M_{Higgs} \ll M_{pl}$ this way (Randall-Sundrum scenarios).
- Can try to find novel $SU\!\!\!/SY$ states at IR end → explain why $M_{SU\!\!\!/SY} \ll M_{pl}$.
- Can use warping to explain why inflationary $H \ll M_{Pl}$, or why cosmic strings have $G\mu \ll 1$.

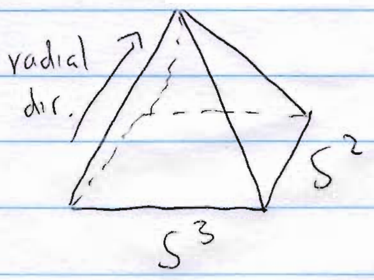
I'll talk about today and a bit about warped inflation in lecture IV.

I. <u>The Conifold</u> (c.f. Klebanov-Strassler, GKP)

A simple but interesting singular non-compact CY_3 is given by

$$\sum_{i=1}^{4} z_i^2 = 0 \quad \subset \mathbb{C}^4$$

Picture of geometry:



"Cone over $S^3 \times S^2$"

Conifold singularities arise at codim 1 in many CY moduli spaces (compact). There is a universal behavior of the periods:

$$\int_A \Omega = z \qquad A \text{ cycle} \simeq S^3$$

$$\int_B \Omega = \frac{z}{2\pi i}\log(z) + \underset{\text{(non-universal)}}{\text{regular}} \qquad B \simeq S^2 \times \text{radial dir.}$$

where $z \to 0$ is the conifold point. Finite z

$\rightarrow$ $\text{Vol}(S^3) \sim z$ at the "tip".

Consider turning on fluxes:

$$\int_A F_3 = M$$

$$\int_B H_3 = -K$$

Then the effective superpotential

$$W = \int (F - \tau H) \wedge \Omega$$

$$= -K\tau z + \frac{M}{2\pi i} z \ln z + \cdots$$

Assume $\frac{K}{g_s}$ is (slightly) large. Then

$$D_z W \sim \frac{M}{2\pi i} \log(z) - i\frac{K}{g_s} + \cdots\cdots$$

negligible if $z \ll 1$, compared to 1^{st} 2 terms

So $D_z W = 0 \Rightarrow$

$$\boxed{z \sim e^{-\frac{2\pi K}{g_s M}}}$$

} Actually M vacua differing by phase

The fluxes $\rightarrow$ a vacuum with a <u>slightly</u> (warped) deformed conifold. [See also <u>Vafa's</u> geometric transition approach].

<u>Two remarks</u>:

i) In compact M_6, τ is dynamical. Naively one would find above that $D_\tau W = 0$ cannot be satisfied at exponentially small z. However, any compact CY has at least one additional A', B' pair of 3-cycles $\rightarrow$ putting

$$\int_{B'} H_3 = -K',$$

one can still find z exponentially small, and fix τ. [See p. 18 of hep-th/0105097].

ii) This is the IR physics of the gravity side of a Non AdS / Non CFT duality. The system with fluxes M, K is dual to an

$$SU(N+M) \times SU(N) \quad \mathcal{N}=1 \text{ QFT}$$

with $N = KM$. This theory has an RG cascade described in Klebanov's lectures. Our W_{flux} is the Veneziano-Yankielowicz superpotential of the pure $SU(M)$ that arises in the IR.

$$\begin{matrix}\text{Warp factor in} \\ \text{IIB sol'n @ tip}\end{matrix} \simeq e^{A}\big|_{tip} = e^{-\frac{2\pi K}{3 g_s M}}$$

II. The $\overline{D3}$ probe of KS

See hepth 0112197
0403123

A simple idea that can → a model with exponentially small $S\cancel{USY}$ scale, is to add $p \ll M$ $\overline{D3}$ branes to the KS geometry. What happens?

The (S-dual frame) $\overline{D3}$ action is:

We'll justify using this @ the end

$$S_{\overline{D3}} = -\frac{\mu_3}{g_s}\int d^4x \, \mathrm{Tr}\sqrt{\det(G_{\parallel})\det(Q)}$$

$$- \mu_3 \int \mathrm{Tr}\left(2\pi i \, i_\Phi i_\Phi B_6 + C_4\right)$$

where

G_{11} = pullback of induced metric along $\overline{D3}$

μ_3 = brane tension

i_Φ is interior derivative

$$i_\Phi i_\Phi B_6 = \Phi^n \Phi^m B_{mnpqrs} \frac{dy^p \wedge \cdots \wedge dy^s}{4!}$$

$$Q^i{}_j = \delta^i{}_j + \frac{2\pi i}{g_s} [\Phi^i, \Phi^k] (G_{kj} + g_s C_{kj})$$

Clearly, Φ parametrizes the $\overline{D3}$ location.

So, what happens?

<u>Step I</u>: The non-commutator terms in the ISD flux background →

$$- \frac{\mu_3}{g_s} \int d^4x \sqrt{g_4} \, e^{4A} \, \mathrm{tr}\left(2 + \frac{1}{2} e^{-2A} \partial_\mu \Phi^i \partial^\mu \Phi^j g_{ij}\right)$$

The leading potential

$$V(r) \sim 2e^{4A(r)}$$

arises by adding the B-I and CS terms (which would <u>cancel</u> for a D3). So $\exists$ a radial force

$$F_r(r) = -2\frac{\mu_3}{g_s}\partial_r e^{4A}(r)$$

This pulls the $\overline{D3}$s to the region with the smallest value of the warp factor -- the tip of the deformed conifold.

<u>Step II</u>: So, lets analyze p coincident branes at the tip. The metric is (following KS)

$$ds^2 \simeq \left(e^{-\frac{2\pi k}{3g_sM}}\right)^2 dx_\mu dx^\mu + R^2 d\Omega_3^2$$

$$+ \left(dr^2 + r^2 d\tilde{\Omega}_2^2\right) \cdot b_0^2 \leftarrow \text{\# of } \mathcal{O}(1)$$

So at $r=0$, it is well approximated by an S^3 of radius

$$R^2 \simeq g_s M$$

where M = # units of F_3 Flux. We roughly expect

$$\int_{S^3} F_3 = M \Rightarrow F_{mnp} = f\,\epsilon_{mnp}$$

warped volume element on S^3

$$f \simeq \frac{2}{\sqrt{g_s^3 M}}$$

Myers effect now occurs -- $\overline{D3}$s blow up into a "fuzzy" 5-brane.

Large $S^3 \Rightarrow C_{kj} \sim \frac{2\pi}{3} F_{kjl}\Phi^l$

$$G_{kj} \sim \delta_{kj} \Rightarrow$$

$$Q^i{}_j = \delta^i{}_j + \frac{2\pi i}{g_s}[\Phi^i,\Phi_j] + i\frac{4\pi^2}{3}F_{kjl}[\Phi^i,\Phi^k]\Phi^l$$

So

$$\mathrm{Tr}\left(\sqrt{\det Q}\right) \simeq p - i\frac{2\pi^2}{3} F_{kjl}\,\mathrm{Tr}\left([\Phi^k,\Phi^j]\Phi^l\right) - \frac{\pi^2}{g_s^2}\,\mathrm{Tr}\left([\Phi^i,\Phi^j]^2\right)$$

using ISD etc, $dB_6 = \frac{1}{g_s^2} *_{10} H_3 = -\frac{1}{g_s} dV_4 \wedge F_3$ (vol of $\mathbb{R}^4$)

The "B_6 term" would cancel the cubic potential for a D3; here instead it adds $\rightarrow$

$$V_{eff}(\Phi) = \frac{\mu_3}{g_s}\left[p - i\frac{4\pi^2 f}{3}\epsilon_{kjl}\,\mathrm{Tr}\left([\Phi^k,\Phi^j]\right.\right.$$
$$\left. - \frac{\pi^2}{g_s^2}\,\mathrm{Tr}\left([\Phi^i,\Phi^j]^2\right) + \cdots\right]$$

really $\times\ e^{-\frac{8\pi K}{3 g_s M}}$ due to $\sqrt{G_{11}}$

Demanding $\frac{\partial V_{eff}}{\partial \Phi^i} = 0 \rightarrow$

$$(\ddot\smile)\quad 0 = [[\Phi^i,\Phi^j],\Phi^j] - i g_s^2 f \epsilon_{ijk}[\Phi^j,\Phi^k]$$

To solve $(\ddot\smile)$, notice that if you take constant matrices Φ^i with

$$[\Phi^i,\Phi^j] = -i g_s^2 f \epsilon_{ijk}\Phi^k \quad (\checkmark)$$

$(\ddot\smile)$ is satisfied.

But up to re-scaling, (V) is just the commutation relation for a $p \times p$ matrix rep of $SU(2)$

$$[J^i, J^j] = 2i\,\epsilon_{ijk} J^k$$

The "landscape" of such p dim'l reps is complicated. The energetically preferred sol'n is the p dim'l irrep, for which

$$V_{eff} \simeq \frac{\mu_3 p}{g_s} \left\{ 1 - \frac{8\pi^2}{3} \frac{(p^2-1)}{M^2} \frac{1}{b_0^{12}} \right\}$$

with b_0 some #.

The <u>radius</u> of the fuzzy S^2 the branes "blow up" into is

$$R^2 = \frac{4\pi^2 (p^2-1)}{b_0^8 M^2} \times R_0^2 \leftarrow S^3 \text{ radius}$$

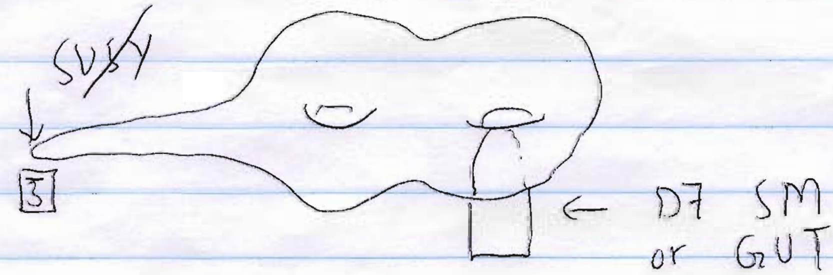
$\Rightarrow$ only trust for $p \ll M$; larger $p \Rightarrow$

a classical instability.

Conclusion: $p \ll M$ $\overline{D3}$s at end of KS (at $\mathbb{Z}_2$ symmetric pt of "baryonic branch") yield a metastable, $\cancel{SUSY}$ state with exponentially small $\cancel{SUSY}$ due to warping.

Comments:

1) May be interesting scenarios



(c.f. 'Mirage mediation' of Choi, Nilles, Nomura, Kitano, ...).

Use warping for $\cancel{SUSY}$, not to explain M_{Higgs} directly ala RS.

2) We used S-dual DBI action for intuitive ease. 'Correct' (obviously appliable) analysis using 5-brane on $S^2 \subset S^3$ w/ induced $\overline{D3}$ charge $\rightarrow$ same conclusions. Apparently

$$\left(\frac{p}{M}\right)_{crit} \simeq \frac{1}{10} \;; \quad \text{larger } \frac{p}{M} \rightarrow \text{instability.}$$

3) The decay process where

$$\int_A F_3 = M \qquad \int_B H_3 = k \qquad p \quad \overline{D3}$$

$$\downarrow$$

$$\int_A F_3 = M-1 \qquad \int_B H_3 = k \qquad M-p \quad D3s \qquad \underline{\underline{\text{SUSY}}}$$

occurs with exponential slowness. Suggests the $\overline{D3}$ states may be metastable states of a large 't Hooft coupling $\mathcal{N}=1$ theory. Similar to ISS, but AdS/CFT instead of

Seiberg duality.

4) In the full moduli space of 'resolved warped deformed conifold' geometries, shown in Dymarsky/Klebanov/Seiberg sect. 14 that $\overline{D3}$s do prefer to sit at KS sol'n; "baryonic branch" modulus gets positive mass in their presence.

Would be very interesting to study more general such examples in other (deformed) conical throats.