Dynamical SUSY Breaking and Meta-Stable Vacua

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Dynamical Supersymmetry Breaking:

- No explicit breaking: $\mathcal{L} = \mathcal{L}_{SUSY}$
- Vacuum spontaneously breaks SUSY.
- SUSY breaking related to some dynamical scale

$$\Lambda = M_{cutoff} e^{-c/g(M_{cutoff})^2} \ll M_{cutoff}$$

Can naturally get hierarchies (Witten).

DSB looks **non-generic**

- Witten index: All SUSY gauge theories with massive, vector-like matter have $Tr(-1)^F \neq 0$ SUSY vacua. So for broken SUSY, need a chiral* gauge theory.
- SUSY breaking is related to breaking global symmetries (Affleck, Dine and Seiberg).
- SUSY breaking requires an R-symmetry (or non-generic superpotential) (Nelson and Seiberg).

*Some vector-like exceptions, with massless matter (KI and Thomas; Izawa and Yanagida).

DSB is hard to analyze

Most of our techniques to analyze SUSY theories are based on holomorphy/chirality/BPS.

But for a detailed analysis of SUSY breaking we need to know the Kahler potential, which is hard to analyze and control.

Since the vacuum is not SUSY, dependence on parameters might not be smooth – can be phase transitions.

“Simplest” example of calculable DSB

(Affleck, Dine, Seiberg)

$SU(3) \times SU(2)$ gauge theory with matter:

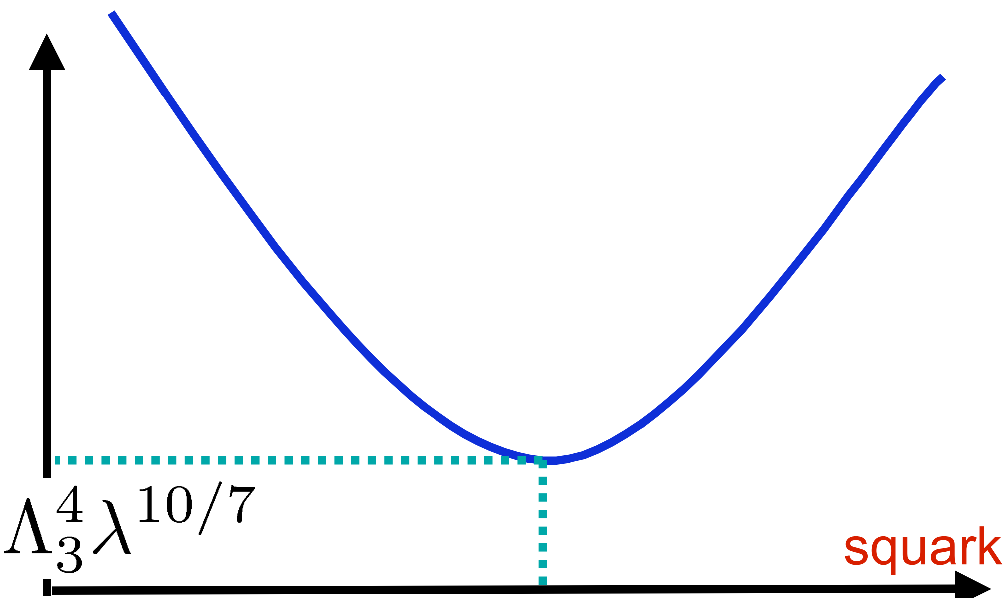
	$SU(3)$	$SU(2)$	$[U(1)_R]$
Q	3	2	-1
$\widetilde{u}_{i=1,2}$	$\bar{3}$	1	0
L	1	2	3

and superpotential

$$W_{tree} = \lambda Q \bar{u}_1 L$$

$\lambda \ll 1$ ensures large vevs (weak coupling). Therefore the theory is calculable:

$$K_{eff} \approx K_{canonical}$$



$$V_{DSB} \sim \Lambda_3^4 \lambda^{10/7}$$

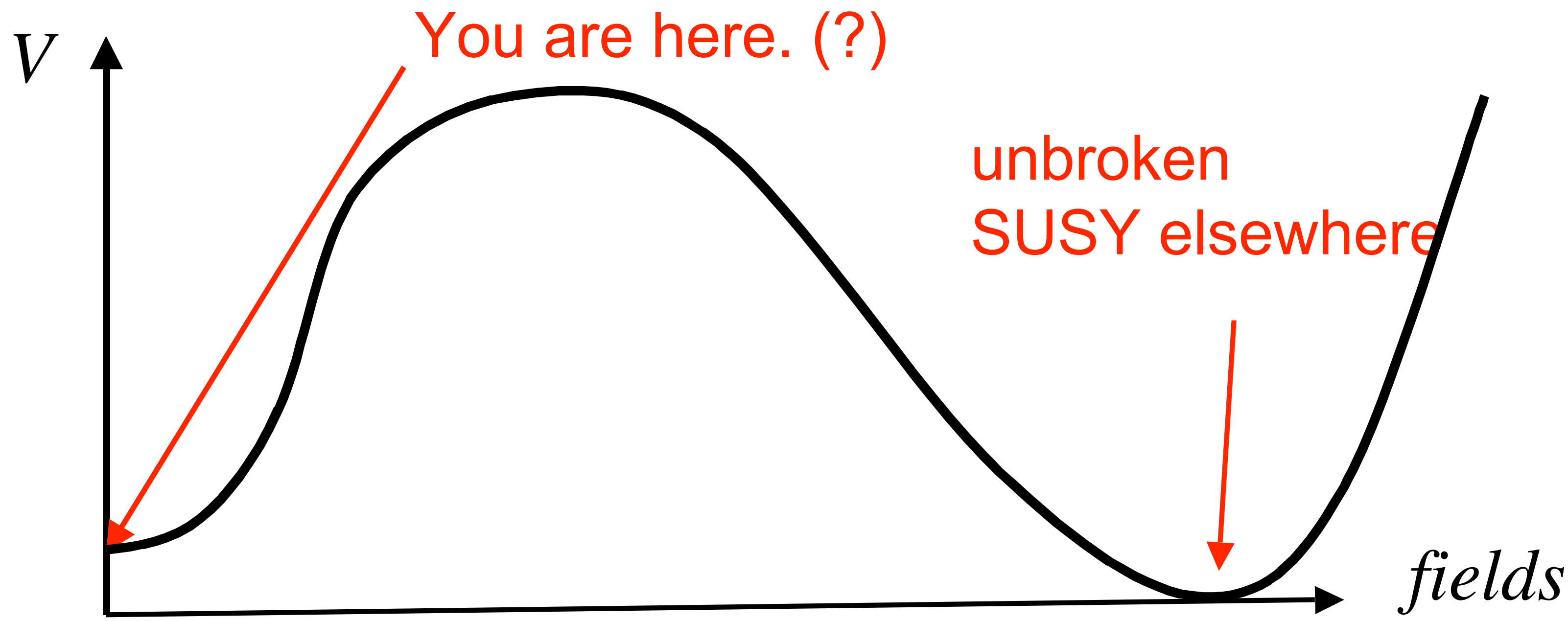
$$v \sim \Lambda_3 \lambda^{-1/7}$$

Mediating susy breaking

Additional structure, complicates the models. Sometimes leads to unwanted vacua, with susy unbroken.

Perhaps we should try a new approach...

Perhaps we live in a long-lived false vacuum



An old idea. Here, also in the SUSY breaking sector.

Find simpler models of DSB. **E.g. good, old SQCD!**

Suggests meta-stable DSB is generic.

Review of N=1 SQCD

	$SU(N_c)$	$[SU(N_f)$	$SU(N_f)]$
Q	N_c	N_f	1
$\tilde{Q}$	$\overline{N}_c$	1	$\overline{N}_f$
$(M = Q\tilde{Q}$	1	N_f	$\overline{N}_f$)

$\beta_e = 3N_c - N_f$

Asymptotically free if $N_f < 3N_c$ (IR free if not)

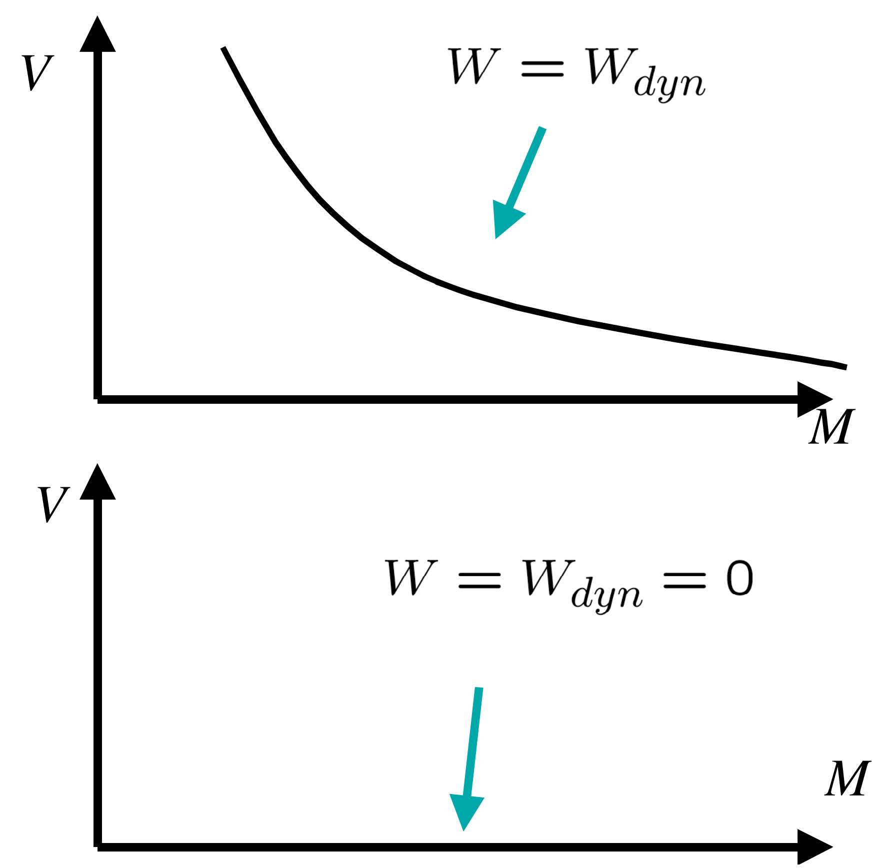
With $N_f < N_c$ massless flavors (Affleck, Dine and Seiberg)

$$W_{dyn} \sim \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}}$$

No static vacuum. Runaway.

With $N_f \geq N_c$ massless flavors

Quantum moduli space of vacua.



$\mathcal{N}$=1 SU(N_c) SQCD with N_f flavors

We will focus on the range of the number of colors and flavors $N_c \leq N_f \leq \frac{3}{2} N_c$ Infra-red free magnetic (Seiberg).

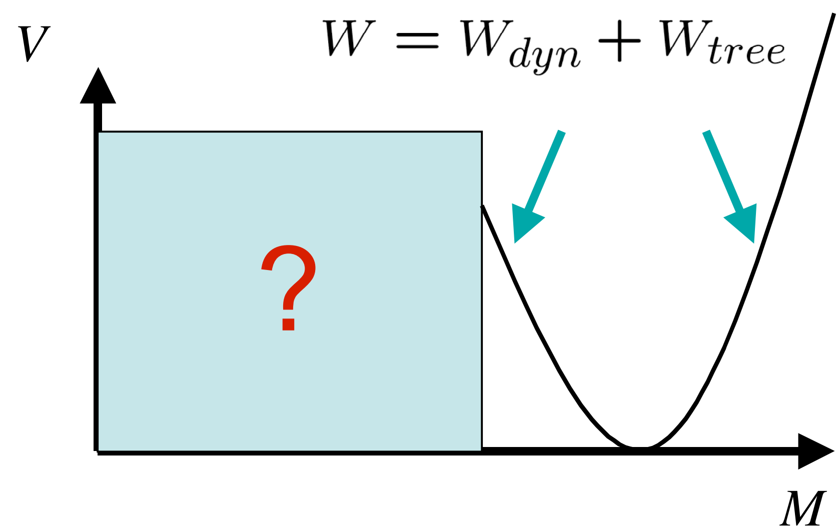
When all the quarks are massive, $W_{tree} = \mathrm{Tr}\, mQ\widetilde{Q} = \mathrm{Tr}\, mM$

there are $Tr(-1)^F = N_c$ SUSY vacua.

For $m = m_0 \mathbb{1}_{N_f}$

$$\langle M \rangle = (m_0^{N_f - N_c} \Lambda^{3N_c - N_f})^{1/N_c} \mathbb{1}_{N_f}$$

Study the limit $m_0 \ll \Lambda$
in the region near the origin.



There, we should use magnetic dual variables…

The magnetic theory (Seiberg)

We will focus on $N_c + 2 \leq N_f \leq \frac{3}{2} N_c$ where the theory is in a free magnetic phase; i.e. the magnetic theory is IR free.

Electric $SU(N_c)$

Λ

Magnetic $SU(N_f - N_c)$

The magnetic theory is

		$SU(N_f - N_c)$	$\times [SU(N_f)$	$\times SU(N_f)]$
q	$\in$	$N_f - N_c$	$\overline{N}_f$	1
$\tilde{q}$	$\in$	$\overline{N_f - N_c}$	1	N_f
Φ	$\in$	1	N_f	$\overline{N}_f$

with $$W_{dual} = \tilde{q} \Phi q$$

The magnetic theory, cont.

$$W_{dual} = \tilde{q}\Phi q$$ where $$\Lambda\Phi = M = \tilde{Q}Q$$

UV cutoff of this IR free theory is Λ .

The Kahler potential for the IR free fields is smooth near the origin and can be taken to be canonical:

$$K_{IR} = \frac{1}{\alpha}\mathrm{Tr}\,\Phi^\dagger\Phi + \frac{1}{\beta}\mathrm{Tr}\,(q^\dagger q + \tilde{q}^\dagger\tilde{q}) + O(\frac{1}{\Lambda^2})$$

Evidence: highly non-trivial 't Hooft anomaly matching.

Key point: The leading Kahler potential is known, up to two dimensionless normalization constant factors.

Rank condition SUSY breaking

Quark masses are described in the magnetic dual by

$$W_{tree} = \mathrm{Tr}\, q\Phi\tilde{q} - \mu^2 \mathrm{Tr}\, \Phi$$

$$\mu^2 = -m_0\Lambda$$

SUSY broken at tree level!

$$F^{\dagger}_{\Phi^g_f} \sim \frac{\partial W_{tree}}{\partial \Phi^g_f} = \tilde{q}^c_g q^f_c - \mu^2 \delta^f_g \neq 0$$

(rank N_f - N_c) (rank N_f)

(using the classical rank of $(q\tilde{q})^f_g$.)
This SUSY breaking is a check of the duality. Otherwise, would have had unexpected, extra SUSY vacua.

Elsewhere: SUSY dynamically restored

For $\langle\Phi\rangle \neq 0$, magnetic q's massive $(W_{dual} = \tilde{q}\Phi q)$ so integrate them out. Then gaugino condensation in dual

$$\Rightarrow \quad W_{dyn} \sim (\det \Phi)^{1/(N_f - N_c)}$$

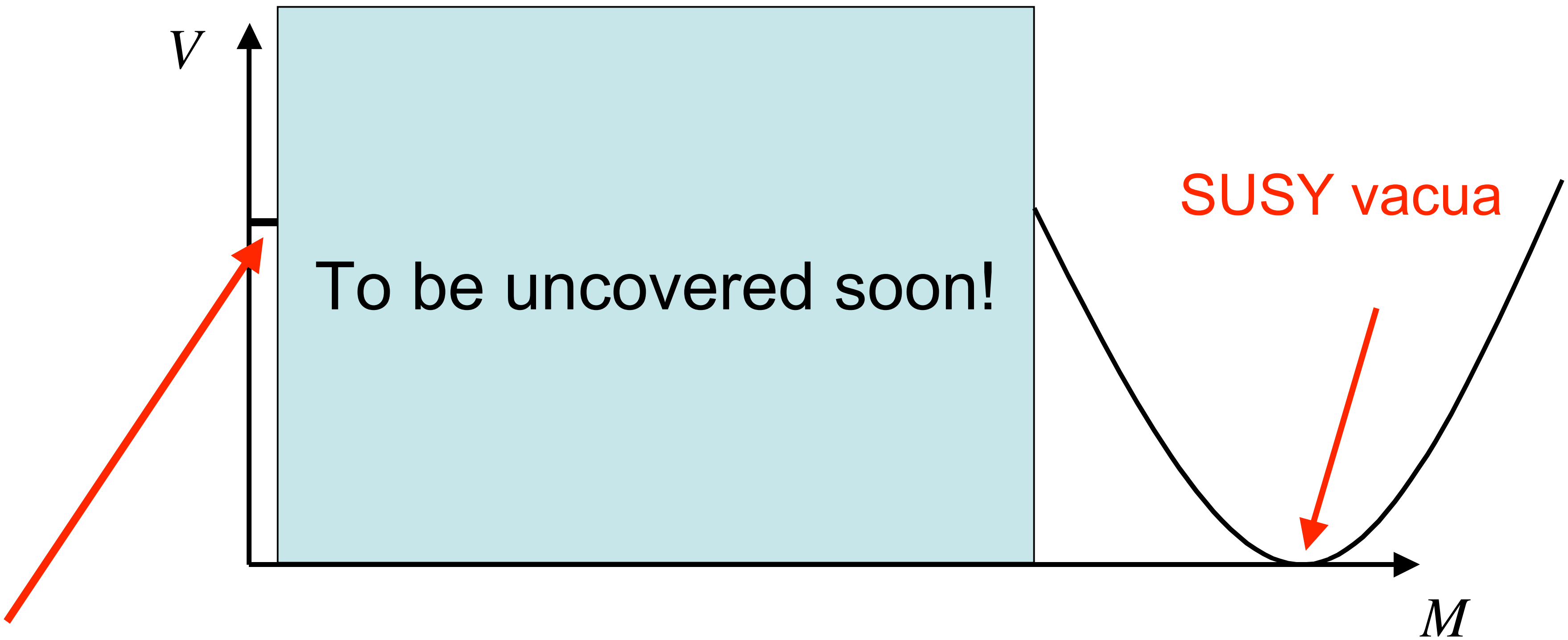
Non-perturbatively restores SUSY in the magnetic theory.

Leads to the expected $Tr(-1)^F = N_c$ susy vacua:

$$\langle M \rangle = (m_0^{N_f - N_c} \Lambda^{3N_c - N_f})^{1/N_c} \mathbb{1}_{N_f} \qquad \langle B \rangle = \langle \tilde{B} \rangle = 0$$

a check (c. 1994) of Seiberg duality.

Summary: the potential with massive flavors



For *M* at the origin SUSY broken by rank condition in the magnetic description. Reliable in free magnetic range: $N_f < \frac{3}{2} N_c$

This ends our review of things understood more than a decade ago.

DSB vacua near the origin, via F.M. dual

$$W_{tree} = \mathrm{Tr}\, q\Phi\tilde{q} - \mu^2 \mathrm{Tr}\,\Phi \qquad \mu^2 = -m_0\Lambda$$

Classical vacua (up to global symmetries) with broken SUSY:

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix} \qquad q = \begin{pmatrix} q_0 \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} \tilde{q}_0 \\ 0 \end{pmatrix} \qquad \tilde{q}_0 q_0 = \mu^2 \mathbb{1}_{N_f - N_c}$$

Pseudo-moduli:

Arbitrary $N_c \times N_c$ and $(N_f - N_c) \times (N_f - N_c)$ matrices

DSB: $V_{min} = N_c \alpha |\mu^4| \neq 0$

Pseudo-flat directions are lifted in the quantum theory.

Typical of tree-level breaking, e.g. O'Raifearteigh model (lect 2).

Pseudo-moduli get a potential at 1-loop in the magnetic theory

Use
$$W_{tree} = \mathrm{Tr}\, q\Phi\tilde{q} - \mu^2 \mathrm{Tr}\,\Phi$$
$$K_{IR} = \frac{1}{\alpha}\mathrm{Tr}\,\Phi^\dagger\Phi + \frac{1}{\beta}\mathrm{Tr}\,(q^\dagger q + \tilde{q}^\dagger\tilde{q}) + O(\frac{1}{\Lambda^2})$$

1-loop effective potential for pseudo-moduli:

$$V_{eff}^{(1)} = \frac{1}{64\pi^2}\mathrm{Tr}(-1)^F \mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2}$$

mass matrices are functions of the pseudo-moduli

1-loop vacuum energy

Higher loops (higher powers of small α, β) are smaller, because the magnetic theory is IR free.

Effect of the one-loop potential for the pseudo-moduli

The effective potential is minimized (up to symmetries):

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad q = \tilde{q}^T = \begin{pmatrix} \mu \mathbb{1}_{N_f - N_c} \\ 0 \end{pmatrix}$$

All pseudo-moduli get non-tachyonic masses at one-loop.
SUSY broken: $V_{min} \approx N_c \alpha |\mu^4| = N_c \alpha |m_0^2 \Lambda^2| > 0$

Vacua (meta) stable (we'll discuss tunneling soon).

Vacua mysterious in electric description. $\langle M \rangle = 0, \ \langle B \rangle \neq 0$
Not semi-classical, very quantum mechanical.

Dynamical SUSY restoration, revisited

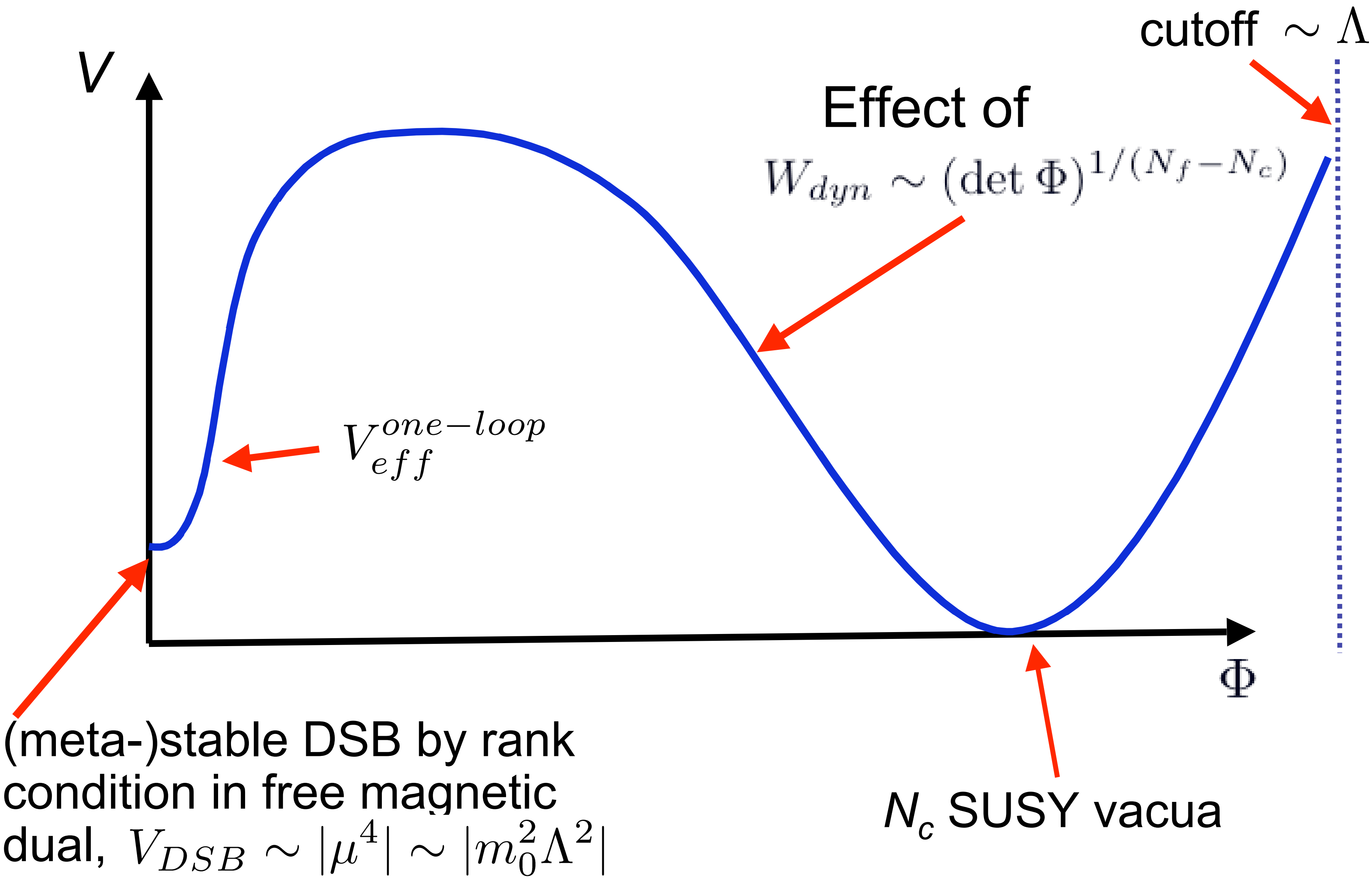
$$W_{dyn} \sim (\det\ \Phi)^{1/(N_f-N_c)} \sim \Phi^{N_f/(N_f-N_c)}$$

In free magnetic range, $N_f < 3N_c/2$, this term is $\sim \Phi^{\# > 3}$, so it is irrelevant for the susy breaking vacua near the origin.

For $\epsilon^2 = |\mu^2/\Lambda^2| = |m_0/\Lambda| \ll 1$, can reliably analyze effect of this term far out on the moduli space, and find the SUSY vacua in the magnetic theory, staying below its cutoff:

$$\Phi \ll \Lambda.$$

Sketch of the full potential



Effects from the microscopic theory

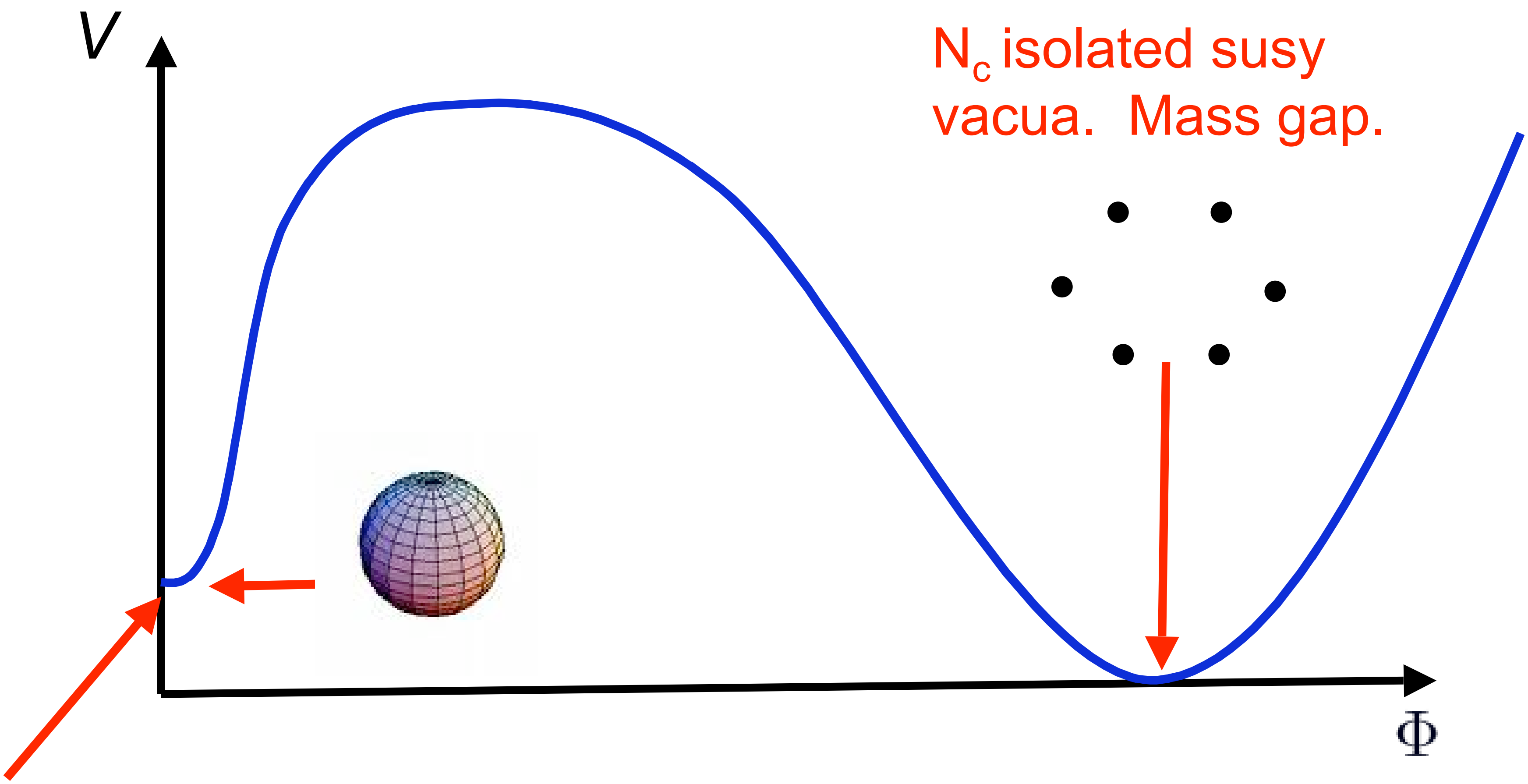
Contributions to the effective potential from modes above $\sim \Lambda$, e.g. from loops of SUSY split massive particles do not change our picture. Uncalculable. Unimportant.

All such contributions can be summarized by corrections to the Kahler potential. Such effects are real analytic in $\mu^2 = -m_0 \Lambda$. Our 1-loop potential is not, because it arises from integrating out modes that are massless as $m_0 \to 0$. This non-analyticity ensures that our DSB vacuum is robust.

Corrections from UV modes are negligible for

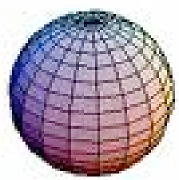
$$\epsilon^2 = |m_0/\Lambda| = |\mu^2/\Lambda^2| \ll 1$$

Spaces of DSB vs SUSY vacua



Compact moduli space of DSB vacua. Massless fermions and scalars: SSB of susy, and some global symmetries.

Compact moduli space of DSB vacua



$$\mathcal{M} = \frac{U(N_f)}{S(U(N_f - N_c) \times U(N_c))} = G/H$$

$$\langle q \rangle = \begin{pmatrix} \mu \mathbb{I}_{N_f - N_c} \\ 0 \end{pmatrix}$$

SSB

$G = SU(N_f)_V \times U(1)_B \cong U(N_f)$

$\downarrow$

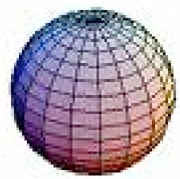
$H = S(U(N_f - N_c) \times U(N_c))$

DSB vacua: $\langle B \rangle \neq 0 \quad \langle M \rangle = 0$ **(Mysterious in electric description!)**

vs **SUSY vacua:** $\langle B \rangle = 0 \quad \langle M \rangle \sim \mathbb{I}_{N_f}$ $\quad G \to G$ $\quad (\mathbf{Z}_{2N_c} \to \mathbf{Z}_2)$

Aside: SSB vs Vafa-Witten thm. OK: squarks, vacua meta-stable.

Moduli space of DSB vacua, cont.



$$\mathcal{M} = \frac{U(N_f)}{S(U(N_f - N_c) \times U(N_c))} = G/H$$

DSB vacua have: exactly massless Goldstone bosons, and Goldstino. Extra massless fermions (from pseudo-moduli). (Electric description: naively **no** massless fields: quarks = massive, and SYM has a mass gap. True in susy vacua.)

$\pi_2(\mathcal{M}) = \mathbf{Z}$: DSB vacua support solitonic strings, topologically (meta) stable.

(v.s. domain walls of susy vacua.)

Mass spectrum of DSB vacua

Most particles have heavy masses $\sim |\Lambda|$.

Some magnetic particles get tree level masses (including the magnetic gauge fields, which are Higgsed)

$$\sim |\frac{g_{YM}}{\sqrt{\alpha}}\mu| \sim |\alpha\mu| \ll |\Lambda| \qquad (K_{IR} = \frac{1}{\alpha}\mathrm{Tr}\Phi^\dagger\Phi + \ldots)$$

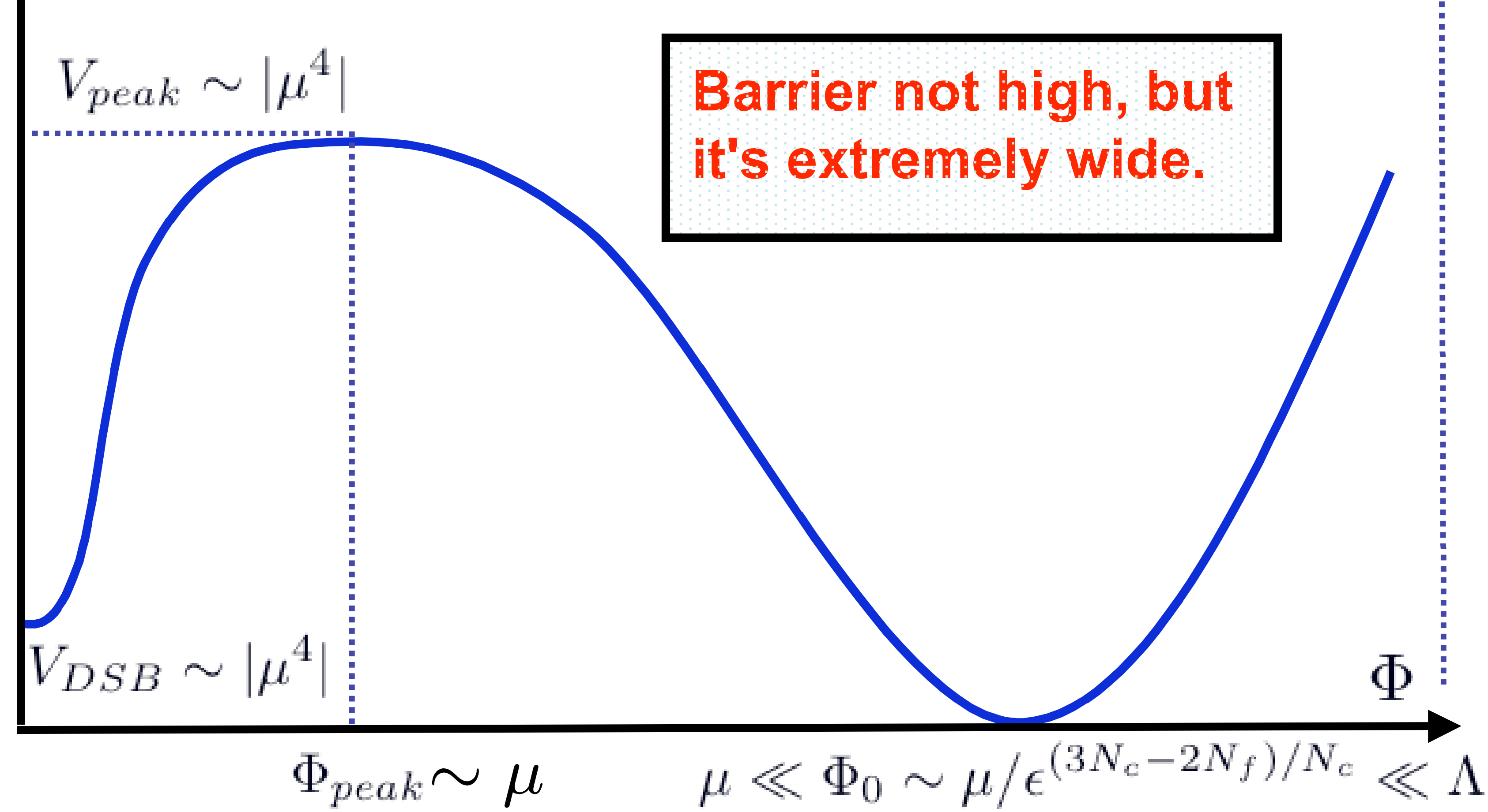
The pseudo-moduli have smaller (one loop) masses

$$\sim |\alpha^{5/2}\mu| < |\alpha\mu|$$

Other particles are exactly massless (before coupling to gravity).

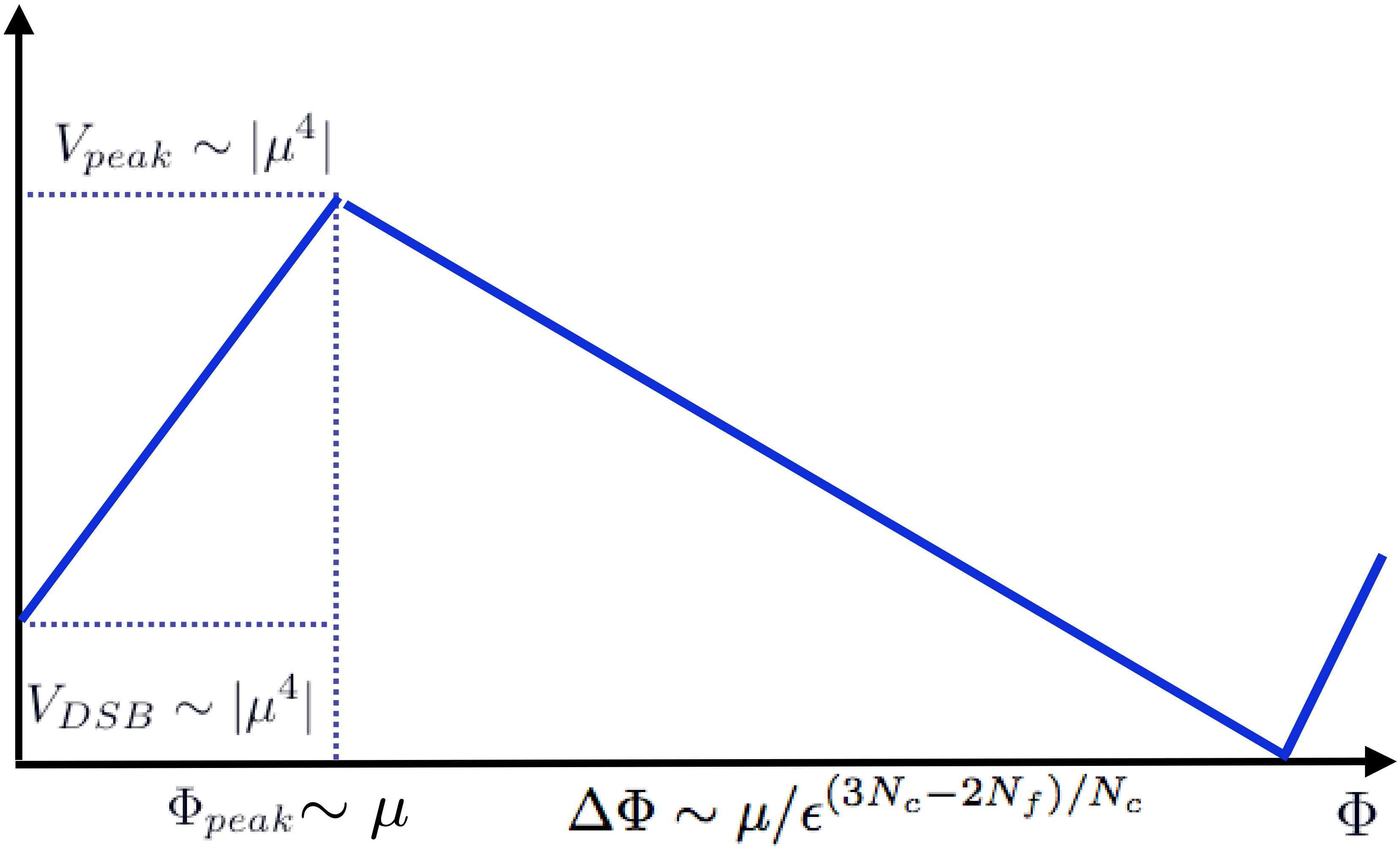
Lifetime of meta-stable DSB vacua

Estimate height and width of potential:



Recall $\mu^2 = -m_0\Lambda \qquad \epsilon = \sqrt{|m_0/\Lambda|} = |\mu/\Lambda| \ll 1$.

Cartoon of the potential



Recall $\mu^2 = -m_0\Lambda \qquad \epsilon = \sqrt{|m_0/\Lambda|} = |\mu/\Lambda| \ll 1$.

Lifetime of DSB vacua, cont.

Decay probability $\sim \exp(-S_{bounce})$ (e.g. Langer,Coleman)

Estimate classical, Euclidean action of bounce:

$$S_{bounce} \sim \frac{|\Delta\Phi|^4}{V_{DSB}} \sim \epsilon^{-4(3N_c-2N_f)/N_c} \gg 1$$

Our meta-stable DSB vacuum is parametrically long-lived for $\epsilon = \sqrt{|m_0/\Lambda|} = |\mu/\Lambda| \ll 1$.

Magnetic description: tunneling suppressed by large $\Delta\Phi$.
Electric description: tunneling suppressed by small V_{DSB} .

Aside: N_f in the conformal window

For $N_f \geq \frac{3}{2} N_c$ the magnetic dual is not IR free. Magnetic gaugino condensation superpotential

$$W_{dyn} \sim (\det \Phi)^{1/(N_f - N_c)} \sim \Phi^{N_f/(N_f - N_c)}$$

then cannot be neglected near the origin, $\sim \Phi^{\# \leq 3}$.

SUSY vacua are then too close to ensure the longevity of the DSB vacua; DSB vacua then not meaningful. Long lived meta-stable DSB vacua only in free magnetic range of N_f.

Easily generalized to $SO(N_c)$ and $Sp(N_c)$ with N_f fundamental flavors

Essentially the same results with $SO(N_c)$ and $Sp(N_c)$ – these phenomena appear to be generic.

SO case: promote to $Spin(N_c)$ and introduce sources – electric and magnetic $\mathbf{Z}_2 \times \mathbf{Z}_2$ order parameters; distinguish Higgs, confining, and oblique confining phases.

- SUSY vacua: dyon condensation, so oblique confining.
- DSB vacua: monopole condensation, so confining.

The vacua are in different phases.

Prospects for Model Building

Several longstanding challenges:

- Naturalness.
- Direct gauge mediation. Landau poles.
- R-symmetry problem.

Let us reconsider each one, in the new context of meta-stable DSB vacua.

Naturalness

Need small parameter: $\epsilon = \sqrt{m_0/\Lambda} \ll 1$. We get

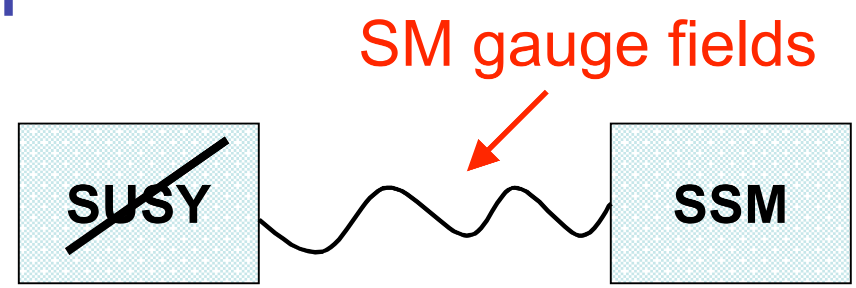
$$M_S^4 \sim |m^2\Lambda^2| \sim |m^2 M_{cutoff}^2| e^{-2c/g^2(M_{cutoff})} \ll m^2 M_{cutoff}^2$$

Good.

Perhaps better if all low-energy scales dynamically generated. Consider similar models, with *m* replaced by a marginal or irrelevant coupling.

Direct gauge mediation and Landau poles.

Direct mediation: longstanding goal to find a nice and simple model.



SQCD has a large global flavor symmetry, can partly gauge and identify with SM or GUT groups

e.g.

$$SU(N_c) \times SU(N_f) \underset{\Lambda}{\rightarrow} SU(N_f - N_c) \times SU(N_f) \rightarrow SU(N_f - N_c) \times SU(N_c)$$

meta-stable DSB

Low-energy gauge fields partly electric and partly magnetic.

Perhaps a scenario like this may help Landau pole problems.

R-symmetry problem

DSB without SUSY vacua: non-generic superpotential or a $U(1)_R$ symmetry. (Affleck, Dine, Seiberg; Nelson, Seiberg).

but...

For nonzero Majorana **gluino masses, $U(1)_R$ should be broken.** To avoid a Goldstone boson, breaking should be explicit, which might restore SUSY. (Gravity may help.)

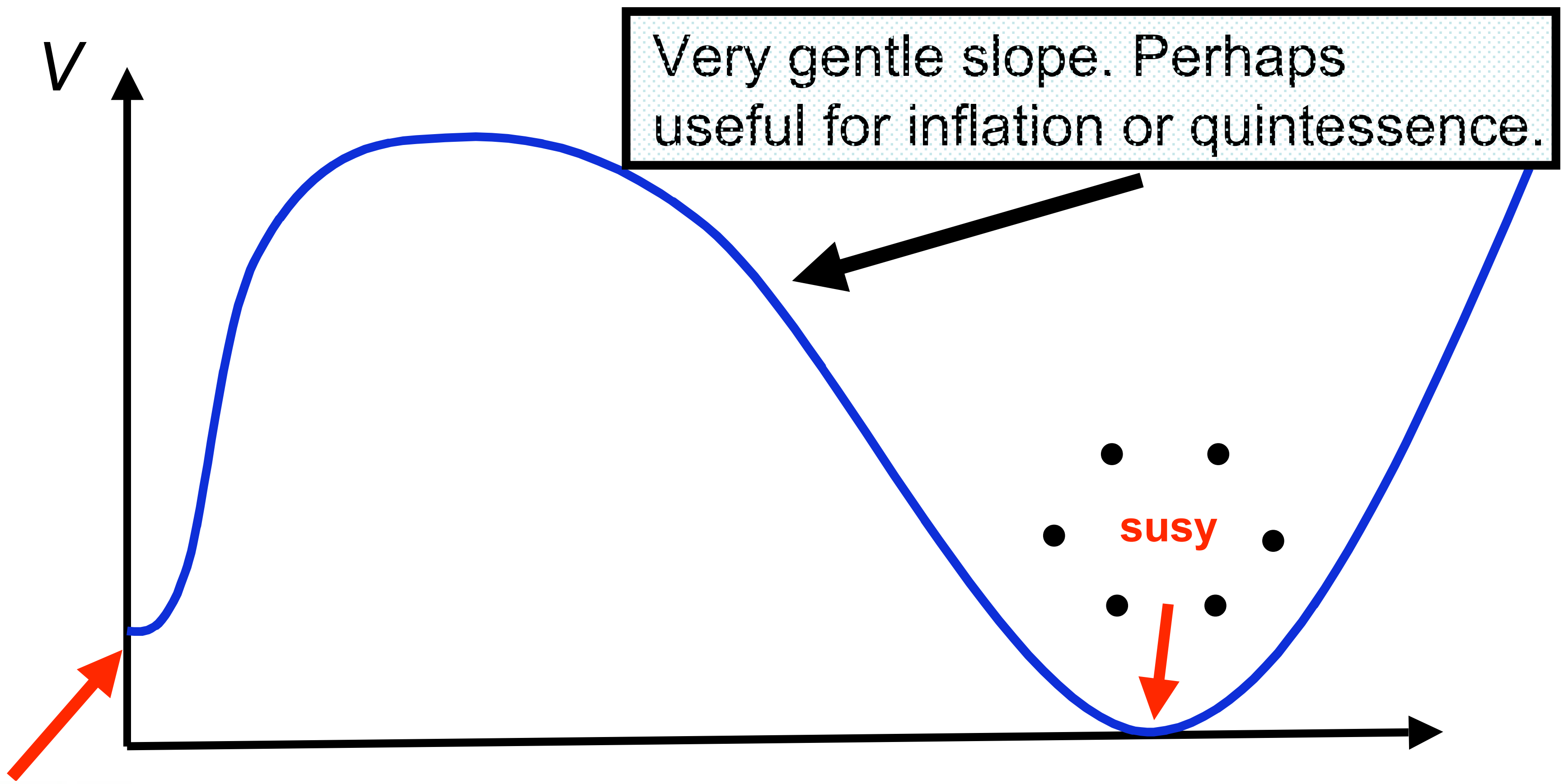
Our examples: **no exact $U(1)_R$**. (Indeed, SUSY vacua.)

Meta-stable DSB vacua have accidental approximate $U(1)_R$.

Perhaps better if that is also spontaneously broken.

(We also have an exact discrete R-symmetry, also bad for gaugino masses. It can be explicitly broken, by added interactions, without harming our DSB vacua.)

Comments on Cosmology



Much larger configuration space of DSB vacua.
Perhaps favors DSB population over susy vacua.
(c.f. Moduli trapping, KLLMMS)

Conclude / Outlook

- Accepting meta-stability leads to surprisingly simple models of DSB. Suggests meta-stable DSB is **generic** in $N = 1$ SUSY field theory, and the landscape of string vacua.
- Many similar models, including string-engineered gauge theories. **Franco & Uranga; Ooguri & Ookouchi**
- Extend to the landscape of string vacua. Account for MS-DSB in counting of susy vs DSB vacua?
- Relate (holographic dual) to anti-D-branes in KS geometry, of KPV, KKLT ? (note: baryonic).