Dynamical SUSY Breaking, Lecture 2

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1. Introduction

As we saw in lecture 1, SQCD with $N_f < N_c$ has a dynamically generated superpotential. Classically there are supersymmetric vacua for all $\langle M \rangle$. In the quantum theory, there is no supersymmetric vacuum for any $\langle M \rangle$, except for $\langle M \rangle \to \infty$. However, this is not considered a good theory of DSB, as there is no vacuum at all for finite $\langle M \rangle$ – there is a tadpole, associated with the runaway potential $V_{dyn} \sim |W'_{dyn}|^2$. We can stop the runaway by lifting the classical moduli space, by adding masses for all of the flavors, via $W_{tree} = \mathrm{Tr} mM$, with m a $N_f \times N_f$ matrix of masses (taking all its eigenvalues to be non-zero). But as you have seen in the exercise, the full superpotential $W_{full} = W_{dyn} + W_{tree}$ now has N_c supersymmetric vacua.

We return to the question of interest for these lectures is *"How generic is dynamical supersymmetry breaking in the landscape of all possible susy gauge theories, and in the landscape of string vacua.* As will be reviewed, theories with no susy vacua seem to be very non-generic. However, theories with meta-stable DSB vacua could be much more common.

2. Dynamical SUSY breaking is non-generic

2.1. Need a Goldstino

Spontaneous SUSY breaking means there is a massless Goldstone fermion. So any candidate theory of DSB must have such a massless fermion in its spectrum. (The goldstino is eaten by the gravitino, which gets a mass, once gravity is included. We will not consider gravity effects in these lectures.)

2.2. Witten Index

All SUSY gauge theories with massive, vector-like matter have $\mathrm{Tr}(-1)_F \neq 0$ SUSY vacua. E.g. for $SU(N_c)$ SYM have $\mathrm{Tr}(-1)^F = N_c$ SUSY vacua. So for broken SUSY need a *chiral* gauge theory. (We'll review some exceptions, with massless, vector-like matter.)

2.3. Susy breaking is related to breaking global symmetries

Affleck, Dine, and Seiberg point out that a *sufficient* conditions for DSB is that

1. All non-compact flat directions are lifted (e.g. by W_{tree})
2. A global symmetry is spontaneously broken, $G \to H$.

Point 2 means there are real massless goldstone bosons, living on the compact space G/H. Point 1 ensures that they can't be promoted to complex chiral superfields, so SUSY must be broken.

2.4. DSB requires an R-symmetry, or non-generic superpotential

This was pointed out by Nelson and Seiberg. Suppose that the low-energy effective theory can be described by a supersymmetric Wess-Zumino effective Lagrangian, without gauge fields. This is the effective description, below the dynamical scale Λ, where the strong gauge dynamics binds the original microscopic fields into composites. Then DSB in the UV theory occurs if there is F-term susy breaking in this effective theory, i.e. if we can not set all $(K^{-1})^{\bar{i}i}(\partial_i W(\Phi^i) = 0$. Assuming that the Kahler metric is non-degenerate (i.e. that the low-energy effective field theory has been properly identified), this means that we can not solve all the equations

$$\frac{\partial W(\Phi^i)}{\partial \Phi^i} = 0 \qquad \text{for all } i = 1 \ldots n. \tag{2.1}$$

But if W is the most generic superpotential, then (2.1) involves n equations for the n quantities Φ^i, so generally they can all be solved. Non-R flavor symmetries do not help, e.g. with a non-R global $U(1)$ symmetry, the equations (2.1) can be written as $n-1$ independent equations for $n-1$ independent unknowns, as seen by writing $W = W(\Phi^i \Phi_n^{-q_i/q_n})$, now for $i = 1 \ldots n-1$. But if there is an R-symmetry, then the equations (2.1) become over-constrained: they are n equations for $n-1$ independent unknowns, as seen by writing $W = \Phi_n^{2/r_n} f(\Phi^i \Phi_n^{-r_i/r_n})$ now for $i = 1 \ldots n-1$, so generically they can not be solved.

These observations fit with what we've already seen for SQCD: turning on $W_{tree} = \mathrm{Tr} mM$ breaks the R-symmetry, and indeed introduces SUSY vacua.

There can still be SUSY breaking without an R-symmetry, as the superpotential can happen to be non-generic. But it is difficult to find examples of that.

Having the R-symmetry be spontaneously broken is a sufficient condition for SUSY breaking, as in the previous subsection.

3. DSB is hard to analyze

Most of our techniques to analyze SUSY theories are based on holomorphy, chirality, BPS. They do not depend on the Kahler potential, which is hard to control. But finding SUSY breaking requires control of the Kahler potential. Also, since the vacuum is not supersymmetric, its dependence on the parameters might not be smooth. There can be phase transitions.

4. Examples of tree-level F-term supersymmetry breaking

Spontaneous supersymmetry breaking requires an exactly massless Goldstino fermion ψ_X. In simple models it originates from a chiral superfield X. The scalar component X can get a mass from either non-canonical Kähler potential terms, or more generally from corrections to the X propagator from loops of massive fields.

4.1. The simplest example

Consider, a theory of a single chiral superfield X, with linear superpotential with coefficient f (with units of mass^2),

$$W = fX, \tag{4.1}$$

and Kahler potential $K = K_{can} = XX^\dagger$. Supersymmetry is spontaneously broken by the expectation value of the F-component of X. The potential is $V = |f|^2$, independent of $\langle X \rangle$, so there are classical vacua for any $\langle X \rangle$. The fermion ψ_X is the exactly massless Goldstino. The complex scalar X is also classically massless. Note that there is a $U(1)_R$ symmetry, with $R(X) = 2$. For $\langle X \rangle \neq 0$ it is spontaneously broken, and the corresponding Goldstone boson is the phase of the field X.

4.2. With more general Kahler potential

Consider again (4.1), but with a more general effective Kähler potential $K(X, X^\dagger)$. The potential, $V = K^{-1}_{XX^\dagger}|f|^2$, is non-vanishing as long as the Kähler metric is non-singular. The fermion ψ_X is the exactly massless Goldstino. The vacuum degeneracy of $K = K_{can} = X^\dagger X$ is lifted by any non-trivial Kähler potential. For example, if near the origin $K = XX^\dagger - \frac{c}{|\Lambda|^2}(XX^\dagger)^2 + \ldots$, then there is a stable supersymmetric vacuum at the origin if $c > 0$. In this vacuum, the scalar component of X gets mass $m_X^2 \approx 4c|f|^2/|\Lambda|^2$. If $c < 0$, the origin is not the minimum of the potential.

The macroscopic, low-energy effective field theory must be under control to determine whether or not supersymmetry is broken. In the example (4.1), a singularity in the Kähler metric signals the need to include additional light degrees of freedom.

4.3. Additional d.o.f. can restore supersymmetry

Suppose that an additional field q becomes massless at a particular value of X, which we can take to be $X = 0$, so

$$W = hXqq + fX. \tag{4.2}$$

For $f = 0$, there is a moduli space of supersymmetric vacua, labelled by $\langle X \rangle$, and q can be integrated out away from the origin. The theory then looks similar to that of the previous subsections, except that the effective Kahler potential is singular, with $1/K_{X\bar{X}} \to 0$, at $X = 0$, corresponding to the additional massless field q there.

Turning on $f \neq 0$ lifts this moduli space. But unlike the theories of the previous subsection, the theory now no longer breaks supersymmetry, as there is a supersymmetric vacuum at $X = 0$, $q = \sqrt{-f/h}$.

Upshot: to determine whether or not supersymmetry is broken requires that the macroscopic low-energy theory be correctly identified.

Note that the theory (4.2) has a $U(1)_R$ symmetry, with $R(X) = 2$ and $R(q) = 0$. Having an R-symmetry is not a sufficient condition for SUSY breaking. The R-symmetry is not spontaneously broken, so the vacuum can be, and is, supersymmetric.

4.4. One-loop lifting of pseudo-moduli

We will be interested in the one-loop effective potential for pseudo-moduli (such as X), which comes from computing the one-loop correction to the vacuum energy

$$V_{eff}^{(1)} = \frac{1}{64\pi^2}\mathrm{STr}\,\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} \equiv \frac{1}{64\pi^2}\left(\mathrm{Tr}\, m_B^4 \log \frac{m_B^2}{\Lambda^2} - \mathrm{Tr}\, m_F^4 \log \frac{m_F^2}{\Lambda^2}\right), \tag{4.3}$$

where m_B^2 and m_F^2 are the tree-level boson and fermion masses, as a function of the expectation values of the pseudo-moduli.[1] In (4.3), $\mathcal{M}^2$ stands for the classical mass-squareds of the various fields of the low-energy effective theory. For completeness, we recall the standard expressions for these masses. For a general theory with n chiral superfields, Q^a, with canonical classical Kähler potential, $K_{cal} = Q_a^\dagger Q^a$, and superpotential $W(Q_a)$:

$$m_0^2 = \begin{pmatrix} W^{\dagger ac}W_{cb} & W^{\dagger abc}W_c \\ W_{abc}W^{\dagger c} & W_{ac}W^{\dagger cb} \end{pmatrix}, \qquad m_{1/2}^2 = \begin{pmatrix} W^{\dagger ac}W_{cb} & 0 \\ 0 & W_{ac}W^{\dagger cb} \end{pmatrix}, \tag{4.4}$$

with $W_c \equiv \partial W/\partial Q^c$, etc., and m_0^2 and $m_{1/2}^2$ are $2n \times 2n$ matrices.

[1] The ultraviolet cutoff Λ in (4.3) can be absorbed into the renormalization of the coupling constants appearing in the tree-level vacuum energy V_0. In particular, $\mathrm{STr}\,\mathcal{M}^4$ is independent of the pseudo-moduli.

4.5. The basic O'Raifeartaigh model

The basic model has three chiral superfields, X, ϕ_1, and ϕ_2, with classical Kähler potential $K_{cl} = X^\dagger X + \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2$, and superpotential

$$W = \tfrac{1}{2} h X \phi_1^2 + h m \phi_1 \phi_2 - h\mu^2 X. \tag{4.5}$$

We denote the coefficient f of the linear term as $f = -h\mu^2$, with μ having dimensions of mass, to make the mass dimension explicit, and to simplify expressions. This theory has a $U(1)_R$ symmetry, with $R(X) = 2$, $R(\phi_1) = 0$, $R(\phi_2) = 2$. The tree-level potential for the scalars is, $V_{tree} = |F_X|^2 + |F_{\phi_1}|^2 + |F_{\phi_2}|^2$, with

$$F_X = h\left(\tfrac{1}{2}\phi_1^2 - \mu^2\right), \quad F_{\phi_1} = h\left(X\phi_1 + m\phi_2\right), \quad F_{\phi_2} = hm\phi_1. \tag{4.6}$$

Supersymmetry is broken because F_X and F_{ϕ_2} cannot both vanish. The X and ϕ_2 equations of motion require that $F_{\phi_1} = 0$, which fixes $\langle \phi_2 \rangle = -\langle X\phi_1/m \rangle$. The minimum of the potential is a moduli space of degenerate, non-supersymmetric vacua, with $\langle X \rangle$ arbitrary. The minimum of the potential depends on the parameter

$$y \equiv \left| \frac{\mu^2}{m^2} \right| \tag{4.7}$$

For $y \leq 1$, the potential is minimized, with value $V = |h^2 \mu^4|$, at $\phi_1 = \phi_2 = 0$ and arbitrary X. (There is a second order phase transition at $y = 1$, where this minimum splits to two minima and a saddle point.) Let us focus on the $y \leq 1$ phase.

The fermion ψ_X is the exactly massless Goldstino. The scalar component of X is a classically pseudo-modulus. The classical mass spectrum of the ϕ_1 and ϕ_2 field can be computed from (4.4). For the fermions, the eigenvalues are

$$m_{1/2}^2 = \frac{1}{4}|h|^2(|X| \pm \sqrt{|X|^2 + 4|m|^2})^2, \tag{4.8}$$

and for the real scalars the mass eigenvalues are

$$m_0^2 = |h|^2 \left(|m|^2 + \tfrac{1}{2}\eta|\mu^2| + \tfrac{1}{2}|X|^2 \pm \tfrac{1}{2}\sqrt{|\mu^4| + 2\eta|\mu^2||X|^2 + 4|m|^2|X|^2 + |X|^4} \right), \tag{4.9}$$

where $\eta = \pm 1$.

The classical flat direction of the classical pseudo-modulus X is lifted by a quantum effective potential, $V_{eff}(X)$. The one-loop effective potential can be computed from the expression (4.3) for the one-loop vacuum energy, using the classical masses (4.8) and (4.9). The pseudo-modulus X is here treated as a background. It is found that the resulting effective potential is minimized at $\langle X\rangle = 0$, so we'll simplify the expressions by just expanding around this minimum: $V_{eff} = V_0 + m_X^2|X|^2 + \ldots$. The one loop corrected vacuum energy is

$$V_0 = |h^2\mu^4|\left[1 + \frac{|h^2|}{64\pi^2}\left(y^{-2}(1+y)^2\log(1+y) + y^{-2}(1-y)^2\log(1-y) + 2\log\frac{|hm|^2}{\Lambda^2}\right)\right]. \tag{4.10}$$

The dependence on the cutoff Λ can be absorbed into the running h. The one-loop quantum mass of the classical pseudo-modulus X is given by

$$m_X^2 = +\frac{|h^4\mu^2|}{32\pi^2}y^{-1}\left(-2 + y^{-1}(1+y)^2\log(1+y) - y^{-1}(1-y)^2\log(1-y)\right). \tag{4.11}$$

The mass (4.11) indeed satisfies $m_X^2 > 0$, consistent with the minimum of the one-loop potential (4.3) being at the origin. For small supersymmetry breaking, $y \to 0$, we have

$$m_X^2 \to \frac{|h^4\mu^4|}{48\pi^2|m|^2}, \qquad \text{for} \qquad |\mu^2| \ll |m^2|. \tag{4.12}$$

In the limit, $y \to 1$, where the supersymmetry breaking is large, we have

$$m_X^2 = \frac{|h^4\mu^2|}{16\pi^2}(\log 4 - 1) \qquad \text{for} \qquad |\mu^2| = |m|^2. \tag{4.13}$$

When the supersymmetry breaking mass splittings are small, the effective potential can alternatively be computed in the supersymmetric low-energy effective theory where we integrate out the massive fields ϕ_1 and ϕ_2. The effective superpotential of the low-energy theory is $W_{low} = -h\mu^2 X$, and the effective Kähler potential, $K_{eff}(X, X^\dagger)$, gets a one-loop correction from integrating out the massive fields. This gives the effective potential

$$V^{(1)} = (K_{eff\ XX^\dagger})^{-1}|h^2\mu^4|. \tag{4.14}$$

This way of computing the effective potential is valid only when the supersymmetry breaking is small, because the true effective potential generally gets significant additional contributions from terms that involve higher super-derivatives in superspace. The effective potential (4.3) gives the full answer, whether or not the supersymmetry breaking is small. In particular, (4.14) only reproduces the effective potential (4.3) to leading order in the $y \to 0$ limit. For example, (4.14) reproduces the mass (4.12) of the small supersymmetry breaking limit, but not the mass (4.13) of the large supersymmetry breaking limit.

5. Dynamical SUSY Breaking

5.1. 3-2 model

The gauge group is $SU(3) \times SU(2)$ and we have chiral superfields: Q in $(\mathbf{3}, \mathbf{2})$, $\tilde{u}$ in $(\bar{\mathbf{3}}, \mathbf{1})$, $\tilde{d}$ in $(\bar{\mathbf{3}}, \mathbf{1})$, L in $(\mathbf{1}, \mathbf{2})$. For $W_{tree} = 0$, the classical moduli space is given by arbitrary expectation values of the gauge invariants

$$X_1 = Q\tilde{d}L \qquad , \qquad X_2 = Q\tilde{u}L \qquad , \qquad Z = QQ\tilde{u}\tilde{d}.$$

We add to the model a tree level superpotential

$$W_{tree} = \lambda Q\tilde{d}L = \lambda X_1. \tag{5.1}$$

The $SU(3)$ dynamics generates

$$W_{dyn} = \frac{\Lambda_3^7}{Z}.$$

The full superpotential is $W = W_{dyn} + W_{tree}$. This theory dynamically breaks supersymmetry. For $\lambda \ll 1$, the vacuum is at large expectation value for the fields, $v \sim \Lambda^3/\lambda^{1/7}$, where the gauge group is very much Higgsed. In this limit, we have $K \approx K_{classical}$, so the Kahler potential is under control. The vacuum energy density at the minimum is $V = M_S^4 = 3.59\lambda^{10/7}\Lambda_3^4$.

5.2. Modified moduli space example

Consider the $SU(N_c)$ theory with $N_f = N_c$ and add fields $S^a_{\tilde{a}}$, b and $\tilde{b}$ and a superpotential

$$W_{tree} = S^a_{\tilde{a}}\tilde{Q}^{\tilde{a}}_i Q^i_a + b\det\tilde{Q} + \tilde{b}\det Q$$

Classically $Q = \tilde{Q} = 0$. In the quantum theory we get the effective superpotential

$$W_{effective} = S^a_{\tilde{a}} M^{\tilde{a}}_a + b\tilde{B} + \tilde{b}B + X(\det M - B\tilde{B} - \Lambda^{2N_c})$$

which breaks SUSY.

Let's consider this for the case $N_f = N_c = 2$, where the fundamentals and anti-fundamentals can be written as $2N_f = 4$ fundamentals Q_{fc}, $f = 1 \ldots 4$, $c = 1, 2$. The gauge invariants are $M_{fg} = Q_{fc}Q_{gc}\epsilon^{cd}$, in the $\mathbf{6}$ of the global $SU(4) \cong SO(6)$ flavor symmetry. Let us write it as $\vec{M}$, to show it is in the vector of $SO(6)$. Seiberg's quantum moduli space constraint for this case is

$$\vec{M} \cdot \vec{M} = \Lambda^4. \tag{5.2}$$

We add singlets $\vec{S}$, also in the **6** of the global flavor $SO(6)$, with superpotential

$$W_{tree} = \lambda \vec{S} \cdot \vec{M}. \tag{5.3}$$

The $\vec{S}$ e.o.m. requires $\vec{M} = 0$, but that is incompatible with (5.2), so susy is broken. Note that there is a $U(1)_R$ symmetry, with $R(M) = 0$ and $R(S) = 2$.

Note that these theories provide examples of non-chiral theories that dynamically break supersymmetry. How is that compatible with the Witten index? It's because the fields $\vec{S}$ are massless. If we add to (5.3) a term $\Delta W = \frac{1}{2}\epsilon \vec{S}^2$, we find the expected $\mathrm{Tr}(-1)^F = 2$ supersymmetric, vacua at $\vec{S}^2 = \lambda^2\Lambda^4/\epsilon^2$. As we take $\epsilon \to 0$, these susy vacua run off to infinity.

At the classical level, this theory has a pseudo-moduli space of flat directions, with susy broken. To see that, note that the constraint (5.2) implies that $SO(6) \to SO(5)$, and write a solution as $\vec{M} = (\sqrt{\Lambda^4 - \vec{v}^2}, \vec{v})$, where $\vec{v}$ is an $SO(5)$ vector. Similarly, write $\vec{S} \equiv (S_1, \vec{s})$, where $\vec{s}$ is an $SO(5)$ vector. Then (5.3) is

$$W = \lambda S_1 \sqrt{\Lambda^4 - \vec{v}^2} + \lambda \vec{v} \cdot \vec{s}. \tag{5.4}$$

The vacua have $\langle S_1 \rangle$ arbitrary, and $\vec{v} = \vec{s} = 0$, with SUSY broken by $F_{S_1} \neq 0$. There is a pseudo-flat direction labeled by $\langle S_1 \rangle$. This is the Goldstino superfield, whose fermionic component is the exactly massless goldstino. The apparent $\langle S_1 \rangle$ pseudomoduli space is lifted in the quantum theory by (4.3), and the susy breaking vacuum is at $\vec{S} = 0$. The complex scalar pseudo-modulus in S_1 gets a positive mass-squared there. Note that the $U(1)_R$ symmetry is not spontaneously broken in the susy breaking vacuum, so there is no massless Goldstone boson.

5.3. An example where susy breaking is an open question

Here is an example that illustrates the need to have the effective theory under control. Consider $SU(2)$ gauge theory with a single matter field Q in the 4 dimensional representation of $SU(2)$ ($j = 3/2$). There is a 1-complex dimensional moduli space of vacua, labeled by the gauge invariant $X = Q^4$. This moduli space is lifted by the tree-level superpotential

$$W = \lambda X. \tag{5.5}$$

There is an anomaly free $U(1)_R$, with $R(Q) = 3/5$. (The $SU(2)$ instanton has 10 fermion zero modes for the field Q, as seen by noting that $\text{Tr}_{j=3/2}T_z^2 = 10\text{Tr}_{j=1/2}T_z^2$). There is a very non-trivial matching of TrR and TrR^3 't Hooft anomalies, between the microscopic $SU(2)$ and Q fields, and the macroscopic field X. This suggests that the effective field theory in the IR is described by the single composite field X as an IR free field. If so, the low-energy theory near the origin has Kahler potential

$$K_{low} = \frac{\alpha}{|\Lambda|^6} X^\dagger X + \dots \qquad \text{near } X = 0 \tag{5.6}$$

where α is a dimensionless number that we cannot determine, the powers of Λ are on dimensional grounds, and the ... are higher order terms, powers of $X^\dagger X$ (far from the origin, we must recover $K \approx K_{cl} \sim (X^\dagger X)^{1/4}$). Then (5.5) dynamically breaks supersymmetry, with $M_S^4 \sim |\lambda^2 \Lambda^6|$.

Note that (5.5) does not preserve the anomaly free $U(1)_R$ symmetry, but there is an accidental $U(1)_R$ symmetry of the IR free low-energy theory, with $R_{accidental}(X) = 0$, which is preserved.

However, the 't Hooft anomaly matching, suggesting an IR free spectrum and (5.6), can be a fluke. There are some known examples of misleading 't Hooft anomaly matching. The theory at the origin might be an interacting SCFT, in which case (5.6) is incorrect. In that case, the superpotential (5.5) is an irrelevant perturbation, and flows to zero in the IR, and susy is certainly unbroken.

It is not yet know which of these two scenarios is correct for this theory.

Intriligator, lecture 2, problem set

The eqn. numbers refer to those in the lecture 2 notes.

1 Verify, for the case $y = 1$, that the mass matrices have the eigenvalues (4.8) and (4.9). If you have access to mathematica, use these in (4.3) to verify (4.13).

2 Show that the superpotential (5.1) preserves a $U(1)_R$ symmetry, which is anomaly free w.r.t. both gauge groups.

3 Show that (5.1) lifts all classical flat directions.

4 Verify the TrR and TrR^3 't Hooft anomaly matching mentioned before (5.6).